Graphs

The Graph of $\theta = \theta_0$

For any value of r, θ is always θ_0

Any point of the form (r , θ_0), where r is any real number, belongs to the graph of the curve

All the following points belongs to that graph:

(1 , θ_0), (2 , θ_0), (3 , θ_0)

(-1 , θ_0) $\equiv$(1, $\theta_0 + \pi$), (-2 , θ_0) $\equiv$ (2 , $\theta_0 + \pi$)

($\sqrt{3}/5$, θ_0)

The graph is a straight line making an angle equal to θ_0 with the polar axis

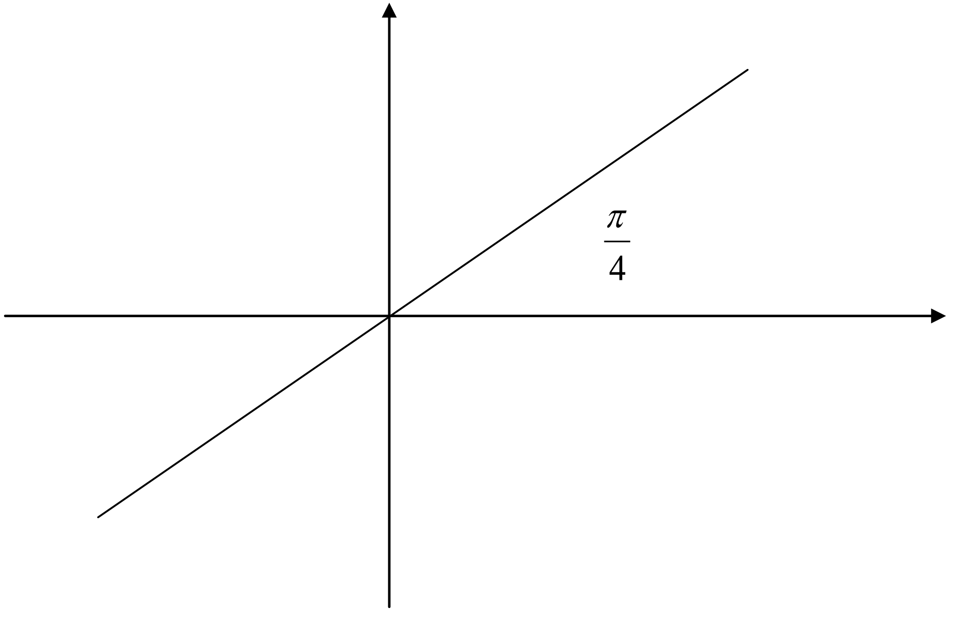
Example (1)

Sketch the graph the following polar equation:

$\theta = \pi/4$

θ	r	(r , θ)
π/4	0	(0 , π/4)
π/4	1	(1 , π/4)
π/4	2	(2 , π/4)
π/4	3	(3, π/4)
π/4	-1	(-1 , π/4)= (1,π+π/4) = (1,5π/4)
π/4	-2	(-2 , π/4)= (2,π+π/4) = (2,5π/4)
π/4	-3	(-3 , π/4)= (3,π+π/4) = (3,5π/4)

The graph is a straight line making an angle equal to π/4 with the polar axis



Reminder

The Cartesian equation of this straight line is $y = x$

We can arrive at this equation, as follows:

We have,

$\theta = \pi/4$

$\rightarrow \tan\theta = 1$

$\rightarrow \sin\theta / \cos\theta = 1$

$\rightarrow r\sin\theta / r\cos\theta = 1$

$\rightarrow y / x = 1 \rightarrow y = x$

The Graph of $r = c\theta$, where c is nonzero constant

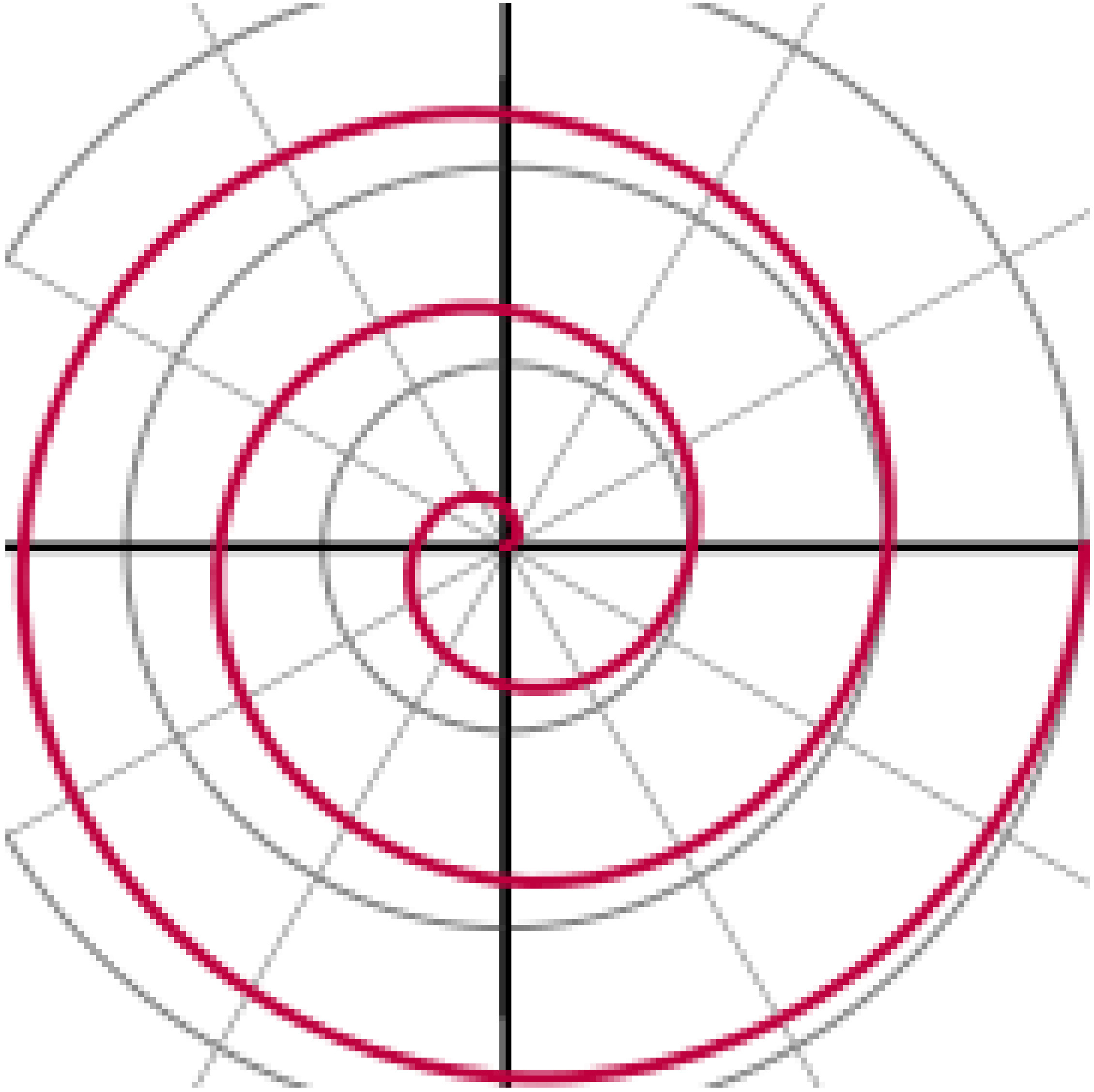
Example (2)

Sketch the graph the following polar equation:

$$r = \theta$$

$r = \theta$

θ	$r = \theta$	(r , θ)
0	0	$(0 , 0)$
$\pi/2$	$\pi/2$	$(\pi/2 , \pi/2)$
π	π	(π , π)
$3\pi/2$	$3\pi/2$	$(3\pi/2 , 3\pi/2)$
2π	2π	$(2\pi , 2\pi)$
$2\pi+\pi/2$	$2\pi+\pi/2$	$(2\pi+ \pi/2 , 2\pi+ \pi/2)$
3π	3π	$(3\pi , 3\pi)$



The graph is a spiral

The Graph of $r = r_0$, where are is a positive number

For any value of θ, r is always r_0

Any point of the form (r_0 , θ), where θ is any angle (or any real number), belongs to the graph of the curve

All the following points belongs to that graph:

$(r_0 , 0)$, $(r_0 , \pi/2)$, (r_0 , π) , $(r_0 , \pi/6)$

$(-r_0 , \pi) \equiv (r_0 , 0)$, $(-r_0 , 3\pi/2)$, $\equiv (r_0 , \pi/2)$

The graph is a circle centered at the origin and of radius equal to r_0

Example (3)

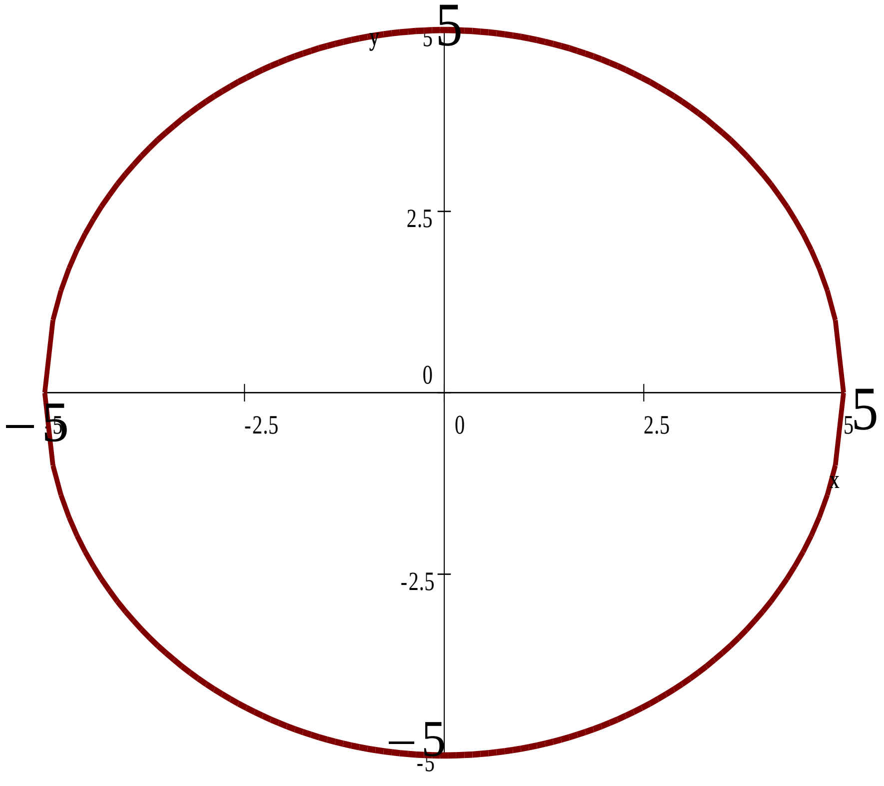
Sketch the graph the following polar equation:

$$r = 5$$

$r = 5$

θ	$r = \theta$	(r , θ)
0	5	$(5 , 0)$
$\pi/2$	5	$(5 , \pi/2)$
π	5	$(5 , \pi)$
$3\pi/2$	5	$(5 , 3\pi/2)$
2π	5	$(5 , 2\pi)$
$2\pi+\pi/2$	5	$(5 , 2\pi+ \pi/2)$
3π	5	$(5 , 3\pi)$

The graph of r = 5 is a circle centered at the origin and of radius 5



The graph of:

$r = c \sin\theta$

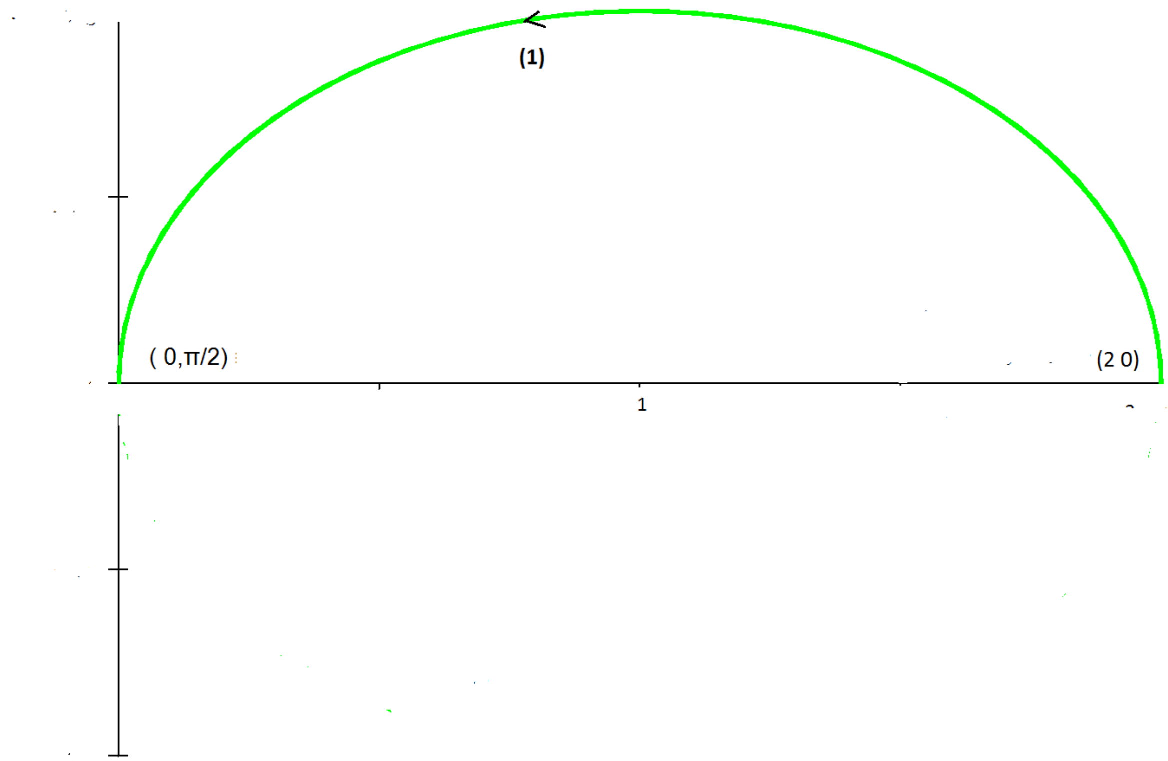
Or

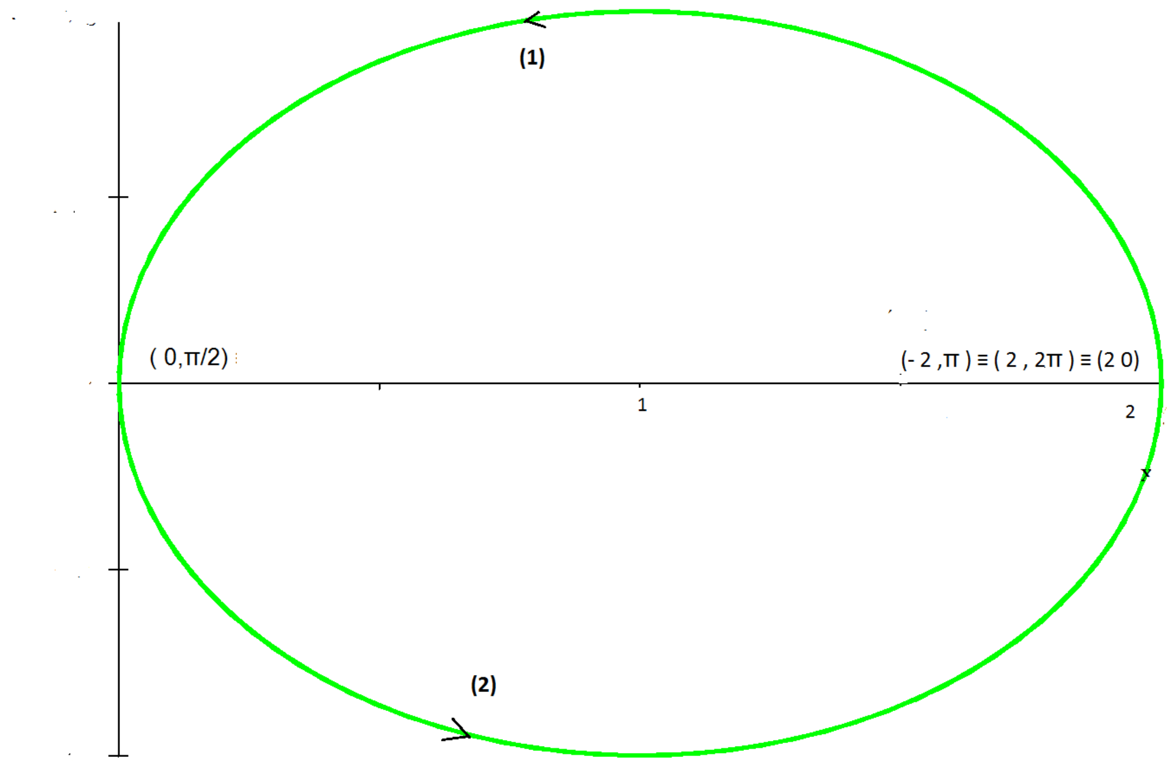
$r = c \cos\theta$

Where c is a nonzero constant

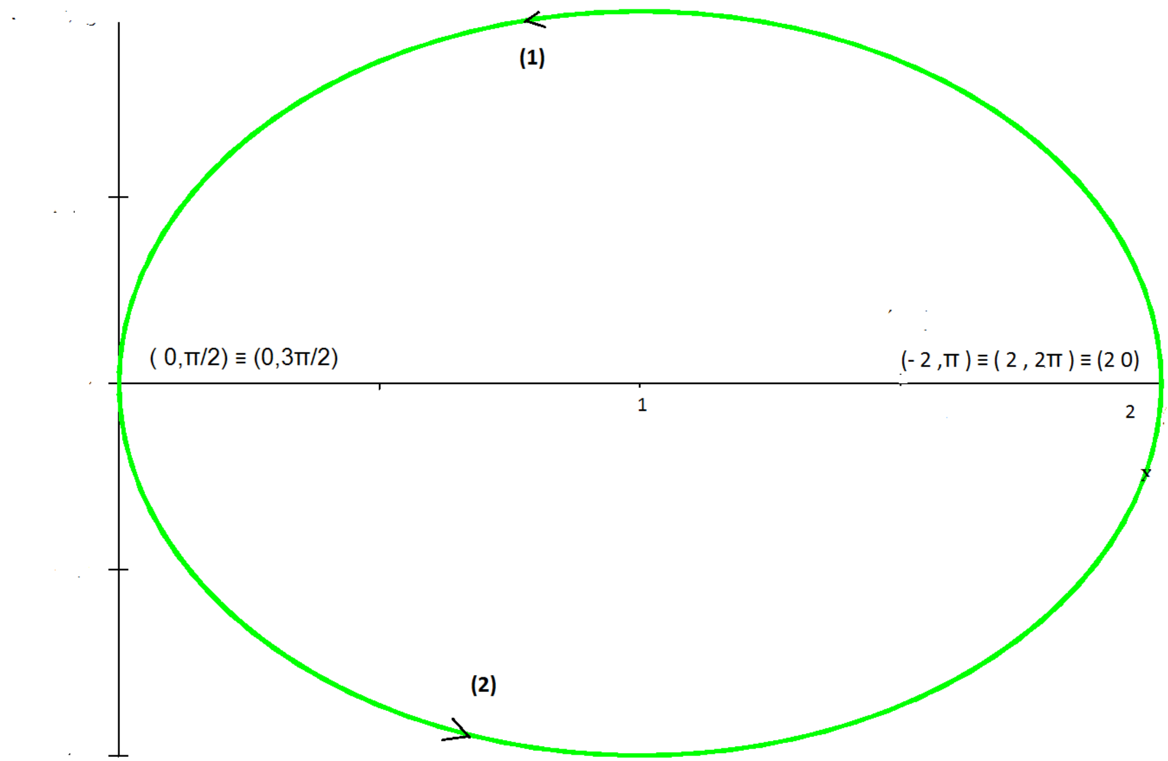
Example (4) - a
$r = 2 \cos\theta$

θ	$r = 2\cos\boldsymbol{\theta}$	**(r, θ)**
0	2	$(2,0)$
$\pi/2$	0	$(0, \pi/2)$
π	-2	$(-2, \pi)$ $\equiv (2, 2\pi)$
$3\pi/2$	0	$(0, 3\pi/2)$
2π	2	$(2, 2\pi)$


