

# STEADY-STATE ANALYSIS OF A SELF-EXCITED INDUCTION GENERATOR INCLUDING TRANSFORMER SATURATION

S. M. Alghuwainem  
Department of Electrical Engineering  
King Saud University  
Riyadh, Saudi Arabia 11421

## Abstract

Due to speed fluctuations of unregulated wind-turbines, the terminal voltage may increase to dangerously high levels which have been reported to cause capacitor failure at windfarms. This paper examines the steady-state analysis and performance characteristics of a stand-alone self-excited induction generator (SEIG) when a transformer is connected to its terminals to supply the load at a different voltage level or to step-up the terminal voltage for transmission. The transformer tends to saturate at higher speeds, and thus absorbs the excess reactive power and limits the increase in terminal voltage and improves voltage regulation. Transformer saturation introduces an additional non-linearity which complicates the analysis considerably. A technique for formulating and solving the system's equations including transformer saturation is presented. The same technique is also applicable when the load itself is nonlinear. Experimental investigation has confirmed the accuracy of the proposed technique.

**Keywords:** - Self-excitation, induction generator, transformer saturation.

## List of Symbols:

$f, \omega, \alpha$	Per-unit stator frequency, per-unit rotor speed, and transformer turns ratio.
$R_s, R_r, R_t$	Stator, referred rotor, and transformer series resistances.
$X_s, X_r, X_c$	Stator, referred rotor, and excitation capacitive reactances at base frequency.
$R_L, X_L$	Load resistance and reactance.
$X_{mm}, X_{mt}$	Machine and transformer magnetizing reactances at base frequency.
$V_g, V_t$	Air-gap and terminal voltages.

PE-1332-EC-0-2-1998 A paper recommended and approved by the IEEE Electric Machinery Committee of the IEEE Power Engineering Society for publication in the IEEE Transactions on Energy Conversion. Manuscript submitted August 27, 1997; made available for printing February 18, 1998.

## 1 INTRODUCTION

It is well known that an induction machine may generate voltage if a capacitor is connected to its stator terminals while its rotor is driven by a prime-mover. In this case, the capacitor provides the lagging magnetizing reactive power which is necessary to establish the air-gap and this configuration is referred to as a self-excited induction generator (SEIG). In recent years, the SEIG has been increasingly used in isolated power systems employing renewable energy sources such as wind and hydro-power [1-8], due to its lower cost, brushless rotor, ruggedness, and ease of maintenance, etc. However, one of the major drawbacks of the stand-alone SEIG is its poor voltage and frequency regulation. Unless the SEIG is connected to a utility grid, its frequency and stator voltage are free to vary with rotor speed and load. The terminal voltage of an isolated SEIG increases considerably for a small increase in speed [6]. In most developing countries, unregulated wind and micro-hydro turbines are often used due to their lower cost, which renders such systems suitable only for supplying power to loads where voltage and frequency need not to be regulated.

Normally, an isolated SEIG is operated such that the value of the excitation capacitance is close to a minimum possible value, in order to prevent overvoltages and overcurrents in the machine which may result with larger capacitance values or at higher speeds [6]. However, capacitor failures due to overvoltages caused by speed fluctuations of unregulated wind or micro-hydro turbines have been reported in the literature [9,10]. Several techniques to improve voltage regulation of the stand-alone SEIG have also been reported [11,12].

A transformer is sometimes connected between the SEIG and the load if a different voltage level is required by the load or to step-up the terminal voltage to a suitable level for transmission via a high voltage cable to a remote load. Because of the relatively large fluctuations in speed, the transformer may become saturated and thus it acts like a saturated reactor and prevents further increase in terminal voltage with speed. The excitation capacitance may be totally or partially connected on the primary or secondary side of the transformer. However, the effective capacitance reflected at the SEIG's terminal is multiplied by the square of the transformer turns ratio. The total effective capacitance of the capacitors and the high-voltage cable must be sufficient to supply the machine, transformer,

and load reactive power requirements. If the transformer or load is saturated, then its reactive power requirement increases non-linearly with the machine's terminal voltage. It is well known that the open-circuit voltage of a SEIG is limited by the saturation of the machine's magnetic circuit which makes the analysis inherently a non-linear problem. If the connected transformer is also saturated, then the terminal voltage is also limited by the saturation of the transformer's magnetic circuit. This additional non-linearity complicates the analysis considerably since the terminal voltage must be taken as an independent variable in addition to the air-gap voltage.

