KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Mid-term Exam II / MATH-244 (Linear Algebra) / Semester 451

Max. Marks: 25 **Max.Time: 1.5 hrs**

Note: Scientific calculators are not allowed.

Question 1: [Marks: (2+3) + 3]

a) Let P_4 denote the vector space of all real polynomials in x with degree ≤ 4 under the usual addition and scalar multiplication**.** Then:

(i) Show that $W = \{a + 2b + (a - b)x + (2a + b)x^3 + (a + b)x^4 \mid a, b \in \mathbb{R}\}$ is a subspace of P_4.

(ii) Find a basis of the above vector space W.

b) Let $\{u_1, u_2, \dots, u_n\}$ be a basis of vector space E. Then show that every element of E has a unique representation a linear combination of the basic vectors $u_1, u_2, \dots, u_n$.

Question 2: [Marks: 3 + 3 + 2]

Let $B = \{u_1, u_2, u_3\}$ be a basis of a vector space V and $C = \{w_1, w_2, w_3\} \subseteq V$ such that:

$$u_1 + w_2 = w_1 + w_3$$
$$u_2 - w_3 = w_1 + w_2$$
$$u_3 + w_1 = w_2 - 2w_3.$$

Then:

a) Show that the set C is a basis of the vector space V.

b) Construct the transition matrix ${}_{C}P_B$ from the above basis B to the basis C.

c) Find the transition matrix ${}_{B}P_C$ by using the matrix ${}_{C}P_B$.

Question 3: [Marks: 3 + (1.5+1.5) + 3]

a) Let $\boldsymbol{M}_2(\mathbb{R})$ the real inner product space of 2×2 matrices with the inner product: $\prec A, B \succ := trace\ (AB^T)$. Let $A, B, C \in \boldsymbol{M}_2(\mathbb{R})$ be the matrices satisfying $A + B = -C$, $||A|| = 3, ||B|| = 5$ and $||C|| = 7$. Then find the angle between the matrices A and B.

b) Show that the set $F = \{(0,1,-1), (1,1,1), (2,-1,-1)\}$ is orthogonal in the Euclidean space $\mathbb{R}^3$. Deduce further that the orthogonal set F is a basis of $\mathbb{R}^3$.

c) Let $G = \{v_1 = (1,1,-1,1), v_2 = (1,1,1,1), v_3 = (-1,1,1,1)\}$ be a basis of vector subspace E of the Euclidean space $\mathbb{R}^4$. Find an orthonormal basis of E by using G.

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