(1)
(0,π/2)
(2 0)
1



(1)
(0,π/2)
(- 2 ,π) ≡ (2 , 2π) ≡ (2 0)
1
2
x
(2)



(1)
(0,π/2) ≡ (0,3π/2)
(- 2 ,π) ≡ (2 , 2π) ≡ (2 0)
1
2
x
(2)

Example (4) - b

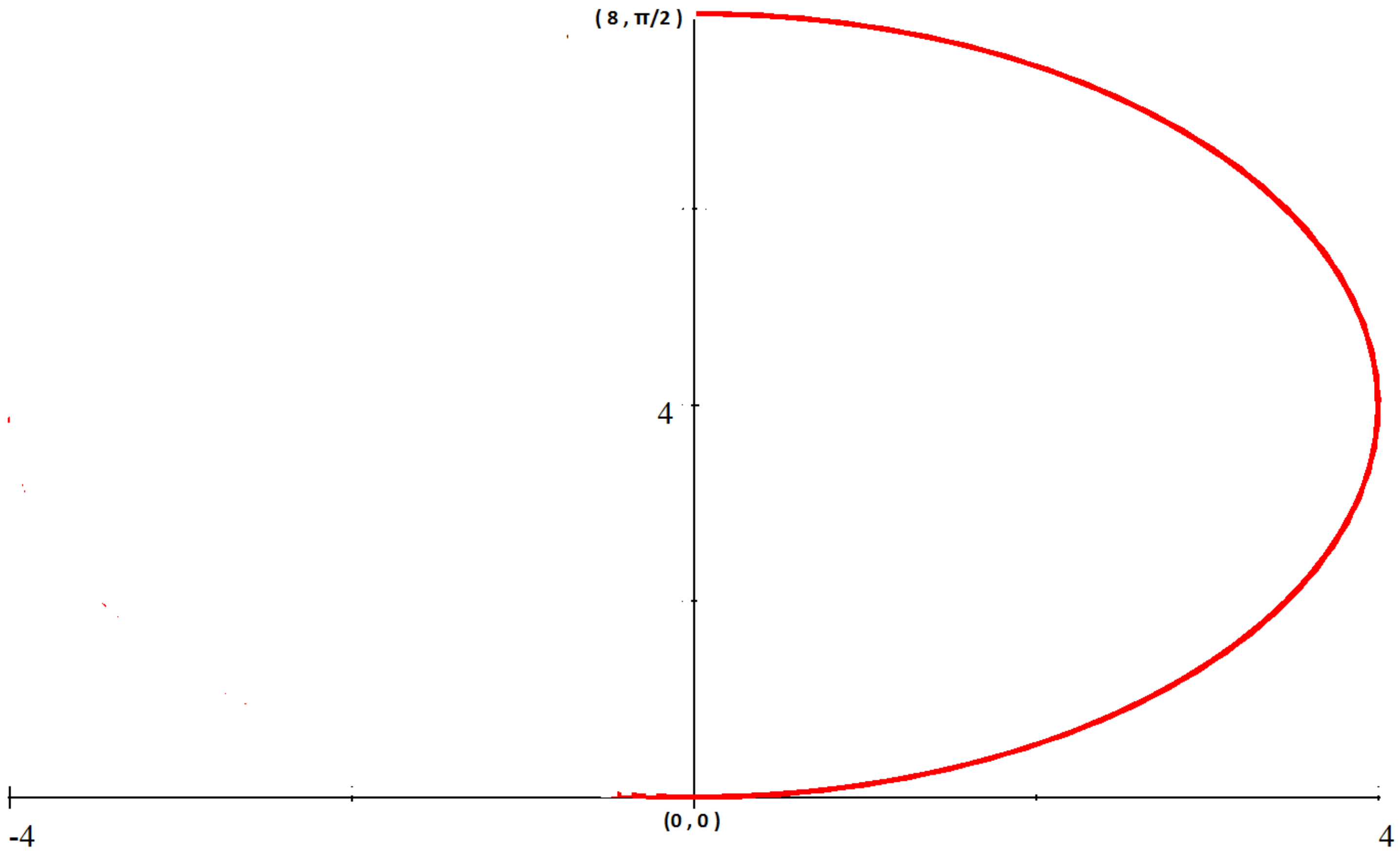
Sketch the graph the following polar equation:

$$r = 8 \sin \theta$$

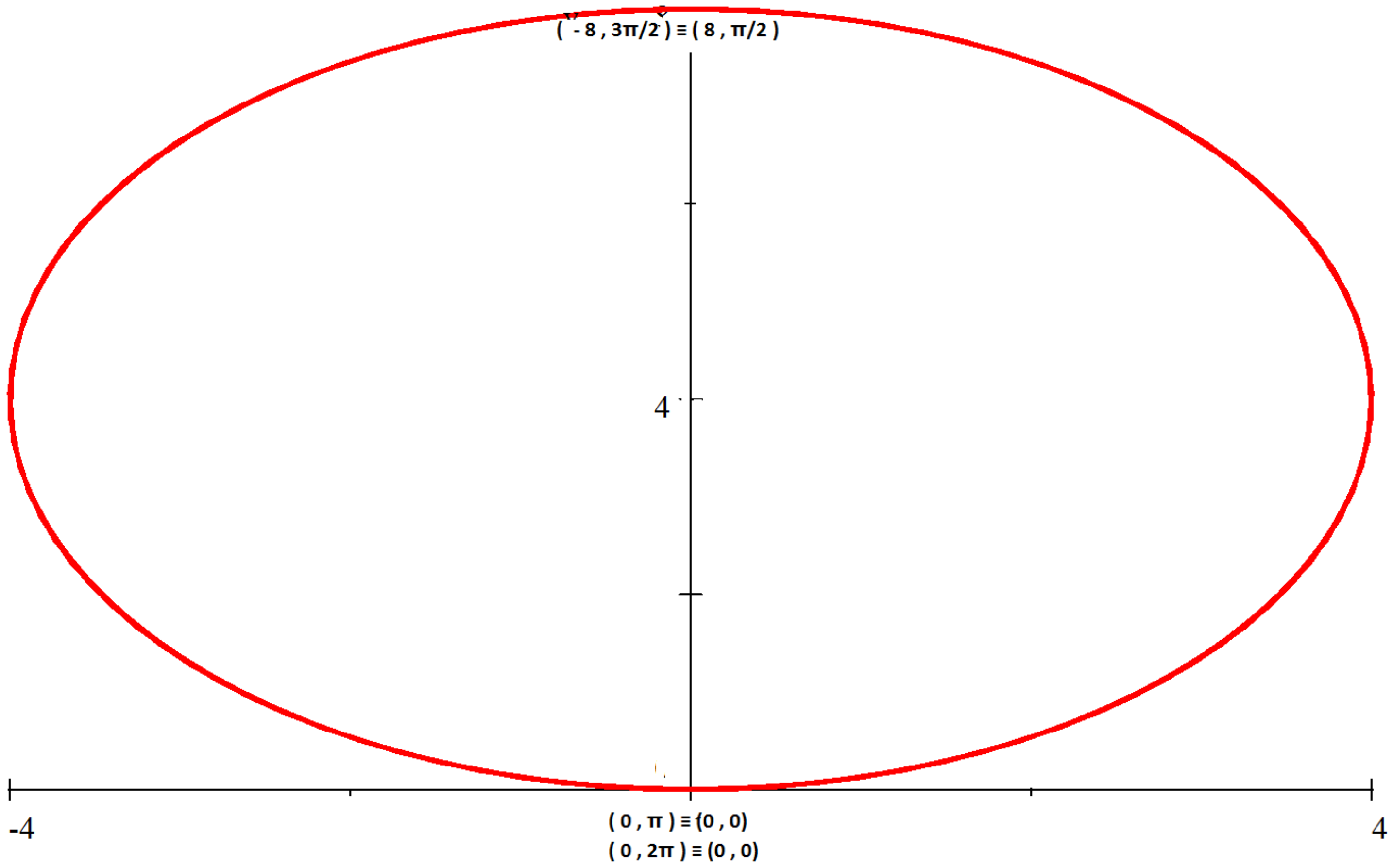
$r = 8 \sin \theta$

θ	$r = 8 \sin \theta$	
0	0	$(0,0)$
$\pi/2$	8	$(8, \pi/2)$
π	0	$(0, \pi)$
$3\pi/2$	-8	$(-8, 3\pi/2) \equiv (8, \pi/2)$
2π	0	$(0, 2\pi) \equiv (0,0)$

$r = 8\sin\theta$



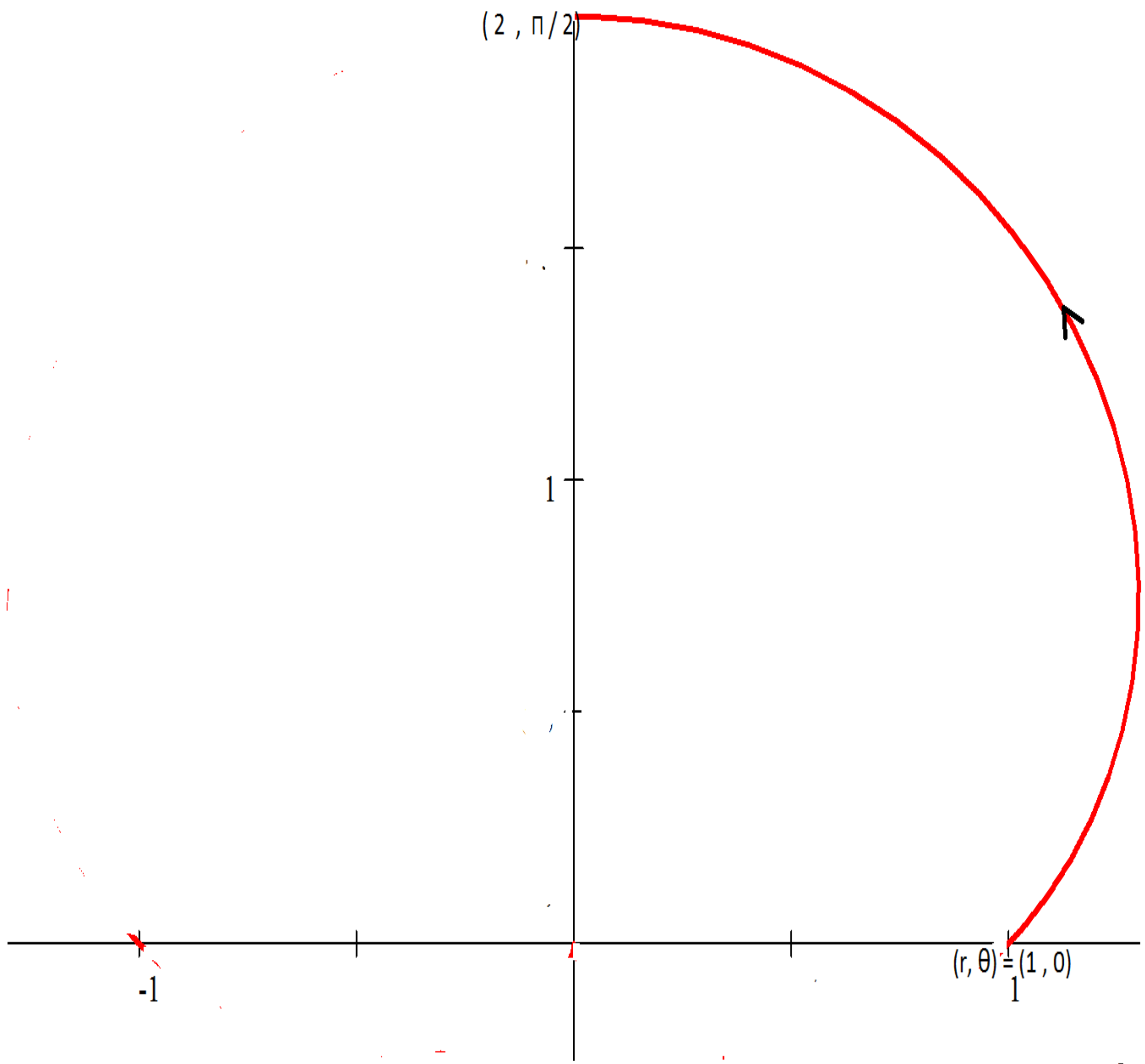
$r = 8\sin\theta$

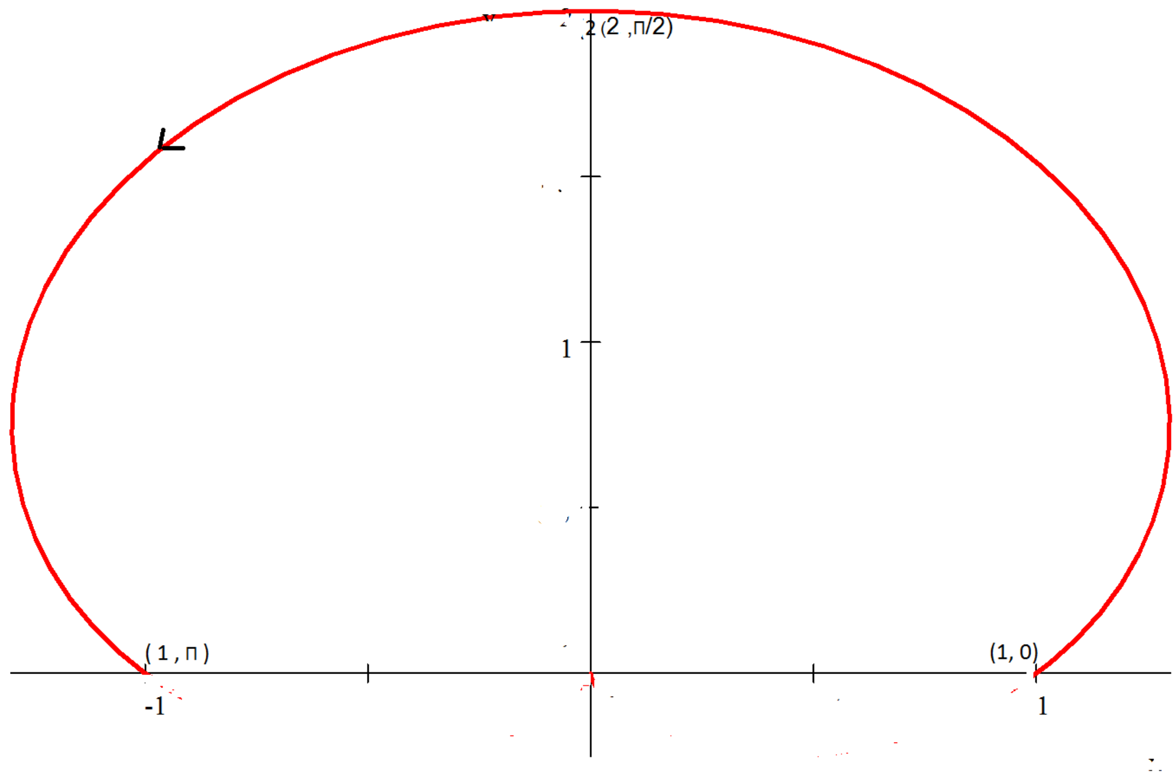


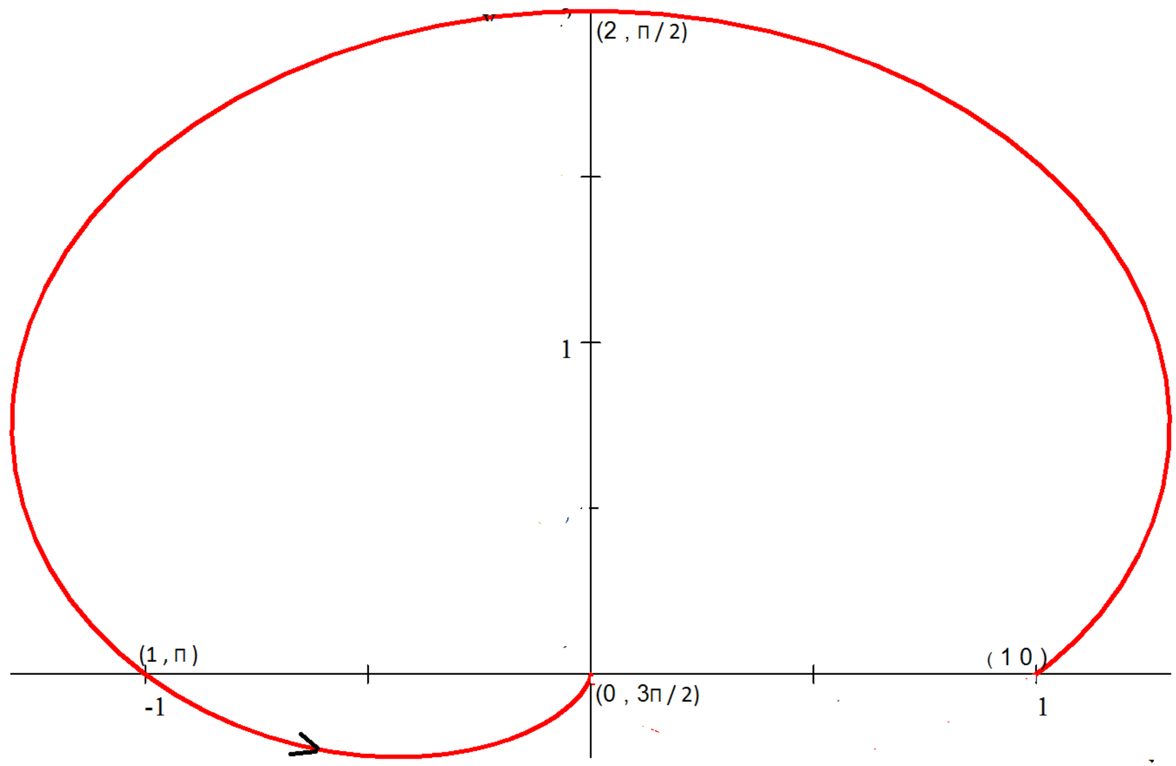
The graph is a circle centered at (0,3) of radius equal to 6/2 = 3