In this paper, the system equations for steady-state analysis including transformer saturation are formulated using nodal analysis of the SEIG's equivalent circuit. The two node equations with complex coefficients are separated into four equations with real coefficients by separating into real and imaginary parts. The resulting real equations are then solved using Newton-Raphson method. Experimental investigation has confirmed the accuracy of the proposed method of analysis.

## 2 System Modelling

Fig. 1 shows a three phase induction machine (IM), which is being driven at speed  $\omega$  by the turbine. It is connected through a transformer (T) to a high-voltage cable which supplies a balanced three-phase  $R-L$  load. The excitation capacitor  $C$  may be connected directly to the machine or through the transformer.

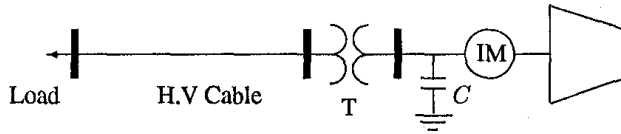


Figure 1: SEIG-transformer system.

Fig. 2 shows the per-phase equivalent circuit of the SEIG referred to the base frequency [13], where the equivalent circuit of the transformer, referred to the machine base is inserted between the load and the machine. The parameters  $f$  and  $\omega$  are the per-unit self-excited electrical frequency and rotor speed respectively.

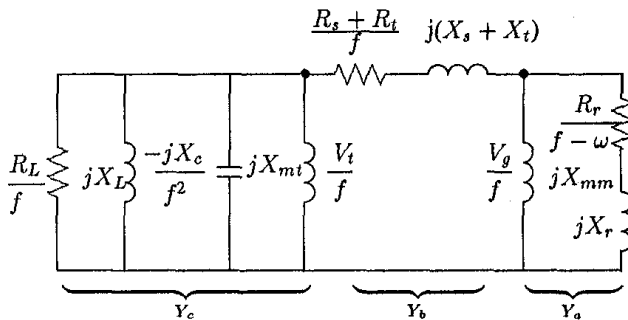


Figure 2: Per-phase equivalent circuit of the SEIG.

The magnetizing reactances  $X_{mm}$  and  $X_{mt}$  of the machine and transformer depend upon the air-gap voltage  $V_g$  and the terminal voltage  $V_t$  respectively, according to their respective magnetic circuits.

Generally, with all other parameters specified, it is possible to solve the equivalent circuit of Fig. 2 for  $f$ ,  $V_g$ , and  $V_t$ . Using circuits nodal analysis, the two node equations are written as

$$(Y_a + Y_b)V_g - Y_b V_t = 0 \quad (1)$$

$$-Y_b V_g + (Y_b + Y_c)V_t = 0 \quad (2)$$

where the admittances  $Y_a$ ,  $Y_b$ , and  $Y_c$ , are as shown in Fig. 2, where

$$Y_a = \frac{1}{jX_{mm}} + \frac{1}{R_r/(f - \omega) + jX_r} \quad (3)$$

$$Y_b = \frac{1}{(R_t + R_s)/f + j(X_t + X_s)} \quad (4)$$

$$Y_c = \frac{1}{jX_{mt}} - \frac{f^2}{jX_c} + \frac{f}{R_L} + \frac{1}{jX_L} \quad (5)$$