Example (5) – A
$r = 1 + \sin\theta$

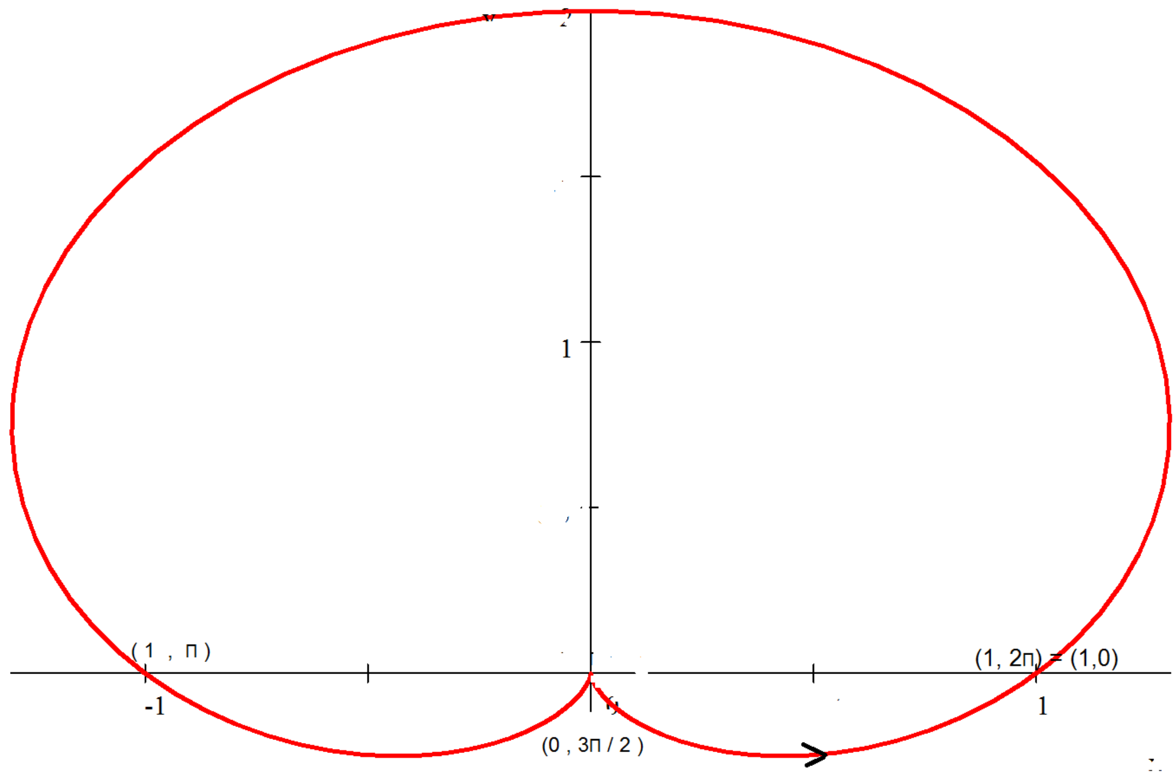
θ	$r = 1 + \sin\theta$	**(r, θ)**
0	1	$(1,0)$
$\pi/2$	2	$(2, \pi/2)$
π	1	$(1, \pi)$
$3\pi/2$	0	$(0, 3\pi/2)$
2π	1	$(1, 2\pi)$


(2 , π / 2)
1
-1
1
(r, θ) = (1 , 0)


2 (2 ,π/2)
1
(1 , π)
(1, 0)
-1
1



(2 , п / 2)
1
(1 , п)
(1 0)
-1
(0 , 3п / 2)
1


2
1
(1 , π)
(1, 2π) = (1,0)
-1
1
(0 , 3π / 2)

Example (5) – B: Graph
$r = 2 - 4\sin\theta$

We find, when r takes:

1. The value 0
2. Takes a maximum value, which is 6, note that $(2 – 4\sin\pi) = 2 – (-4) = 6$,
3. Takes a minimum value, which is -2, note that $(2 – 4\sin 0) = 2 – (4) = -2$

Solution:

We have

$r = 2 – 4\sin\theta = 2 (1 – 2 \sin\theta)$

$r = 0$ if $\sin\theta = ½$

$\pi/6 = 5\pi/6 - \pi = \pi/6$, $\theta =$ That’s $r = 0$ if θ

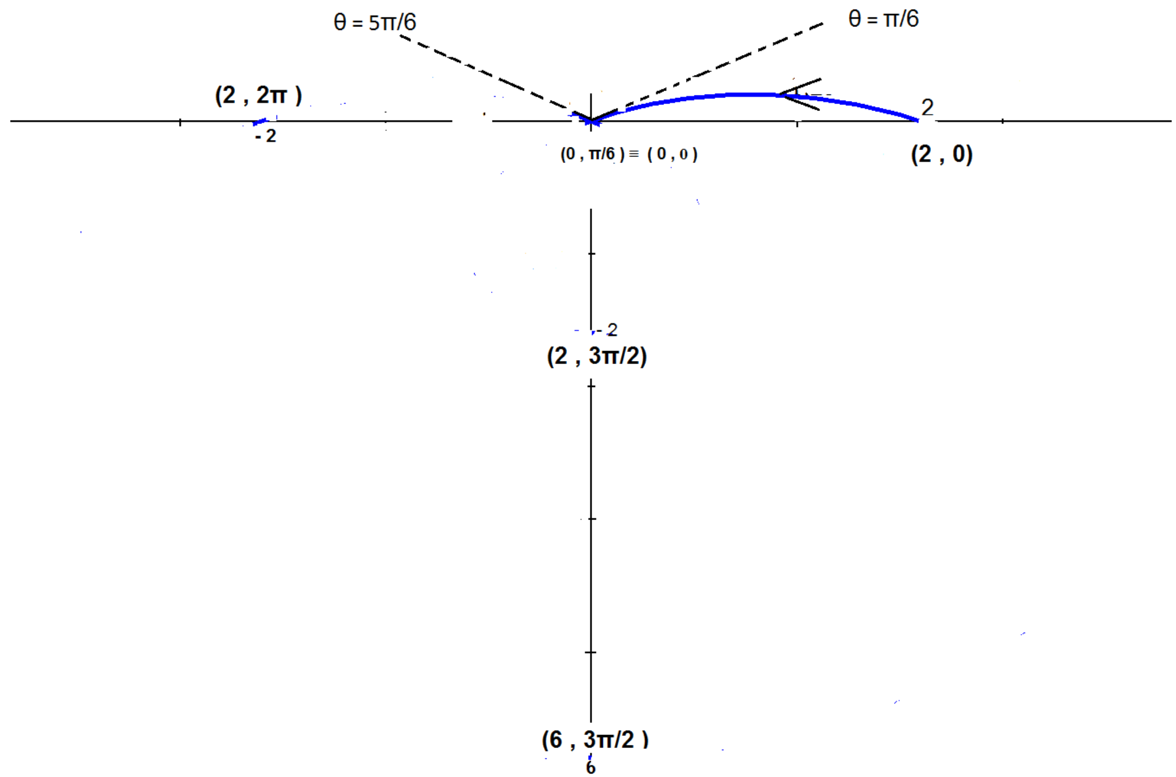
Thus, we must pay attention to these values, when graphing

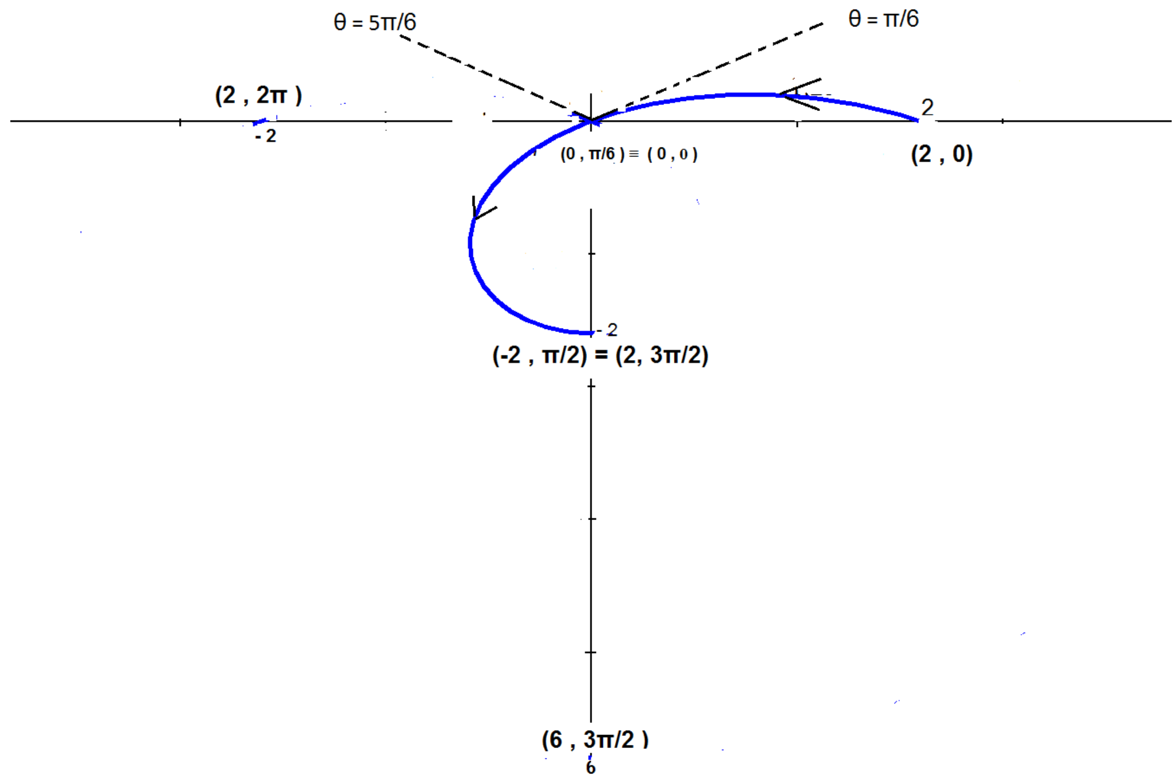
The Graph of $r = 2(1 - 2\sin\theta)$

θ	**$r = \theta$**	**(r, θ)**
0	2	$(2, 0)$
$\pi/6$	0	$(0, \pi/6)$
$\pi/2$	-2	$(-2, \pi/2) = (2, 3\pi/2)$
$5\pi/6$	0	$(0, 5\pi/6)$
π	2	$(2, \pi)$
$3\pi/2$	6	$(6, 3\pi/2)$
2π	2	$(2, 2\pi)$

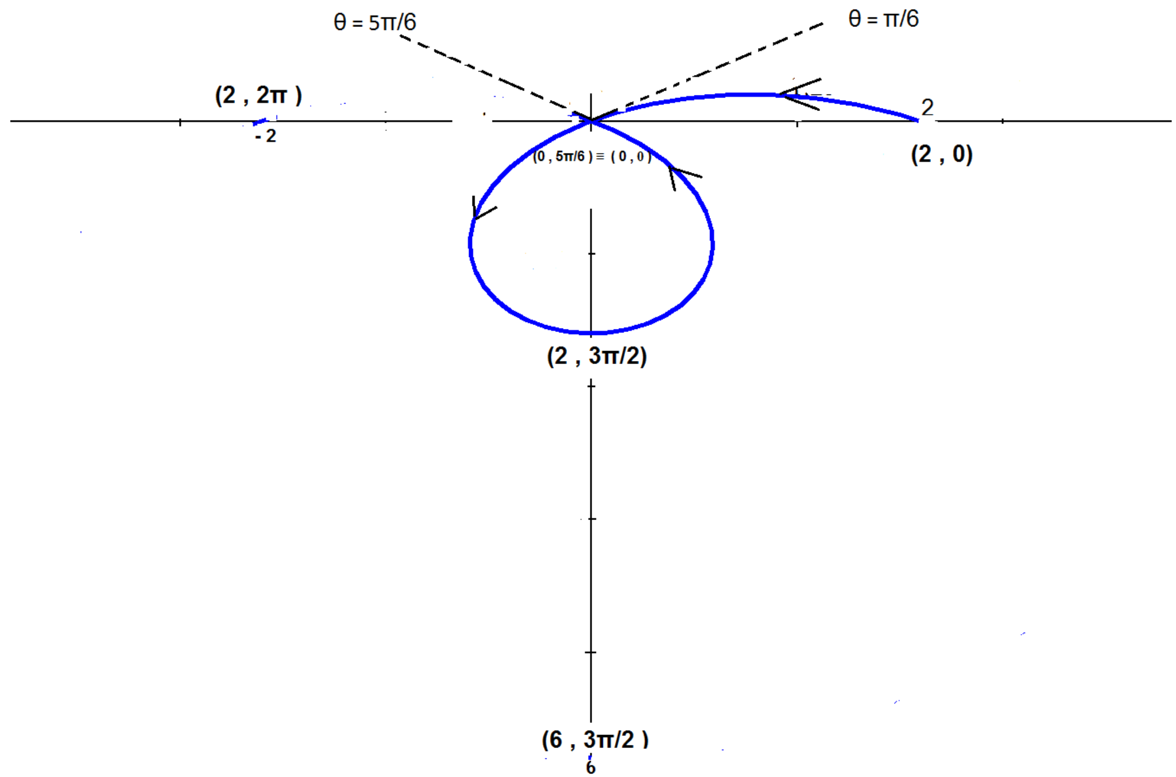
Please, notice the following copy error in the Graph

The point to the left of the pole (the origin) , named erroneously (2 , 2π) while it is, of course, (2 , π)

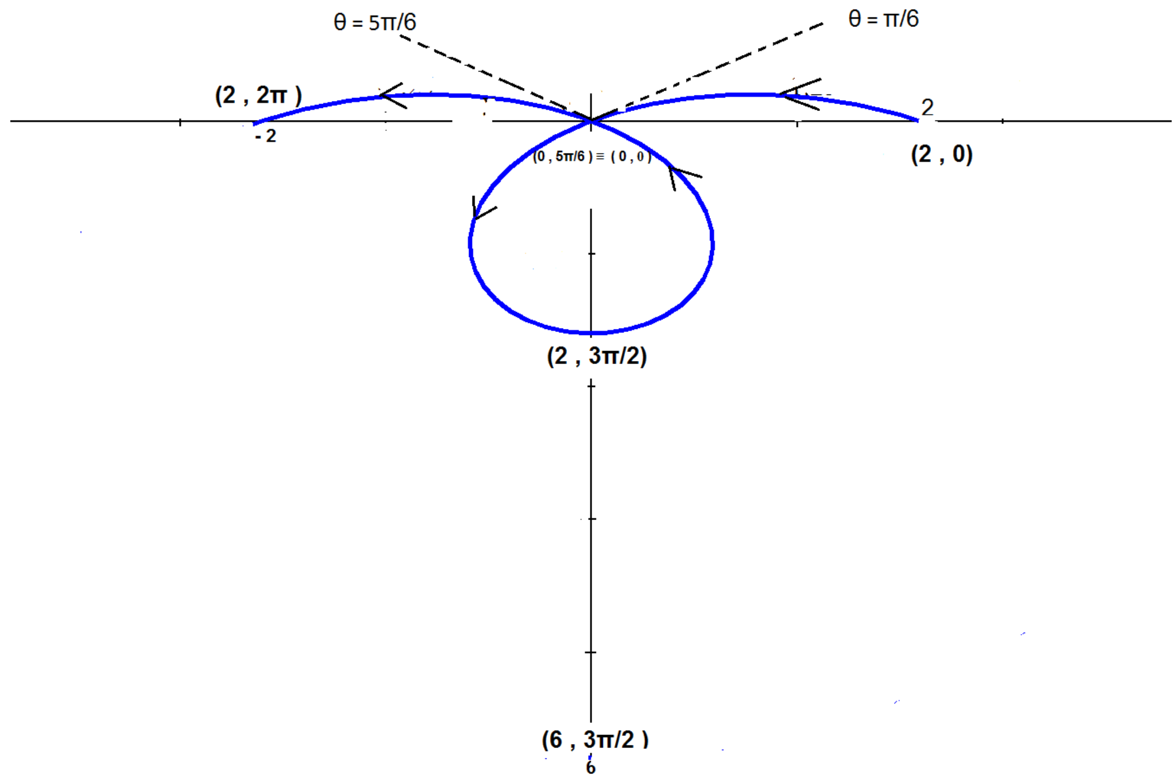

θ = 5π/6
θ = π/6
(2 , 2π)
- 2
2
(0 , π/6) ≡ (0 , 0)
(2 , 0)
- 2
(2 , 3π/2)
(6 , 3π/2)
6



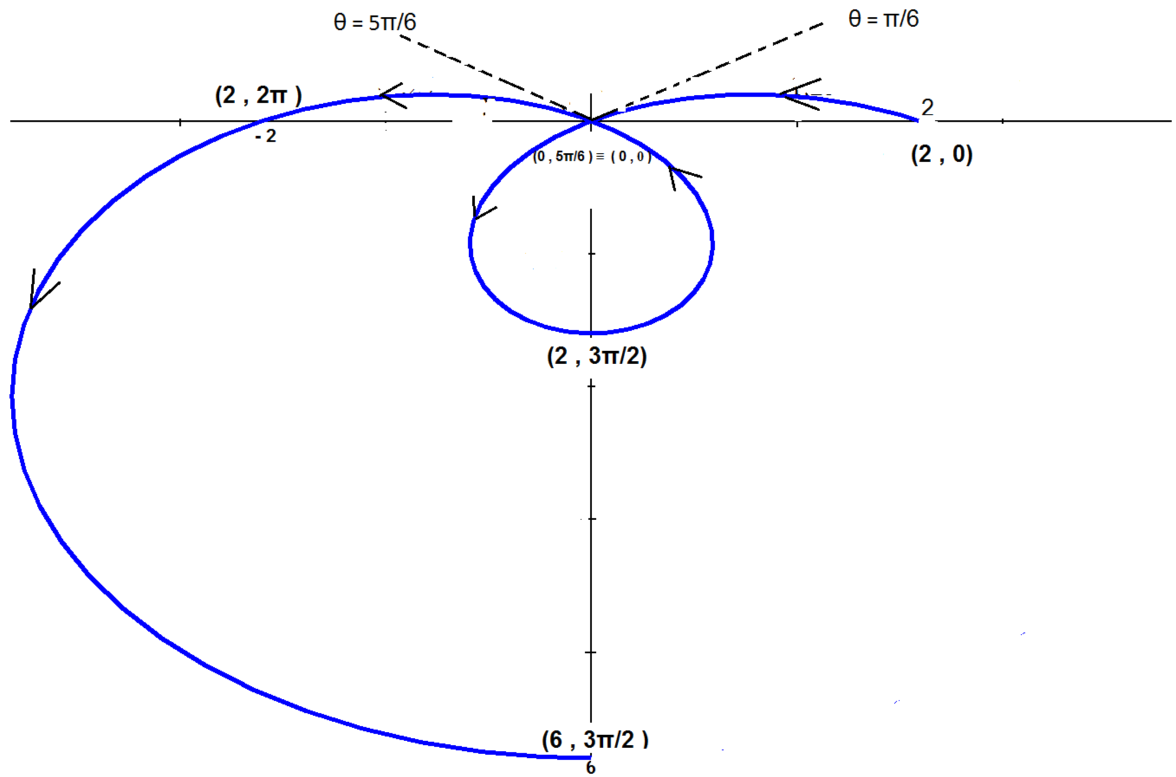
θ = 5π/6
θ = π/6
(2 , 2π)
2
- 2
(0 , π/6) ≡ (0 , 0)
(2 , 0)
- 2
(-2 , π/2) = (2, 3π/2)
(6 , 3π/2)
6



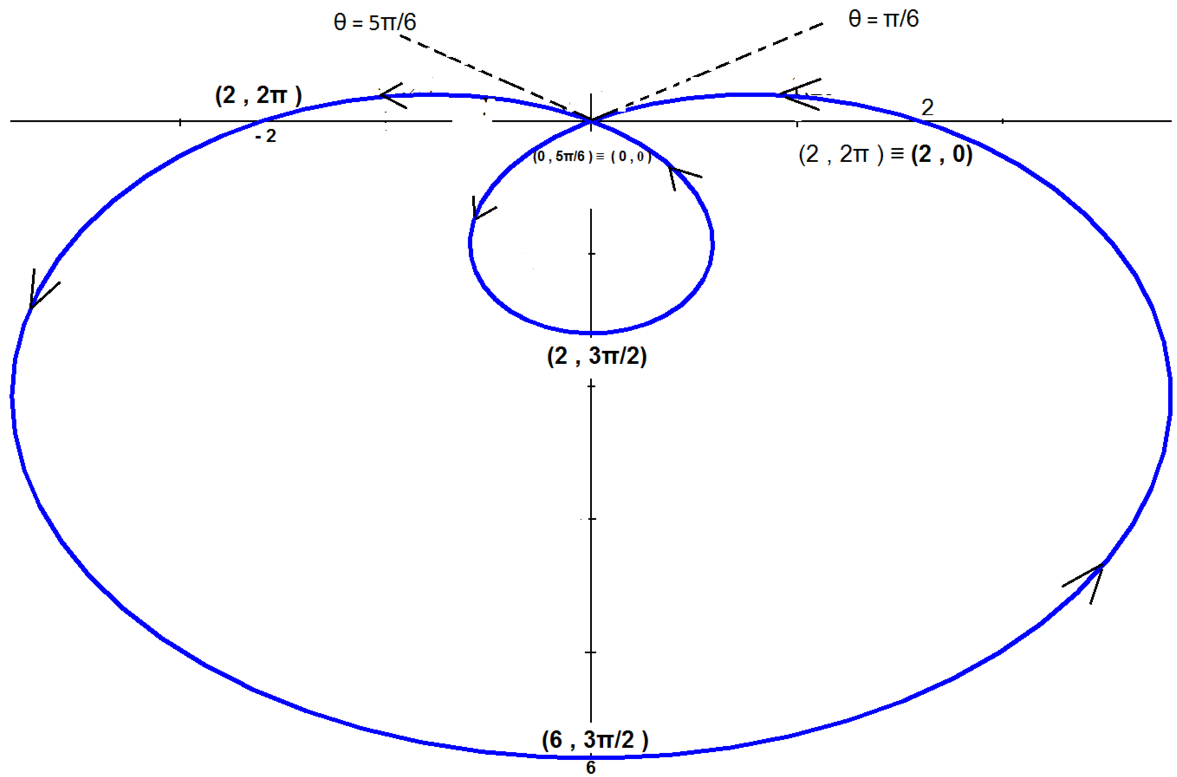
θ = 5π/6
θ = π/6
(2 , 2π)
- 2
2
(2 , 0)
(0 , 5π/6) ≡ (0 , 0)
(2 , 3π/2)
(6 , 3π/2)
6



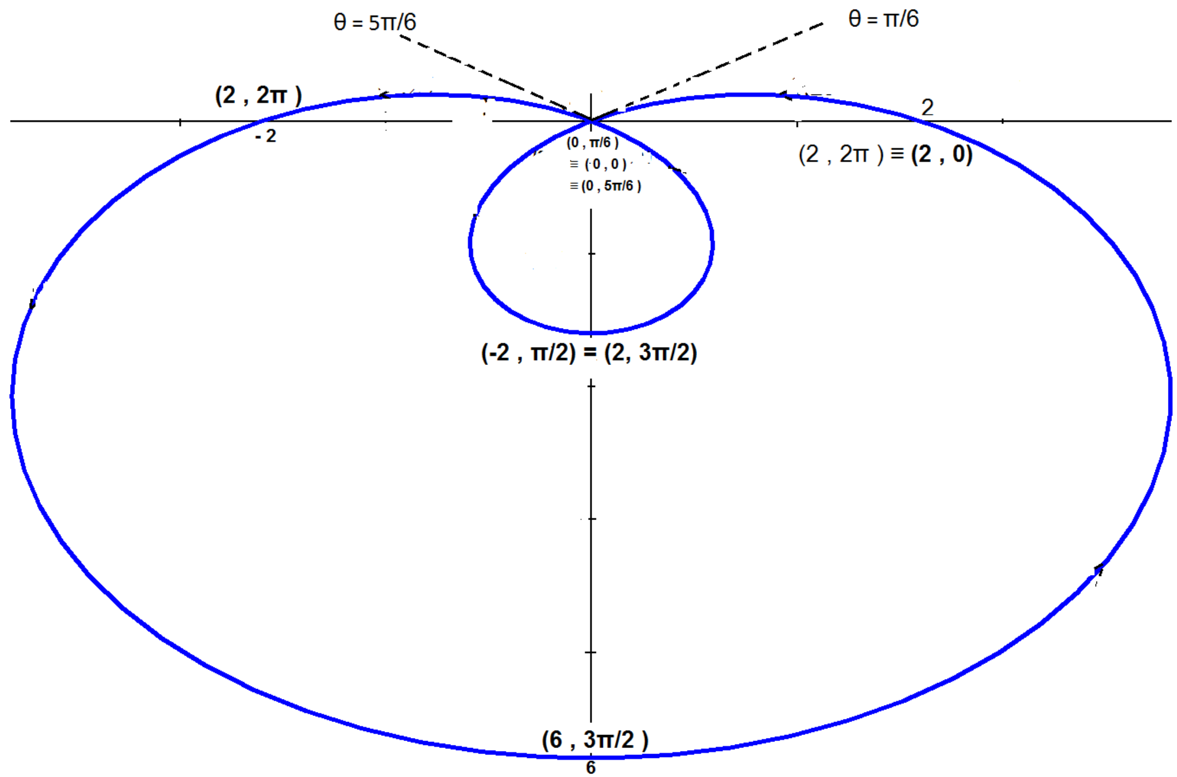
θ = 5π/6
θ = π/6
(2 , 2π)
- 2
2
(2 , 0)
(0 , 5π/6) ≡ (0 , 0)
(2 , 3π/2)
(6 , 3π/2)
6



θ = 5π/6
θ = π/6
(2 , 2π)
2
- 2
(0 , 5π/6) ≡ (0 , 0)
(2 , 0)
(2 , 3π/2)
(6 , 3π/2)
6



θ = 5π/6
θ = π/6
(2 , 2π)
2
- 2
(0 , 5π/6) ≡ (0 , 0)
(2 , 2π) ≡ (2 , 0)
(2 , 3π/2)
(6 , 3π/2)
6



θ = 5π/6
θ = π/6
(2 , 2π)
2
- 2
(0 , π/6)
≡ (0 , 0)
≡ (0 , 5π/6)
(2 , 2π) ≡ (2 , 0)
(-2 , π/2) ≡ (2, 3π/2)
(6 , 3π/2)
6

Example (6) – A : Graph $r = \cos 2\theta$

We find when: $2\theta = n\pi$ and when $2\theta = n(\pi/2)$

$r = 1$ if $\cos 2\theta = 1$

$2\theta = 0, 2\pi, 4\pi, 6\pi, \ldots\ldots\ldots$

$\rightarrow\ \theta =$ **$0, 2\pi/2 = \pi, 4\pi/2 = 2\pi$** , ...

$r = -1$ if $\cos 2\theta = -1$

$2\theta = \pi, 3\pi, 5\pi, \ldots\ldots\ldots$

$\rightarrow\ \theta =$ **$\pi/2$, $3\pi/2$** ,..........

$r = 0$, which if $\cos 2\theta = 0$

$2\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2, \ldots\ldots\ldots$

$\rightarrow\ \theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/6, \ldots\ldots.$

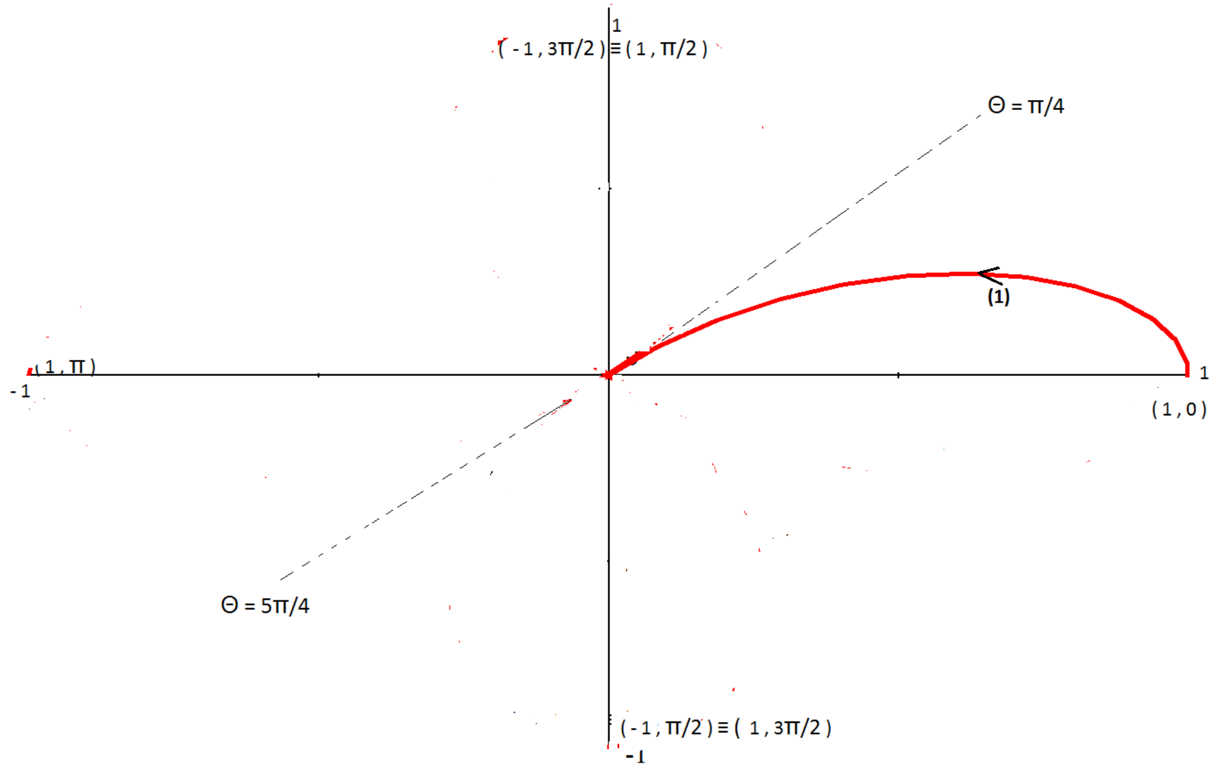
Example (6) – A

$r = \cos 2\theta$

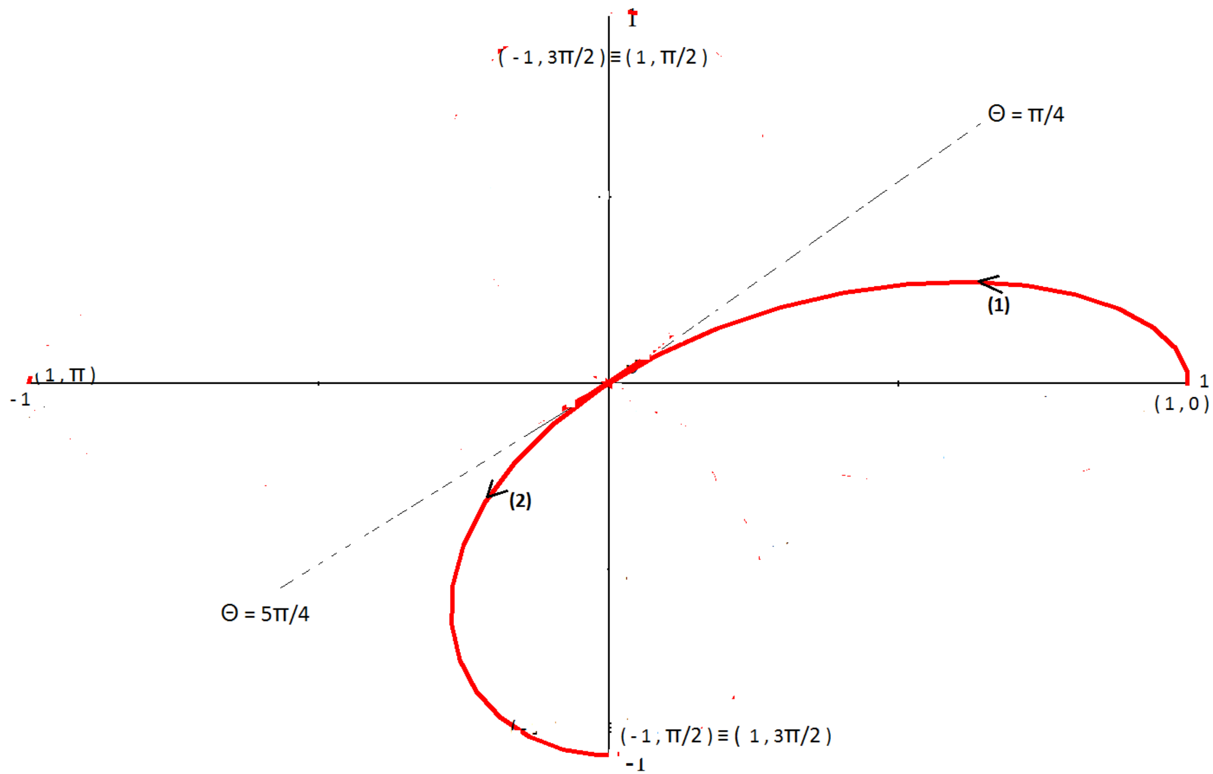
θ	$r = \cos 2\theta$	**(r, θ)**
0	1	(1,0)
$\pi/4$	0	$(0 , \pi/4)$
$\pi/2$	- 1	$(-1 , \pi/2)$ $\equiv (1 , 3\pi/2)$
$3\pi/4$	0	$(0 , 3\pi/4)$
π	1	$(1, \pi)$