Equations (1) and (2) contain complex coefficients and they can be separated into four real equations by equating their real and imaginary parts to zero. Selecting  $V_g$  as a reference phasor, the real unknowns are  $f$ ,  $V_g$ ,  $V_t$  and phase angle of  $V_t$ . Let the real parts of (1) and (2) be denoted by  $f_1$  and  $f_2$  while their imaginary parts be denoted by  $g_1$  and  $g_2$ , then the resulting real equations are as follows:-

$$f_1 = (Y_{ar} + Y_{br})V_g - V_t \cos(\delta)Y_{br} + V_t \sin(\delta)Y_{bi} = 0 \quad (6)$$

$$f_2 = (Y_{cr} + Y_{br})V_t \cos(\delta) - (Y_{ci} + Y_{bi})V_t \sin(\delta) - V_g Y_{br} = 0 \quad (7)$$

$$g_1 = (Y_{ai} + Y_{bi})V_g - V_t \cos(\delta)Y_{bi} - V_t \sin(\delta)Y_{br} = 0 \quad (8)$$

$$g_2 = (Y_{ci} + Y_{bi})V_t \cos(\delta) + (Y_{cr} + Y_{br})V_t \sin(\delta) - V_g Y_{bi} = 0 \quad (9)$$

where  $Y_{ar}$ ,  $Y_{br}$ , and  $Y_{cr}$  are the real parts of  $Y_a$ ,  $Y_b$  and  $Y_c$ . Similarly,  $Y_{ai}$ ,  $Y_{bi}$  and  $Y_{ci}$  are their imaginary parts, and  $\delta$  is the phase angle of  $V_t$ . It should be noted that  $Y_{ar}$  and  $Y_{ai}$  are functions of  $V_g$  and  $f$ , while  $Y_{cr}$  and  $Y_{ci}$  are functions of  $V_t$  and  $f$ , whereas  $Y_{br}$  and  $Y_{bi}$  are functions of  $f$  only. Equations (6-9) can be solved using the Newton-Raphson method. The elements of the resulting Jacobian matrix  $[J]$ , can be calculated as follows:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial f} & \frac{\partial f_1}{\partial V_g} & \frac{\partial f_1}{\partial V_t} & \frac{\partial f_1}{\partial \delta} \\ \frac{\partial f_2}{\partial f} & \frac{\partial f_2}{\partial V_g} & \frac{\partial f_2}{\partial V_t} & \frac{\partial f_2}{\partial \delta} \\ \frac{\partial g_1}{\partial f} & \frac{\partial g_1}{\partial V_g} & \frac{\partial g_1}{\partial V_t} & \frac{\partial g_1}{\partial \delta} \\ \frac{\partial g_2}{\partial f} & \frac{\partial g_2}{\partial V_g} & \frac{\partial g_2}{\partial V_t} & \frac{\partial g_2}{\partial \delta} \end{bmatrix} \quad (10)$$

### 3 Experimental Verification

The behavior of a model SEIG/transformer/excitation capacitor system is investigated in the laboratory. A 3-phase, 380 V, 2.9 A, 4-pole, 60 Hz, Y-connected, induction machine is coupled to a dc motor drive. The induction machine has the following measured parameters in per-unit:-  $R_s = 0.092$ ,  $R_r = 0.064$ ,  $X_s = X_r = 0.21$ . Its magnetizing reactance  $X_{mm}$  is measured by performing an open-circuit test, where the machine is driven at synchronous speed and a 60 Hz variable voltage is applied to the stator. From the measured data, the equation for  $X_{mm}$  (pu) at base frequency ( $f = 1$  pu), in terms of the open-circuit voltage (pu), is obtained using regression analysis.

$$X_{mm} = 3.46 - 6.5V_g + 9.51V_g^2 - 4.77V_g^3 \quad (11)$$

The test transformer is a 3-phase, 380 V/220 V, Y-Y, which is used in the step-down or step-up modes. It has the following, measured parameters, in per-unit, on the machine base:-  $R_t = 0.012$ ,  $X_t = 0.0285$ , turns ratio (low voltage/high voltage)  $\alpha = 0.593$ . Its magnetizing reactance  $X_{mt}$  is measured by performing an open-circuit test by applying a 60 Hz (1 pu), variable voltage. From the measured data, the equation for  $X_{mt}$  (pu), in terms of the open-circuit voltage (pu), is obtained using regression analysis.

$$X_{mt} = 88.5 + 11.3V_t - 110.31V_t^2 + 35.8V_t^3 \quad (12)$$

Fig. 3 is a plot of  $X_{mm}(V_g)$  and  $X_{mt}(V_t)$  with respect to  $V_g$  and  $V_t$  respectively.

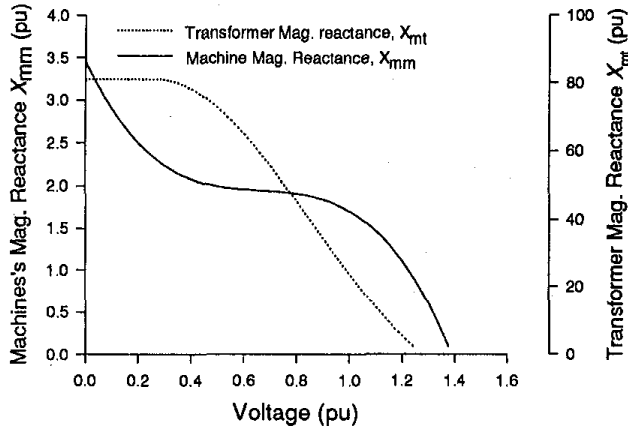


Figure 3: Variation of machine and transformer magnetizing reactances with their respective applied voltages.

Fig. 4 shows the calculated as well as the measured variations of the terminal voltage, capacitor current, and frequency  $f$  with speed when a capacitor  $C = 25.5 \mu\text{F}$  per-phase is connected to the stator. A very good agreement can be noticed between the calculated and the measured values.

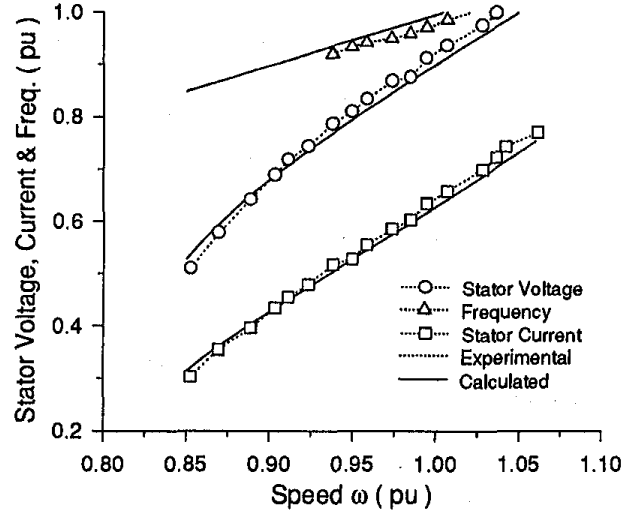


Figure 4: Variation of machine terminal voltage, frequency, and capacitor current with speed, at no-load.

Fig. 5 shows the calculated and measured variations of terminal voltage, capacitor current, and stator current with speed, when a  $75 \mu\text{F}$  capacitor per-phase is connected to the machine through the step-down transformer. A good agreement is observed between the two sets of values.

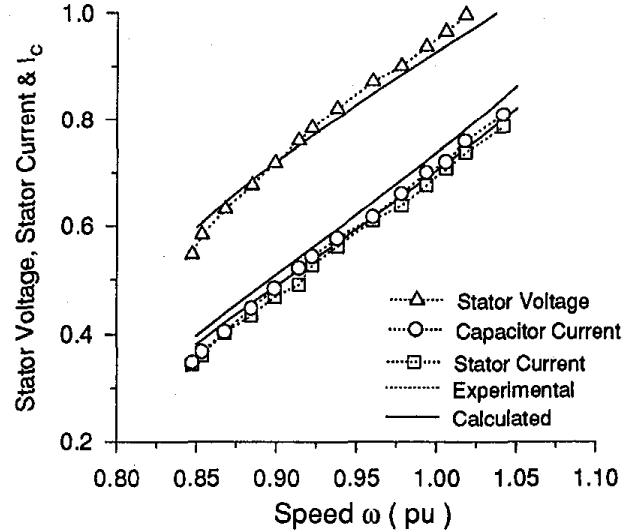


Figure 5: Variation of machine terminal voltage, stator current, and capacitor current with speed, when a  $75 \mu\text{F}$  capacitor per-phase is connected to the machine through the step-down transformer.