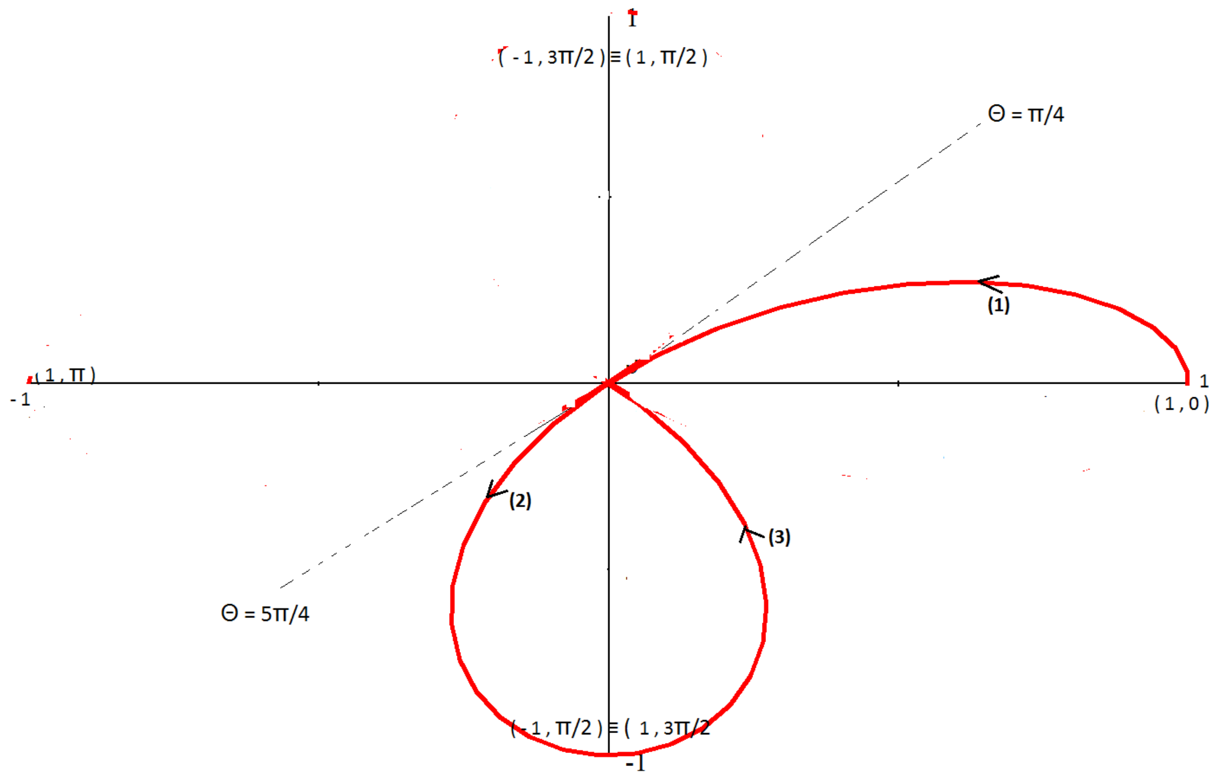
$r = \cos 2\theta$

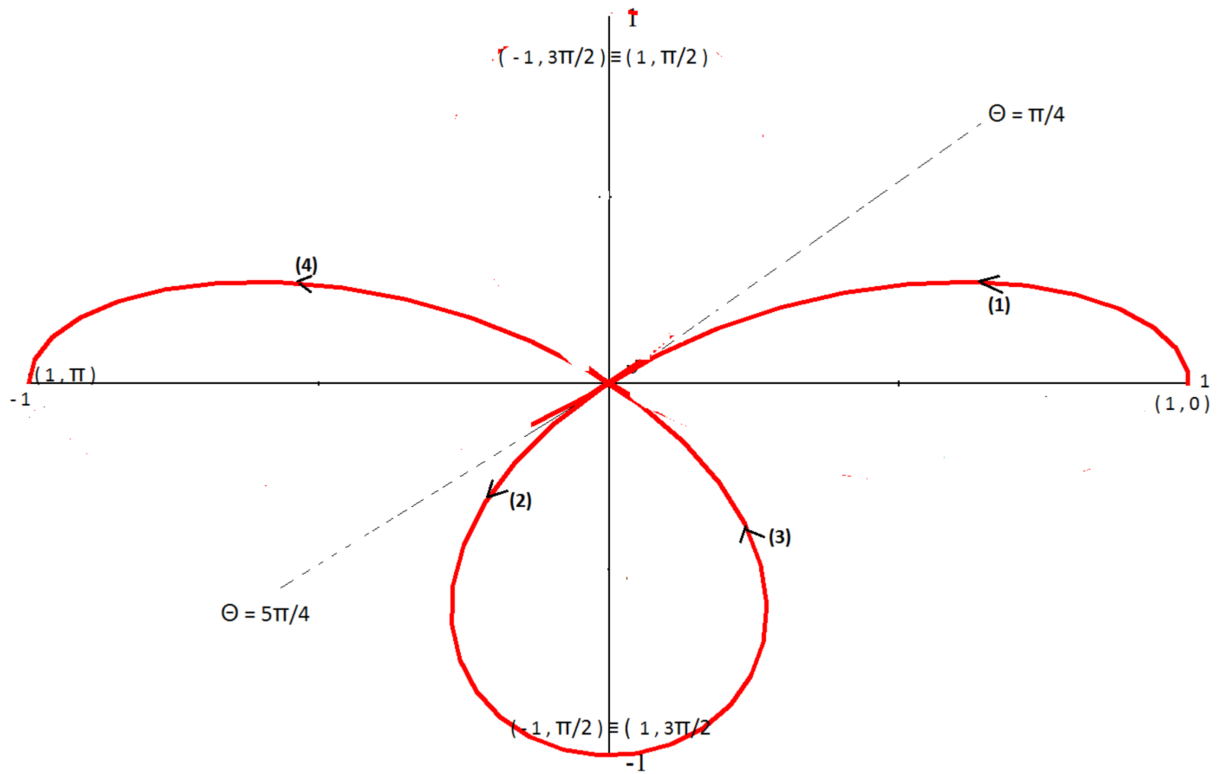
θ	$r = \cos 2\theta$	**(r , θ)**
$5\pi/4$	0	$(0, 5\pi/4)$
$3\pi/2$	$- 1$	$(- 1 , 3\pi/2)$ $\equiv (1 , \pi/2)$
$7\pi/4$	0	$(0 , 7\pi/4)$
2π	1	$(0 , 2\pi)$



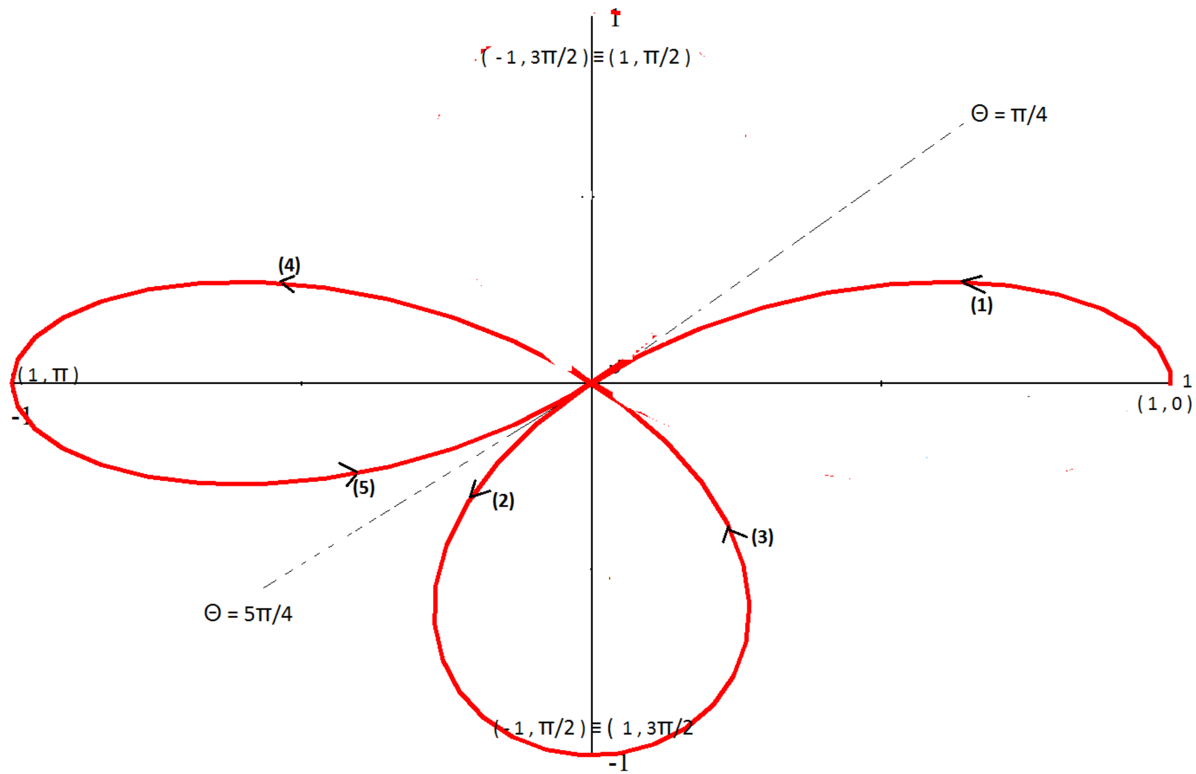
1
(- 1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(1)
(1 , π)
- 1
1
(1 , 0)
Θ = 5π/4
(- 1 , π/2) ≡ (1 , 3π/2)
- 1

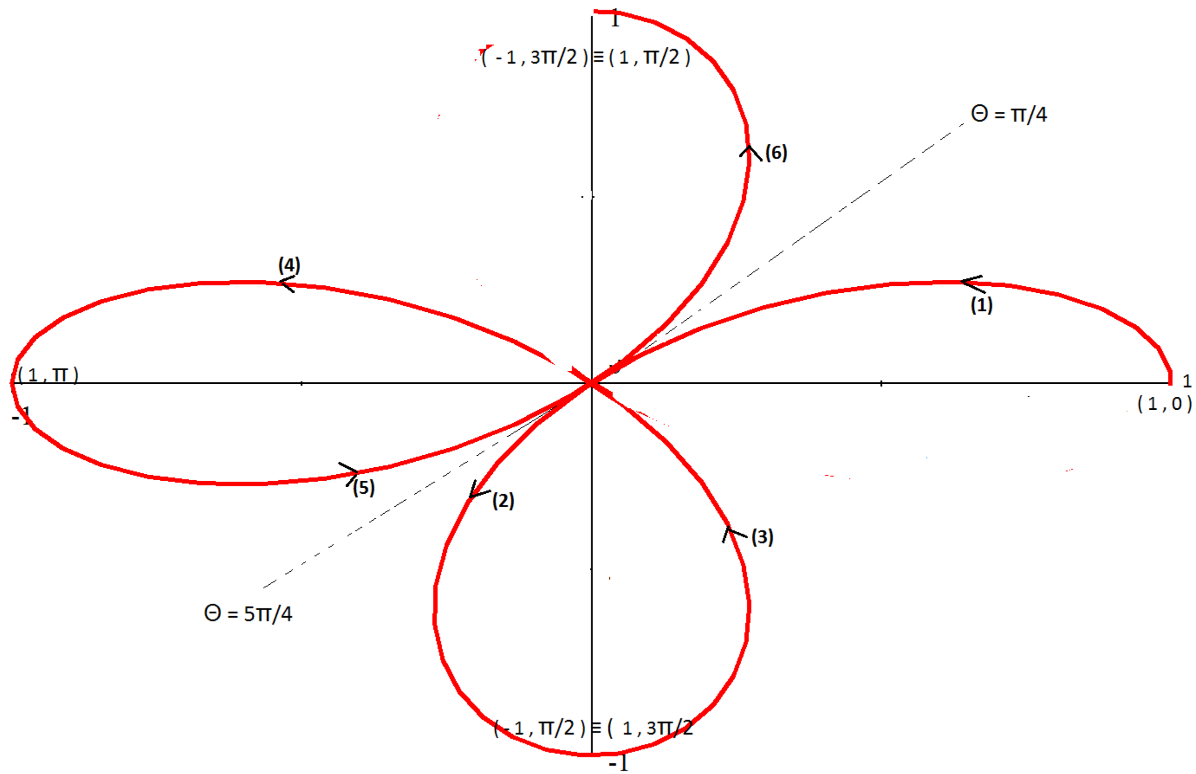

1
(- 1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(1)
(1 , π)
- 1
1
(1 , 0)
(2)
Θ = 5π/4
(- 1 , π/2) ≡ (1 , 3π/2)
- 1


1
(- 1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(1)
(1 , π)
- 1
1
(1 , 0)
(2)
(3)
Θ = 5π/4
(- 1 , π/2) ≡ (1 , 3π/2
-1

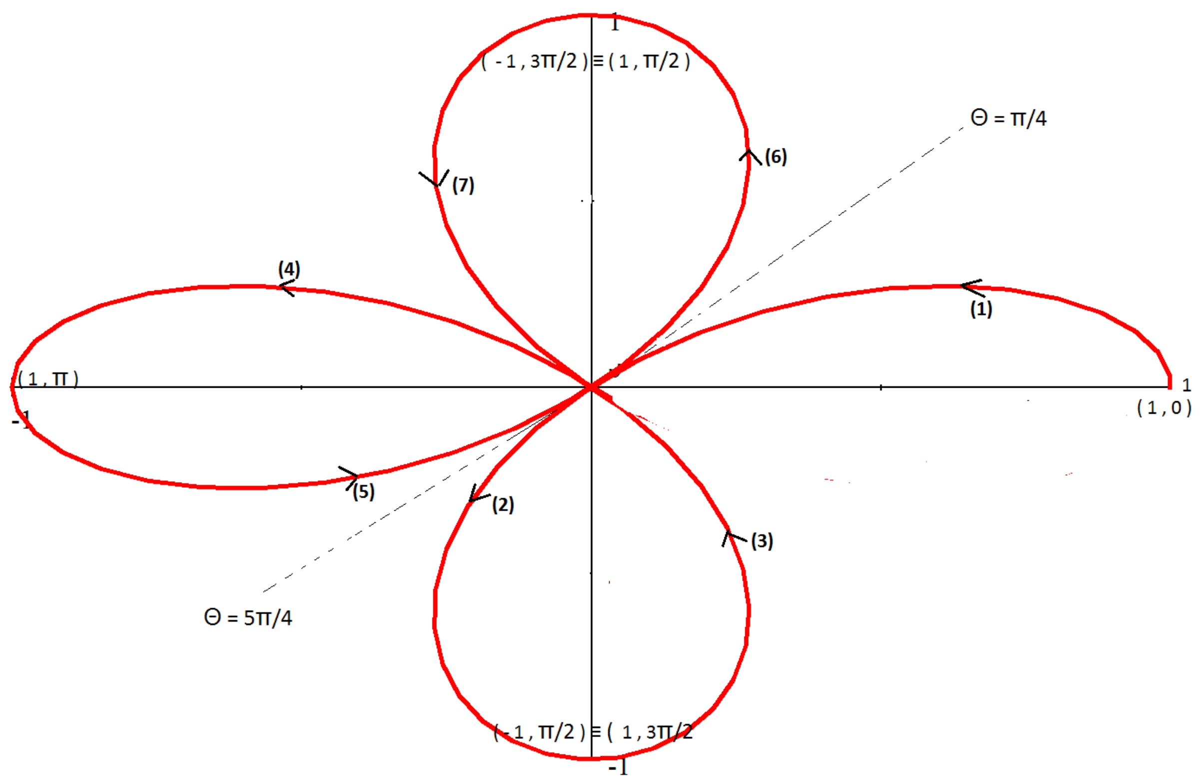


1
(-1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(4)
(1)
(1 , π)
-1
1
(1 , 0)
(2)
(3)
Θ = 5π/4
(-1 , π/2) ≡ (1 , 3π/2
-1

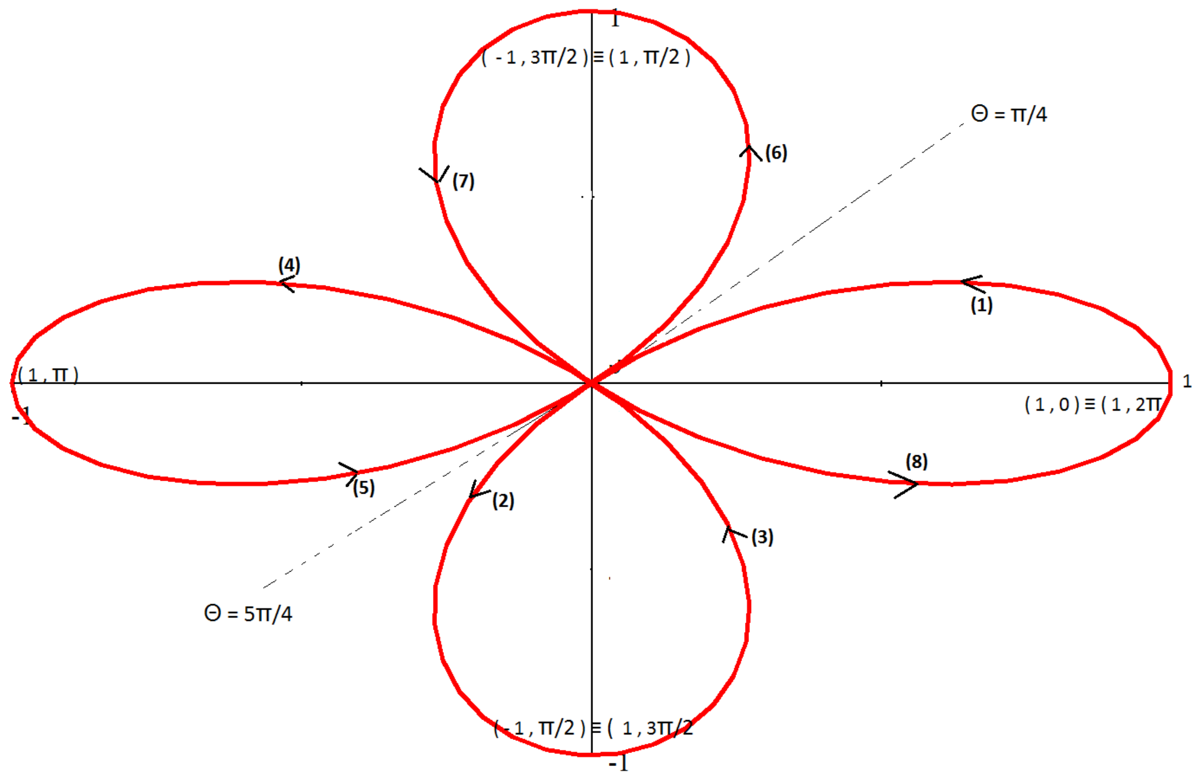

1
(- 1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(4)
(1)
(1 , π)
-1
1
(1 , 0)
(5)
(2)
(3)
Θ = 5π/4
(-1 , π/2) ≡ (1 , 3π/2
-1



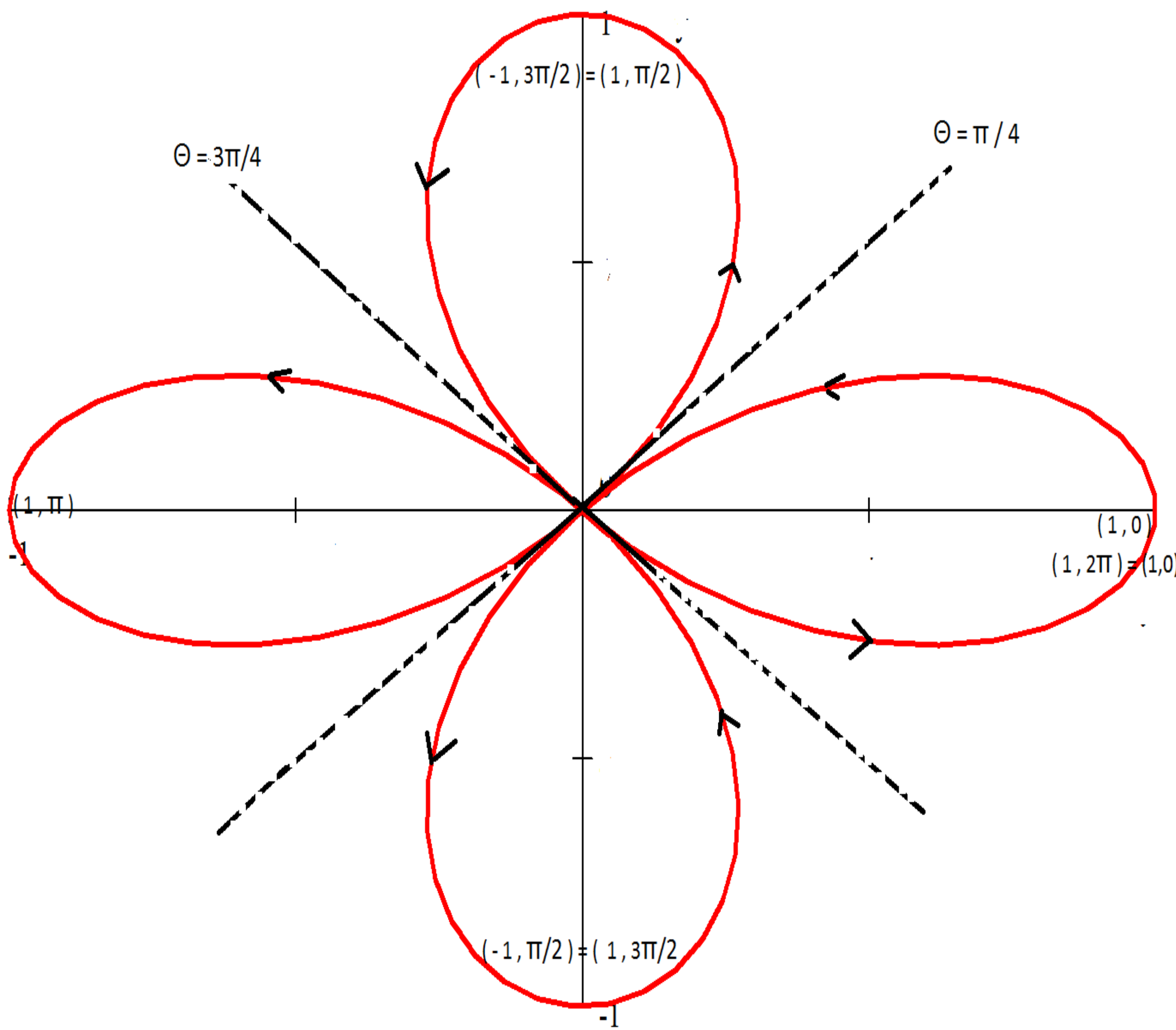
(- 1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(6)
(4)
(1)
(1 , π)
1
(1 , 0)
(5)
(2)
(3)
Θ = 5π/4
(-1 , π/2) ≡ (1 , 3π/2
-1



1
(-1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(6)
(7)
(4)
(1)
(1 , π)
-1
1
(1 , 0)
(5)
(2)
(3)
Θ = 5π/4
(-1 , π/2) ≡ (1 , 3π/2
-1



1
(-1 , 3π/2) ≡ (1 , π/2)
Θ = π/4
(6)
(7)
(4)
(1)
(1 , π)
1
(1 , 0) ≡ (1 , 2π
-1
(8)
(5)
(2)
(3)
Θ = 5π/4
(-1 , π/2) ≡ (1 , 3π/2
-1



1
(-1 , 3π/2) = (1 , π/2)
Θ = π / 4
Θ = 3π/4
(1 , π)
-1
(1 , 0)
(1 , 2π) = (1,0)
(-1 , π/2) = (1 , 3π/2
-1

Example (6)-B

$r = 4\sin 3\theta$

The graph of $r = 4\sin 3\theta$

We find when: $3\theta = n\pi$ and when $3\theta = n(\pi/2)$

$r = 0$ if $\sin 3\theta = 0$

$3\theta = 0, \pi , 2\pi, 3\pi , 4\pi, \ldots\ldots$

$\rightarrow\ \theta =$ **$0, \pi/3 , 2\pi/3, 3\pi/3=\pi , 4\pi/3,$**$\ldots$

$r = 4$, which is maximum if $\sin 3\theta = 1$

$3\theta = \pi/2 , 5\pi/2 , 9\pi/2$

$\rightarrow\ \theta = (\pi/2)/3=$**$\pi/6$,** $(5\pi/2)/2=$ **$5\pi/6$, …..**

$r = - 4$, if $\sin 3\theta = -1$

$3\theta = 3\pi/2 , 7\pi/2 , 11\pi/2,\ldots\ldots\ldots$

$\rightarrow\ \theta = (3\pi/2)/3=$ **$\pi/2$** , $(7\pi/2)/3= 7\pi/6$, $(11\pi/2)/3=11\pi/6$

The graph of $r = 4\sin 3\theta$

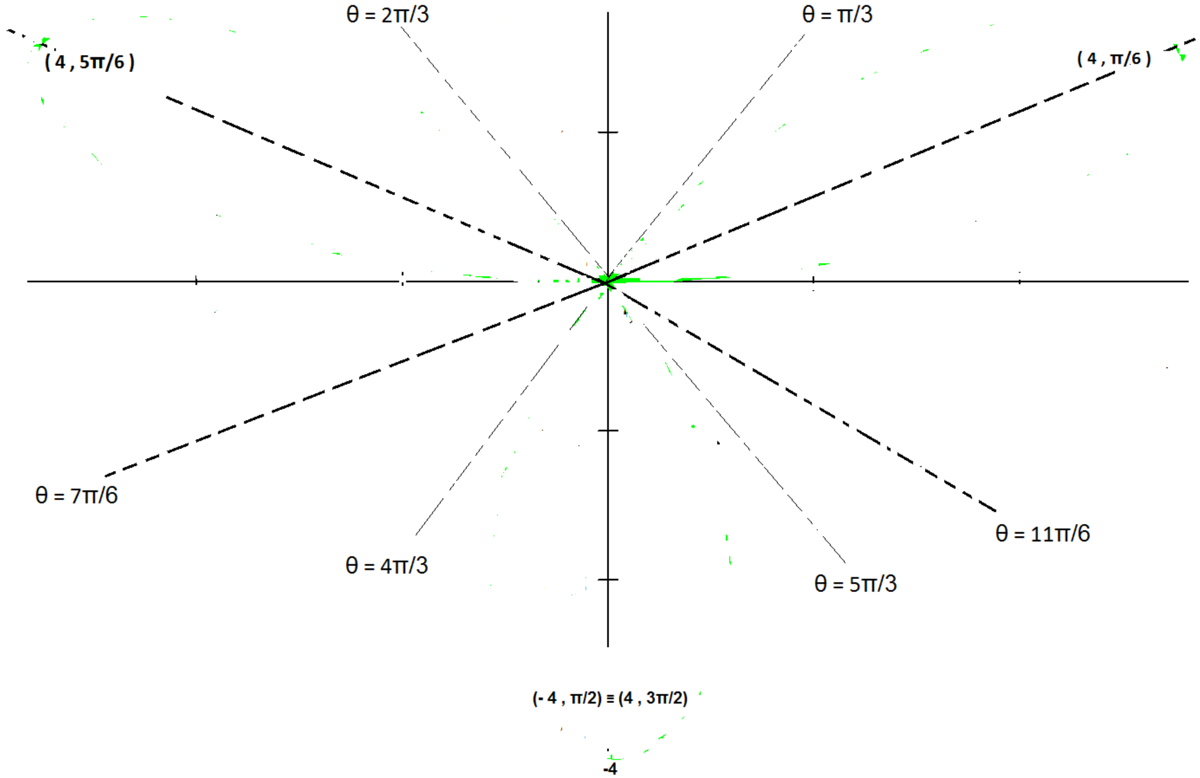
θ	**r =** $4\sin 3\theta$	**(r , θ)**
0	0	(0 , 0)
$\pi/6$	4	$(4, \pi/6)$
$\pi/3$	0	$(0, \pi/3) \equiv (0,0)$
$\pi/2$	- 4	$(-4, \pi/2) \equiv (4, 3\pi/2)$
$2\pi/3$	0	$(0, 2\pi/3) = (0,0)$
$5\pi/6$	4	$(4, 5\pi/6)$
π	**0**	**$(0, \pi) = (0,0)$**

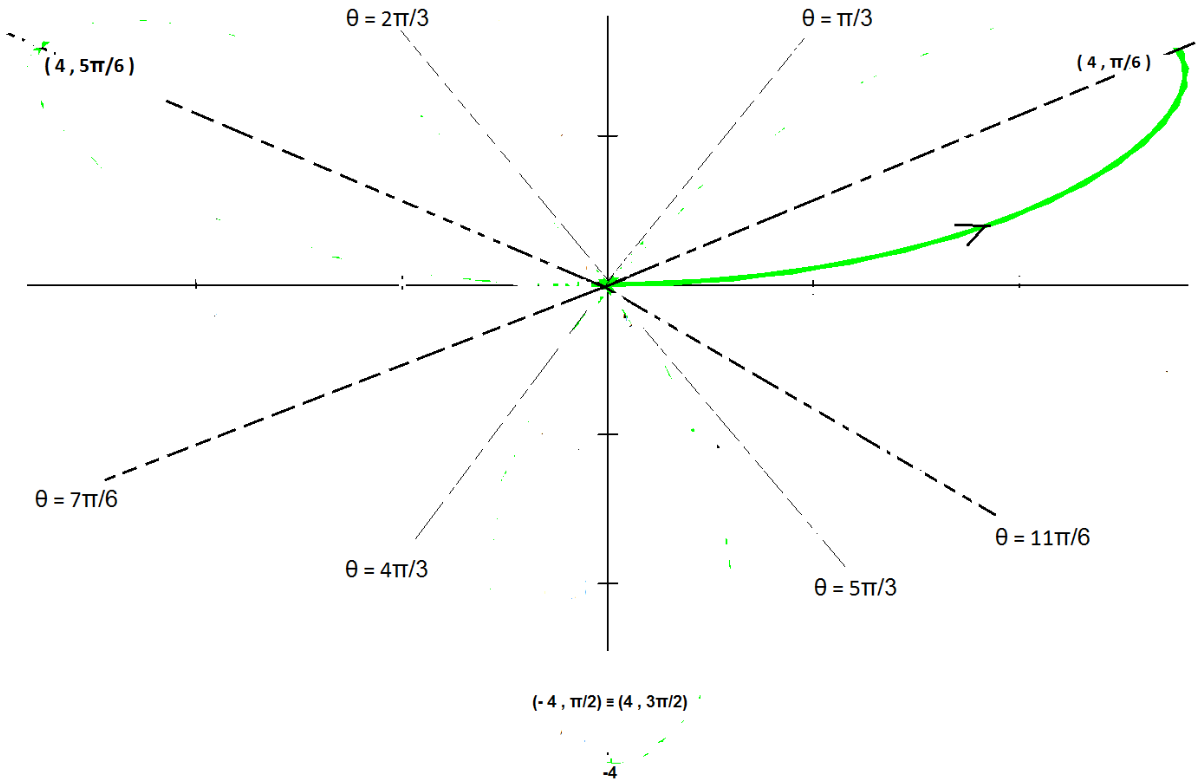
The graph of $r = 4\sin 3\theta$

Values of v grater than **π** will not result in new points

It will just trace the curve again and again

θ	**r =** $4\sin 3\theta$	**(r , θ)**
π	**0**	**(0 , π) = (0,0)**
7π/6	- 4	(- 4 , 7π/6) ≡ (4 , π/6)
4π/3	0	(0, 4π/3) = (0, π/3) = (0,0)
3π/2	4	(4 , 3π/2)
5π/3	0	(0 , 5π/3) ≡ (0, 2π/3) = (0,0)
11π/6	- 4	(- 4 , 11π/6) ≡ (4, 5π/6)


θ = 2π/3
θ = π/3
(4 , 5π/6)
(4 , π/6)
θ = 7π/6
θ = 11π/6
θ = 4π/3
θ = 5π/3
(- 4 , π/2) ≡ (4 , 3π/2)
-4