Fig. 6 shows the calculated and measured variations of stator current, machine's terminal voltage and capacitor current with  $\omega$  when a  $17.9 \mu\text{F}$  is connected through the step-up transformer, from which a reasonable agreement between the two sets of data is obvious.

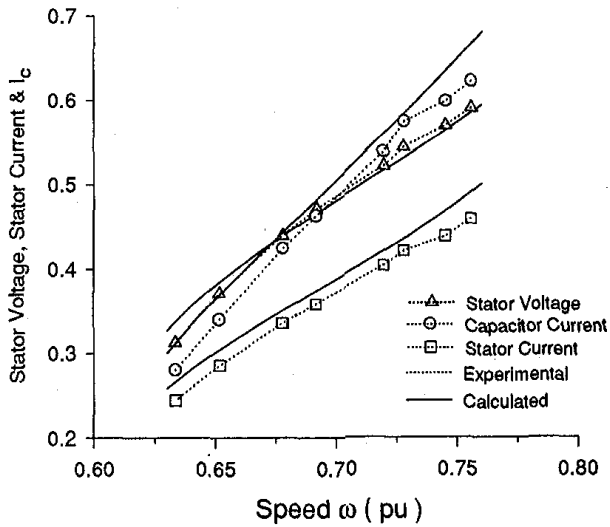


Figure 6: Variation of machine's terminal voltage, stator current, and capacitor current when a 17.9  $\mu\text{F}$  capacitor per-phase is connected to the machine through the step-up transformer.

#### 4 Effect of Transformer Saturation

If the transformer core becomes saturated then its magnetizing reactance becomes very low and its magnetizing reactive power becomes large. The transformer magnetizing reactance is effectively in parallel with the excitation capacitance. Therefore the reactive power becomes less since part of the reactive power supplied by the excitation capacitor is consumed by the transformer's magnetizing reactance. Therefore, it is very important to incorporate magnetic saturation of the transformer (if present) in the analysis of the SEIG equivalent circuit for a better solution accuracy since the solution point is strongly affected by such a saturation.

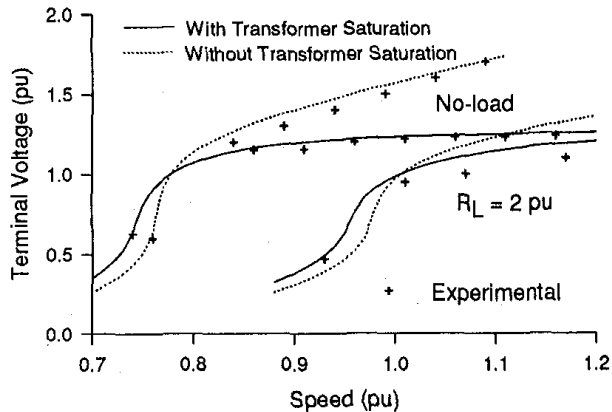


Figure 7: Variation of terminal voltage with speed, when  $C = 30 \mu\text{F}$  is connected to the stator, with and without transformer saturation.

Fig. 7 shows variation of the terminal voltage with speed variation when  $C = 30 \mu\text{F}$  is connected to the stator, with and without transformer saturation. Fig. 8 shows variation of the terminal voltage with excitation capacitance at  $\omega = 1$  per-unit, with and without transformer saturation.

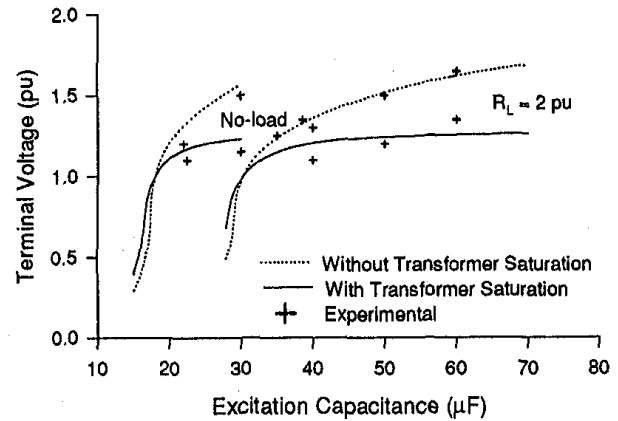


Figure 8: Variation of terminal voltage with excitation capacitance, at  $\omega = 1$  pu, with and without transformer saturation.

For the case when transformer saturation is ignored,  $X_{mt}$  is assumed to remain constant at its value which corresponds to  $V_t = 1$  per-unit. It is clear from Fig. 7 that, without transformer saturation, the terminal voltage increases almost linearly with speed, in the region from  $V_t = 1$  pu to  $V_t = 1.6$  pu. However, with transformer saturation the terminal voltage does not increase higher than  $V_t = 1.2$  pu, which is the saturation voltage level of the transformer. It is also clear from Fig. 8 that, without transformer saturation, the terminal voltage increases almost linearly with capacitance, in the region from  $V_t = 1$  pu to  $V_t = 1.6$  pu. However, with the transformer saturation the terminal voltage does not increase higher than  $V_t = 1.2$  pu.

#### 5 EFFECT OF LOAD IMPEDANCE

The load impedance has a significant influence on the operating point of the SEIG since it is directly in parallel with the excitation capacitance. The load power-factor is specially important since it affects the overall capacitive reactance. Fig. 9 shows variation of the terminal voltage with load conductance for resistive load, at  $\omega = 1$  pu, and  $C = 30 \mu\text{F}$ . Fig. 10 shows the effect of load power-factor on the terminal voltage, for constant load resistance, speed, and excitation capacitance.

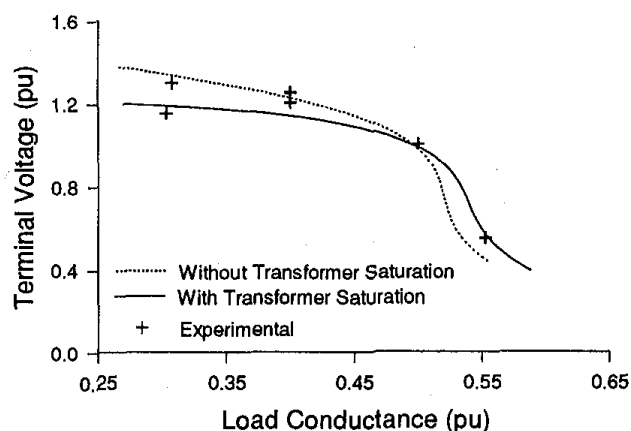


Figure 9: Variation of the terminal voltage with load conductance for resistive load,  $\omega = 1$  pu,  $C = 30$   $\mu$ F.

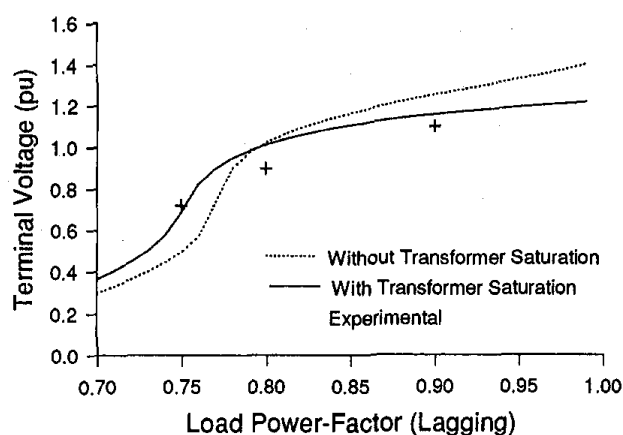


Figure 10: Variation of the terminal voltage with lagging power-factor load, for  $\omega = 1$  pu,  $C = 45$   $\mu$ F.