θ = 2π/3
θ = π/3
(4 , 5π/6)
(4 , π/6)
θ = 7π/6
θ = 11π/6
θ = 4π/3
θ = 5π/3
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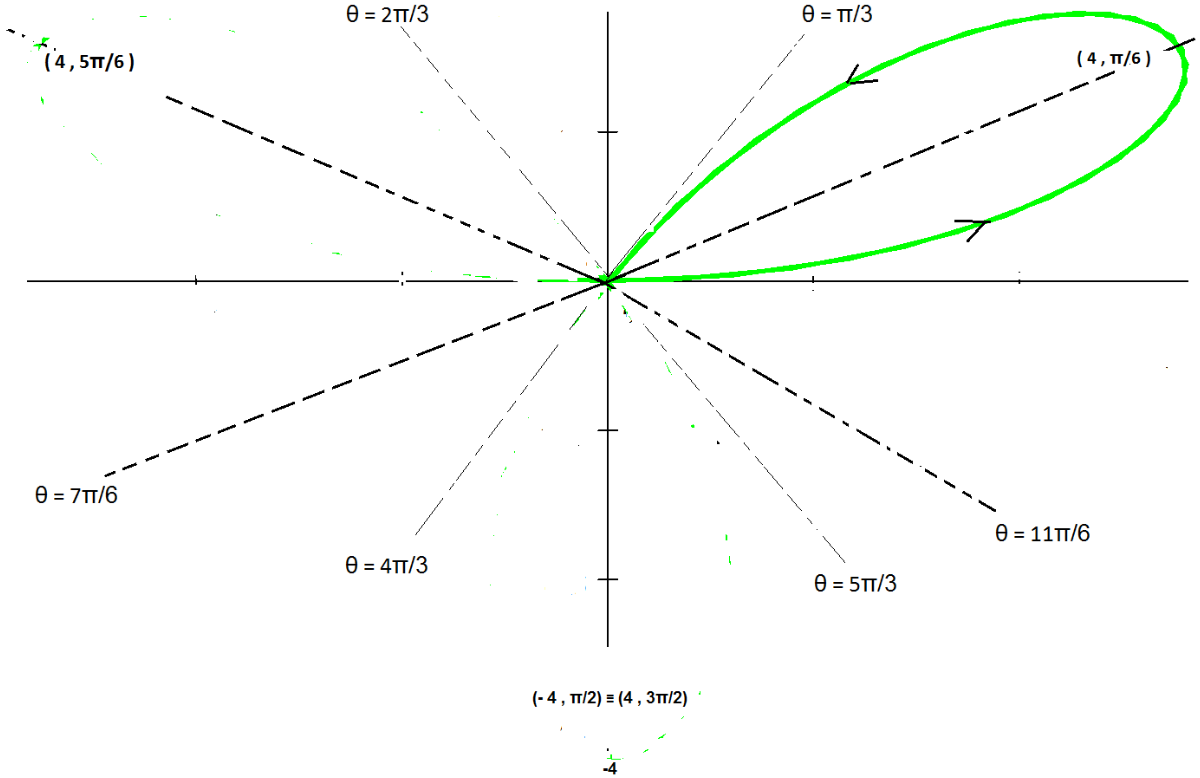
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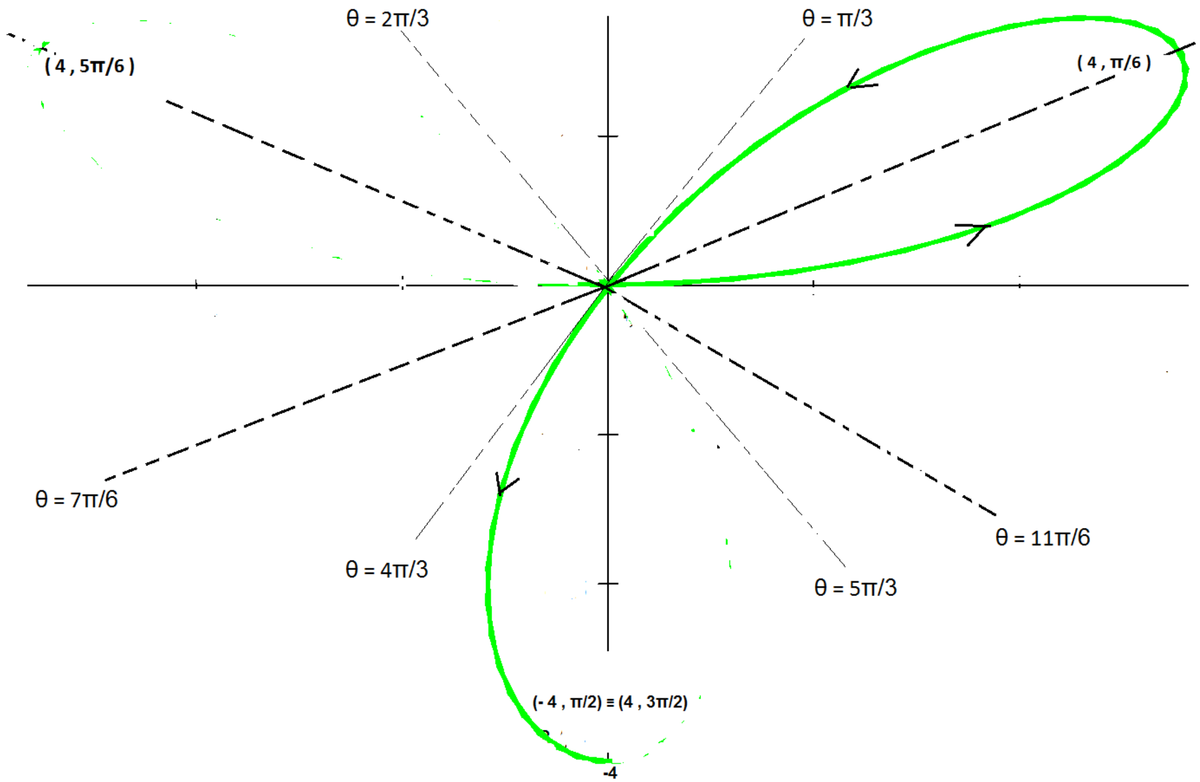
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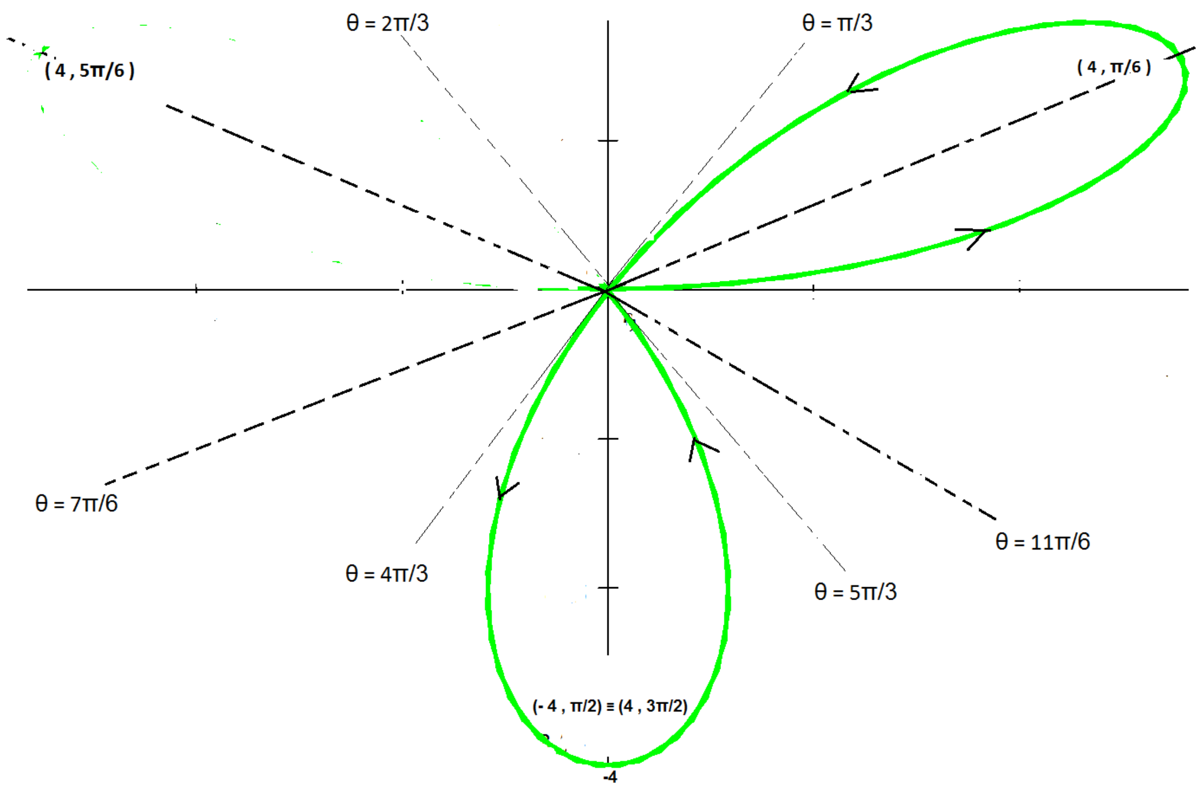
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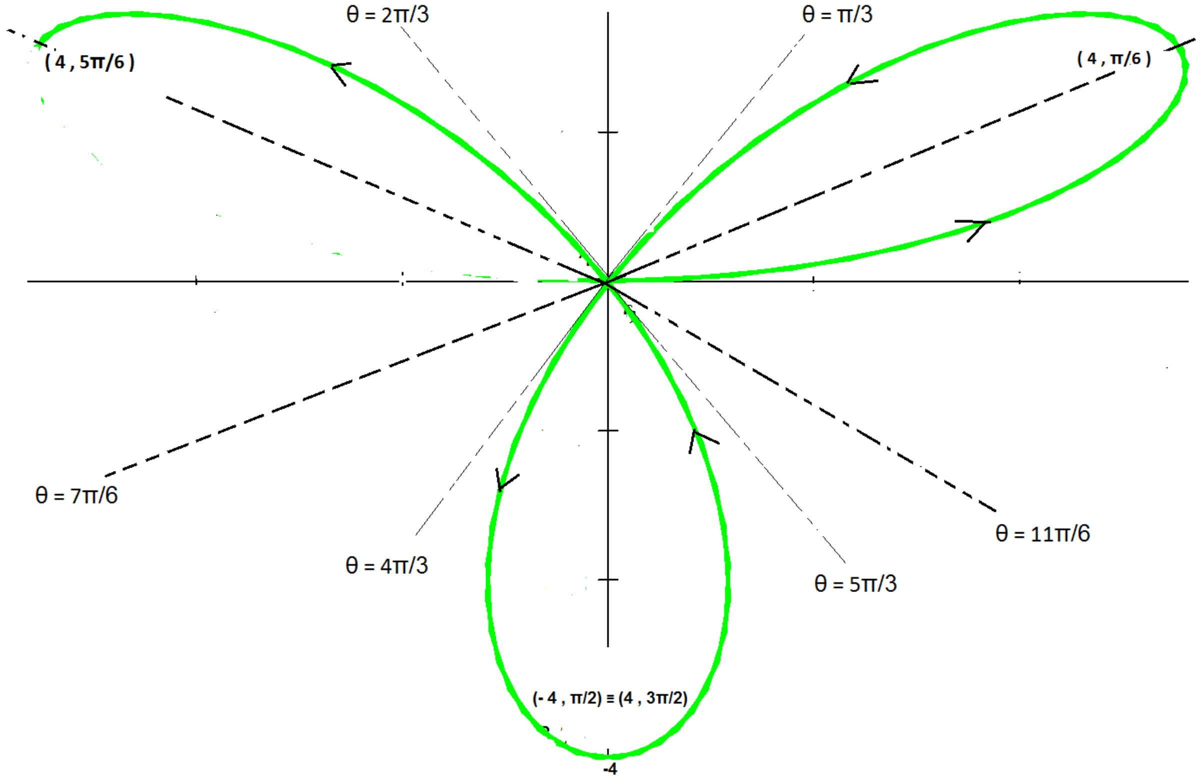
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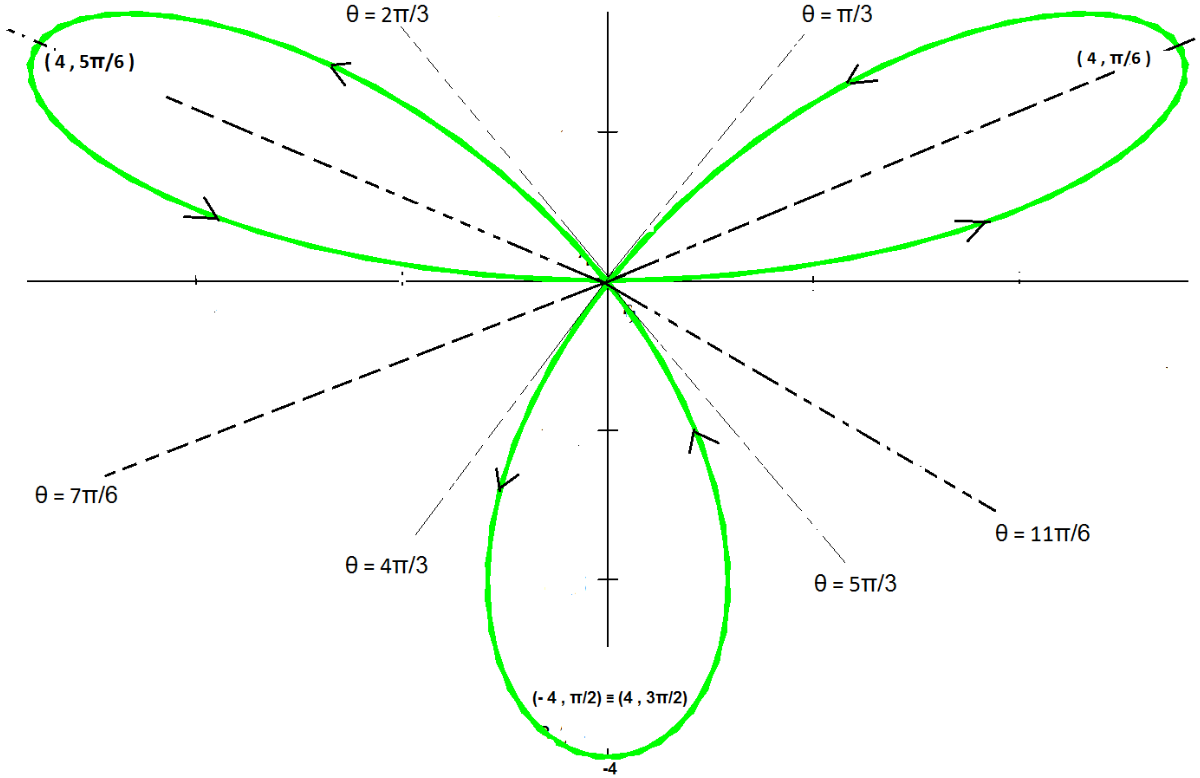

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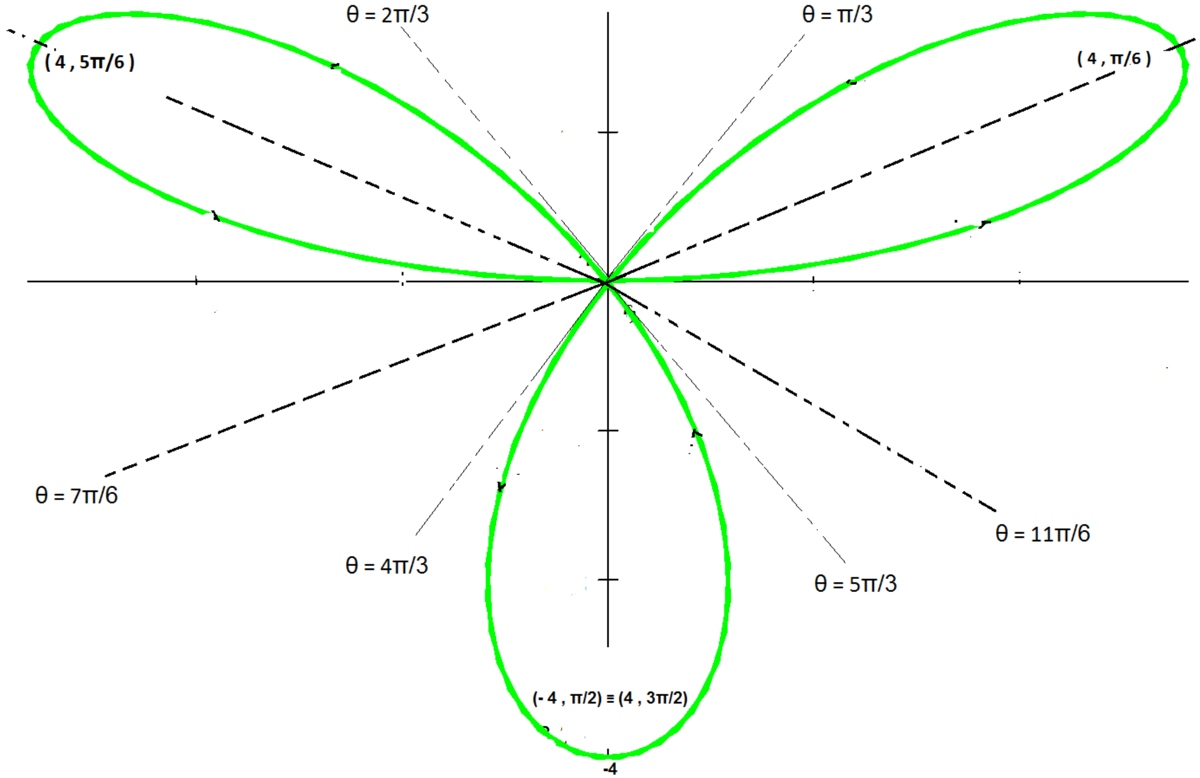

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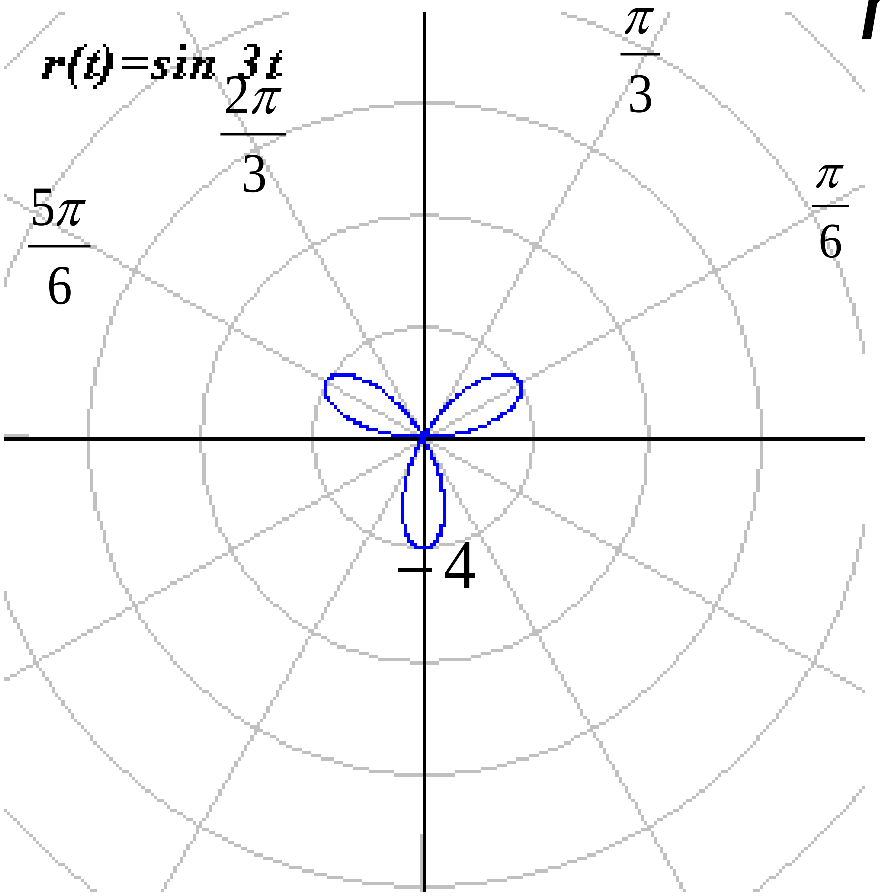


$$r = 4\sin 3\theta$$

Remark

- The graph of:

$r = a \sin n\theta$

Or

$r = a \cos n\theta$

A rose curve consists of:

n leaves, if n is odd

2n leaves, if n is even

Sec 7

Arch Length

Let r = f(θ), and let dr/dθ be continuous on $[\theta_1 , \theta_2]$. Then, the arc length L of the curve from $\theta = \theta_1$ to $\theta = \theta_2$ is:

$$L = \int_{\theta_1}^{\theta_2} \sqrt{f^{2}(\theta) + [f'(\theta)]^{2}}\, d\theta$$

Provided no part of the graph is traced more than once

on the interval $[\theta_1 , \theta_2]$.