## 6 Voltage Control by Turns Ratio

When a SEIG is loaded, its terminal voltage decreases even for fixed values of excitation capacitance and speed, as shown in Fig. 11 for different values of transformer turns ratio  $\alpha$ . One of the methods to overcome this shortcoming is to increase the excitation capacitance as the load current is increased. Fig. 12 shows an example of computed values of  $C$  with load current for a purely resistive load, at constant speed  $\omega = 1$  pu. Increasing the excitation capacitance as the load increase may have some practical limitations. However, it is possible to connect all or part of the excitation capacitance to the machine through a tap-changing transformer (or a variac). By adjusting the transformer turns ratio, the effective capacitance as seen by the SEIG is controlled since it is multiplied by the square of the turns ratio. It is possible to reduce the required transformer rating by connecting the major capacitance directly to the machine's terminal and connecting only a small, controllable capacitance through the transformer.

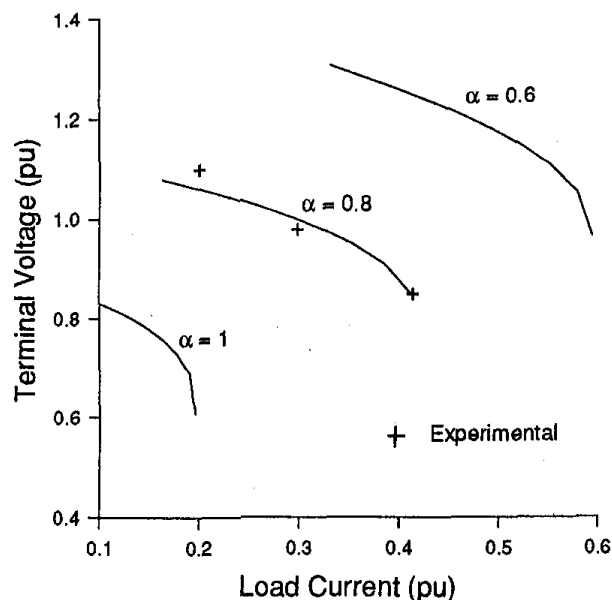


Figure 11: Variation of the terminal voltage with load for different transformer turns ratio, at constant speed and constant excitation capacitance.

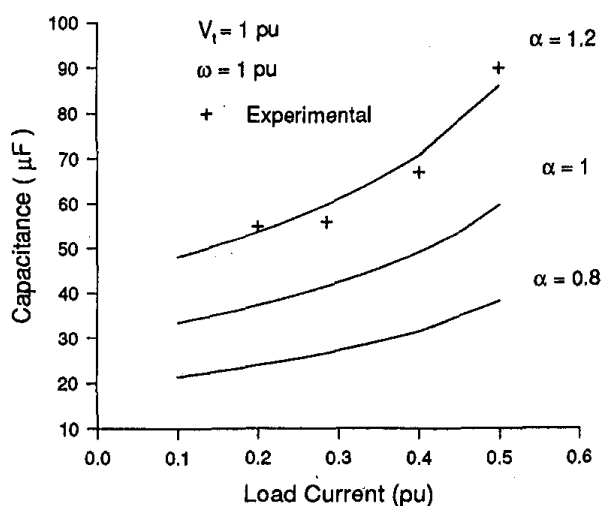


Figure 12: Variation of capacitance with load current at constant terminal voltage and constant speed.

## 7 CONCLUSIONS

In this paper, the advantages of connecting a saturable transformer or reactor to the terminals of an isolated self-excited induction generator are highlighted. These advantages include improved voltage regulation and protection against over-voltages which are the major cause of excitation capacitor failure at wind farms which use unregulated wind-turbines. The presence of the saturable element will also protect against over-voltages caused by increase in capacitance. Capacitance is normally increased in order to compensate for voltage drop

due to increased load current, but a large increase in capacitance may lead to over-excitation and hence over-voltage. The additional nonlinearity introduced by the transformer or reactor complicates the analysis of the SEIG considerably. A method to analyze the steady-state performance characteristics of the SEIG with two nonlinearities is presented.

## 8 REFERENCES

- [1] D. B. Watson, J. Arrillaga and T. Densem, "Controllable dc Power Supply from Wind-Driven Self-Excited Induction Machines," *Proc. IEE*, Vol. 126, No. 12, 1979, pp. 1245-1248.
- [2] G. Raina and O. P. Malik, "Wind Energy Conversion Using a Self-Excited Induction Generator," *IEEE Trans. on Power App. & Sys.*, Vol. 102, No. 12, 1983, pp. 3933-3936.
- [3] N. H. Malik and A. H. Al-Bahrani, "Influence of the Terminal Capacitor on the Performance Characteristics of a Self-Excited Induction Generator," *IEE Proceedings-C*, Vol. 137, No. 2, 1990, pp. 168-173.
- [4] S. S. Murthy, O. P. Malik, and A. K. Tandon, "Analysis of Self-Excited Induction Generators," *IEE Proceedings-C*, Vol. 129, No. 6, 1982, pp. 260-265.
- [5] L. Ouazene and G. McPherson, Jr., "Analysis of the Isolated Induction Generator," *IEEE Trans. on Power App. & Sys.*, Vol. 102, No. 8, 1983, pp. 2793-2798.
- [6] A. K. Al Jabri and A. I. Alolah, "Limits on the Performance of Three-Phase Self-Excited Induction Generators," *IEEE Trans. on Energy Conversion*, Vol. 5, No. 2, 1990, pp. 350-356.
- [7] S. M. Alghuwainem, "Performance Analysis of a PV Powered DC Motor Driving a 3-phase Self-Excited Induction Generator," *IEEE Trans. on Energy Conversion*, Vol. 11, no. 1, pp. 155-161, March 1996.
- [8] S. M. Alghuwainem, "Speed Control of a PV Powered DC Motor Driving a Self-Excited 3-Phase Induction Generator for Maximum Utilization Efficiency," *IEEE PES Summer Meeting*, July 28 - August 1, 1996, Denver, Colorado, Paper 96 SM 570-2 EC.
- [9] L. Tang and R. Zavadil, "Shunt Capacitor Failures Due to Windfarm Induction Generator Self-Excitation Phenomenon," *IEEE Trans. on Energy Conversion*, Vol. 8, No. 3, 1993 pp. 513-519.
- [10] J. C. Smith, J. I. Herrera and M. F. McGranaghan, "Analysis of Capacitor Failure at the Kamoia Windfarm on the Island of Hawaii," *Solar Energy Research Institute*, (SERI, now known as NREL), August 1989.
- [11] L. Shridhar, B. Singh, C. Jha, B. P. Singh and S. S. Murthy, "Selection of Capacitors for the Self Regulated Short Shunt Self Excited Induction Generator," *IEEE Trans. on Energy Conversion*, Vol. 10, No. 1, 1995, pp. 10-17.
- [12] T.F. Chan, "Steady State Analysis of Self-Excited Induction Generators," *IEEE Trans. on Energy Conversion*, Vol. 9, No. 2, 1994, pp. 288-296.
- [13] M. G. Say, *Alternating Current Machines*, London: Pitman, 5th Ed., 1983, pp. 333-336.



**Saad M. Alghuwainem** received B.Sc. in Electrical Engineering from University of Riyadh in 1974. From 1974 to 1976 he worked as a teaching assistant in the EE Department, Riyadh University. He received M.Sc. degree in Electrical Engineering from University of Colorado, Boulder in 1978.

From 1979 to 1981 he worked in the EE Department, King Saud University as a research assistant. From 1982 to 1986 he attended The University of Michigan, Ann Arbor where he received Ph.D. in Electrical Engineering. Since 1986 he has been with the Department of Electrical Engineering, King Saud University, where he is currently an associate professor. His interests include renewable energy sources, energy conversion systems, power system protection, and electromagnetic transients.