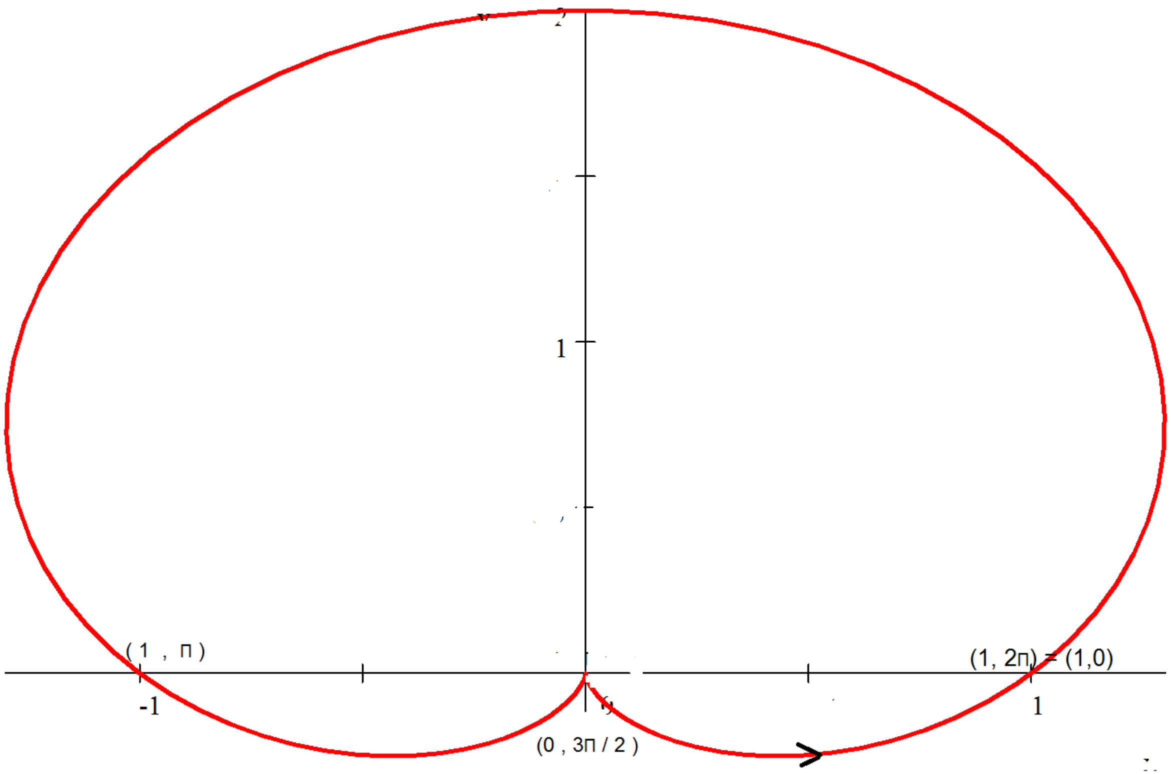
Examples

Find the length of the curve :

- $r = 1 + \sin\theta$
- $r = 2 - 2\cos\theta$
- $r = 2 + 2\cos\theta$

Solutions

r = 1 + sinθ



1. r = 1 +sinθ

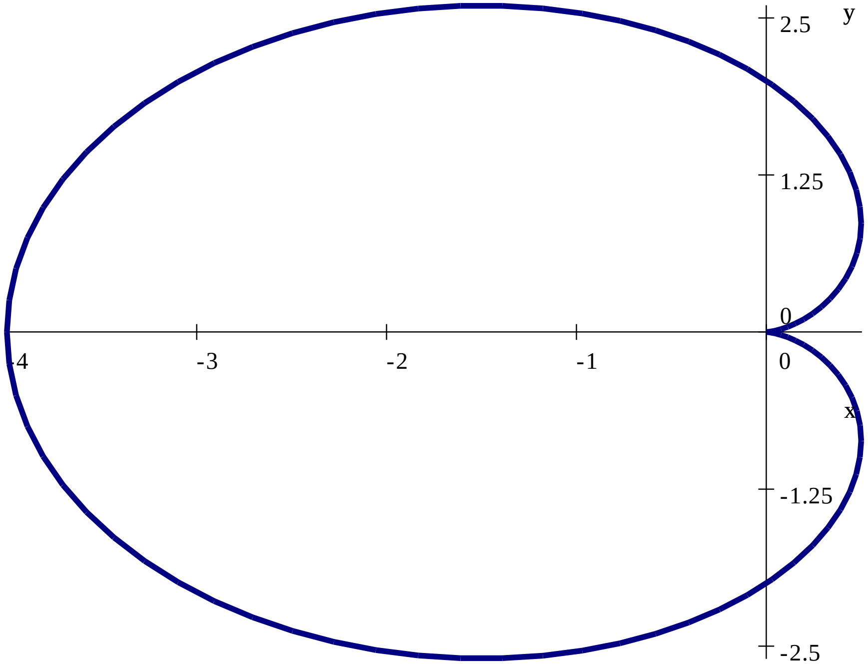
$Let\ L\ be\ the\ arch\ length\ of\ the\ given\ curve$

$$L = \int_0^{2\pi} \sqrt{(1+\sin\theta)^2 + [\cos\theta]^2}\, d\theta$$

$$= L = \int_0^{2\pi} \sqrt{2+2\sin\theta}\ d\theta$$

$$= 8$$

$r = 2 - 2\cos\theta$



1. r = 2 – 2cosθ

Let L be the arch length of the given curve

$$L = \int_0^{2\pi} \sqrt{(2-2\cos\theta)^2 + [2\sin\theta]^2}\, d\theta$$

$$= 2\int_0^{2\pi} [1 - 2\cos\theta + \cos^2\theta + \sin^2\theta]^{\frac{1}{2}}\, d\theta$$

$$= 2\int_0^{2\pi} [2 - 2\cos\theta]^{\frac{1}{2}}\, d\theta$$

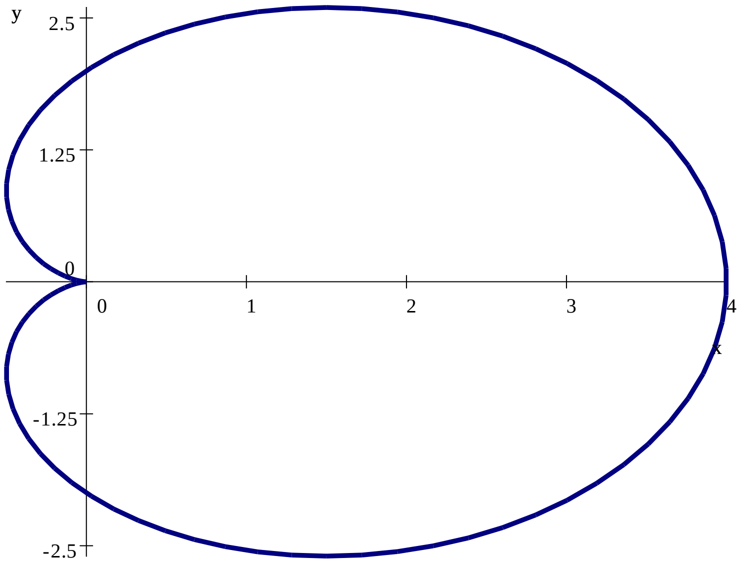
$$= 2\sqrt{2}\int_0^{2\pi} [1 - \cos\theta]^{\frac{1}{2}}\, d\theta$$

$$= 2\sqrt{2}\int_0^{2\pi} \sqrt{2\sin^2\tfrac{\theta}{2}}\, d\theta = 4\int_0^{2\pi} \left|\sin\tfrac{\theta}{2}\right| d\theta$$

$$= 4\int_0^{2\pi} \sin\tfrac{\theta}{2}\, d\theta \ldots\cdots\cdots\cdots\ldots Why?$$

$$= 4[(-2\cos\tfrac{\theta}{2})_0^{2\pi}] = -8[-1-1] = 16$$

2. $r = 2 + 2\cos\theta$



2. r = 2 + 2cosθ

Let L be the arch length of the given curve

$$L = \int_0^{2\pi} \sqrt{(2+2\cos\theta)^2 + [-2\sin\theta]^2}\, d\theta$$

$$= 2\int_0^{2\pi} [1+2\cos\theta + \cos^2\theta + \sin^2\theta]^{\frac{1}{2}}\, d\theta$$

$$= 2\int_0^{2\pi} [2+2\cos\theta]^{\frac{1}{2}}\, d\theta$$

$$= 2\sqrt{2}\int_0^{2\pi} [1+\cos\theta]^{\frac{1}{2}}\, d\theta$$

$$= 2\sqrt{2}\int_0^{2\pi} \sqrt{2\cos^2\tfrac{\theta}{2}}\, d\theta = 4\int_0^{2\pi} \left|\cos\tfrac{\theta}{2}\right| d\theta$$

$$= 4\left[\int_0^{\pi} \cos\tfrac{\theta}{2}\, d\theta + \int_{\pi}^{2\pi} (-\cos\tfrac{\theta}{2})\, d\theta\right] \ldots. Why?$$

$$= 4\left[(2\sin\tfrac{\theta}{2})_0^{\pi} - (2\sin\tfrac{\theta}{2})_{\pi}^{2\pi}\right]$$

$$= 8\,[(1-0)-(0-1)]$$

$$= 8(2) = 16$$

Another Method

From tthe graph, by symmetry, we have,

$$L = 2\int_0^{\pi} \sqrt{(2+2\cos\theta)^2 + [-2\sin\theta]^2}\, d\theta$$

$$= 4\int_0^{\pi} [1+2\cos\theta + \cos^2\theta + \sin^2\theta]^{\frac{1}{2}}\, d\theta$$

$$= 4\int_0^{\pi} [2+2\cos\theta]^{\frac{1}{2}}\, d\theta$$

$$= 4\sqrt{2}\int_0^{\pi} [1+\cos\theta]^{\frac{1}{2}}\, d\theta$$

$$= 4\sqrt{2}\int_0^{\pi} \sqrt{2\cos^2\tfrac{\theta}{2}}\, d\theta = 8\int_0^{\pi} \left|\cos\tfrac{\theta}{2}\right| d\theta$$

$$= 8\int_0^{\pi} \cos\tfrac{\theta}{2}\, d\theta \cdots\cdots\cdots\cdots\cdots ...Why?$$

$$= 8\,(2\sin\tfrac{\theta}{2})_0^{\pi}$$

$$= 16\,[1-0]$$

$$= 16$$

Sec 8

Area

- Let $r = f(\theta)$ and $0 < \theta_2 - \theta_1 \leq 2\pi$
- Let $r = f(\theta)$ be continuous and either

$f(\theta) \geq 0$ or $f(\theta) \leq 0$ on $[\theta_1 , \theta_2]$

- Then the area A of the region enclosed by

the curve $r = f(\theta)$ and the lines $\theta = \theta_1$

and $\theta = \theta_2$ is:

$$\int_{\theta_1}^{\theta_2} \tfrac{1}{2} f^2(\theta)\,d\theta$$

Examples

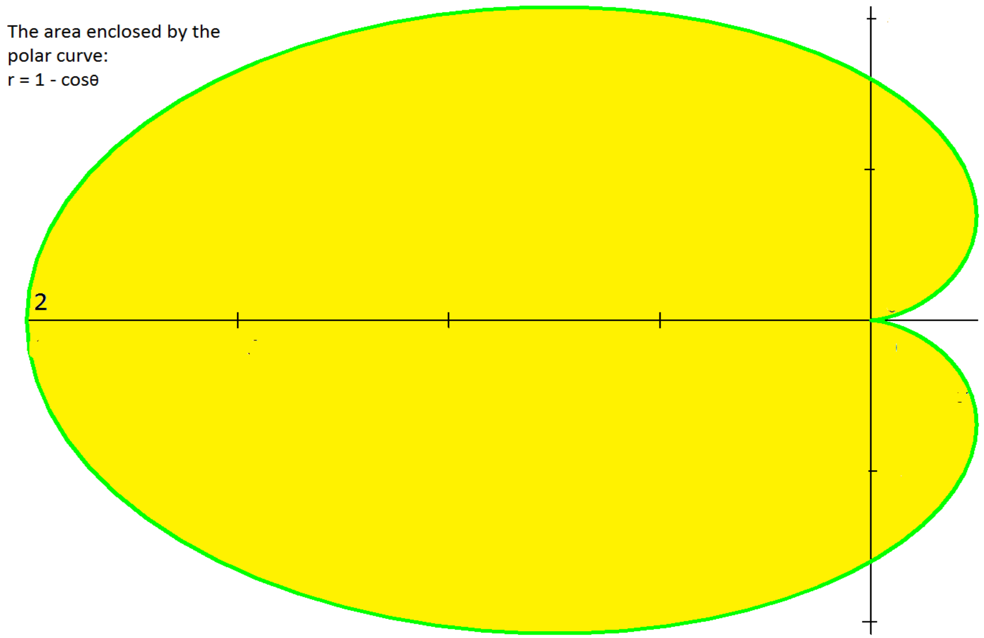
Find the area enclosed by :

- The curve $r = 1 - \cos\theta$
- The area enclosed by the curve:
- The curve $r = \cos 2\theta$
- The curve $r = 4 + 4\cos\theta$, but outside the circle $r=6$
- The curve $r = 1 - \cos\theta$, the positive x-axis, the positive y-axis
- The curve $r = 3\cos\theta$, but outside $r= 1 + \sin\theta$ *

Solutions

Example (1)

The area enclosed by r = 1 – cosθ



The required area A is the area enclosed by the given curve and the lines $\theta = 0$ *and* $\theta = 2\pi$

and so,

$$A = \int_0^{2\pi} \tfrac{1}{2}(1-\cos\theta)^2\, d\theta = \frac{3\pi}{2}$$

See details on next slide

$$A=\int_0^{2\pi}\frac{1}{2}(1-\cos\theta)^2\,d\theta$$

$$=\int_0^{2\pi}\frac{1}{2}(1-2\cos\theta+\cos^2\theta)d\theta$$

$$=\int_0^{2\pi}\frac{1}{2}\left(1-2\cos\theta+\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$=\int_0^{2\pi}\left(\frac{1}{2}-\cos\theta+\frac{1}{4}+\frac{1}{4}\cos 2\theta\right)d\theta$$

$$=\int_0^{2\pi}(\frac{3}{4}-\cos\theta+\frac{1}{4}\cos 2\theta)d\theta$$

(*Notice that* $\int_0^{2\pi}\frac{1}{4}\cos 2\theta\,d\theta=\frac{1}{4}\,\frac{1}{2}\int_0^{2\pi}cos2\theta\;2d\theta$)

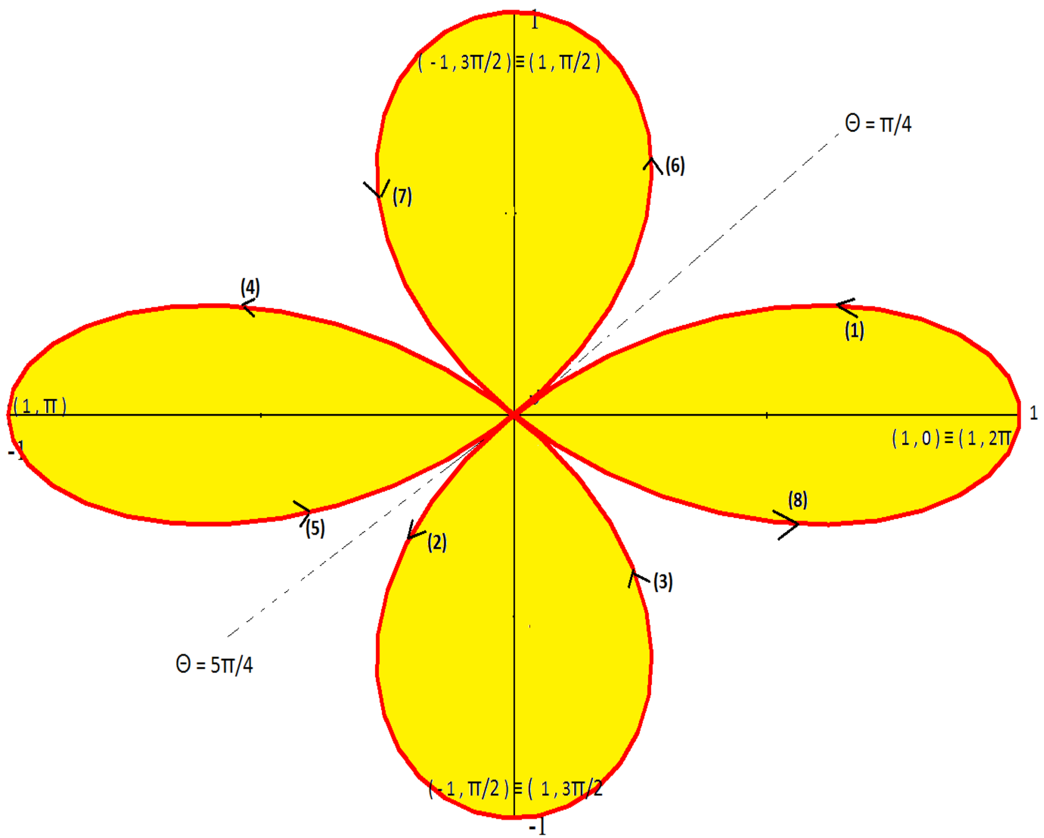
$$=\left(\frac{3}{4}\theta-\sin\theta+\frac{1}{8}\sin 2\theta\right)\Bigg|_0^{2\pi}$$

$$=\left(\frac{3}{4}(2\pi)-\sin 2\pi+\frac{1}{8}\sin 2(2\pi)\right)-\left(\frac{3}{4}(0)-\sin(0)+\frac{1}{8}\sin 2(0)\right)$$

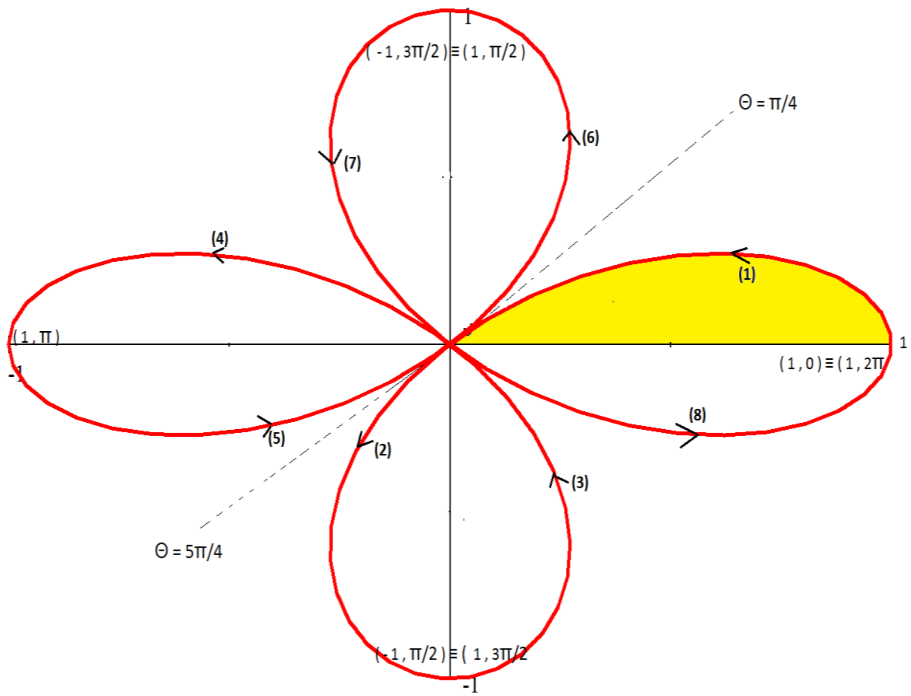
$$=\left(\frac{3\pi}{2}-0+0\right)-)+(0+0+0)=\frac{3\pi}{2}$$

Example (2)

The area enclosed by the rose curve r = cos2θ



3. The area enclosed by the rose curve $r = \cos 2\theta$ is eight times the area enclosed by a half leave



By symmetry,

the required area A is eight times the area enclosed by the given curve

and the lines $\theta = 0$ *and* $\theta = \frac{\pi}{4}$

and so,

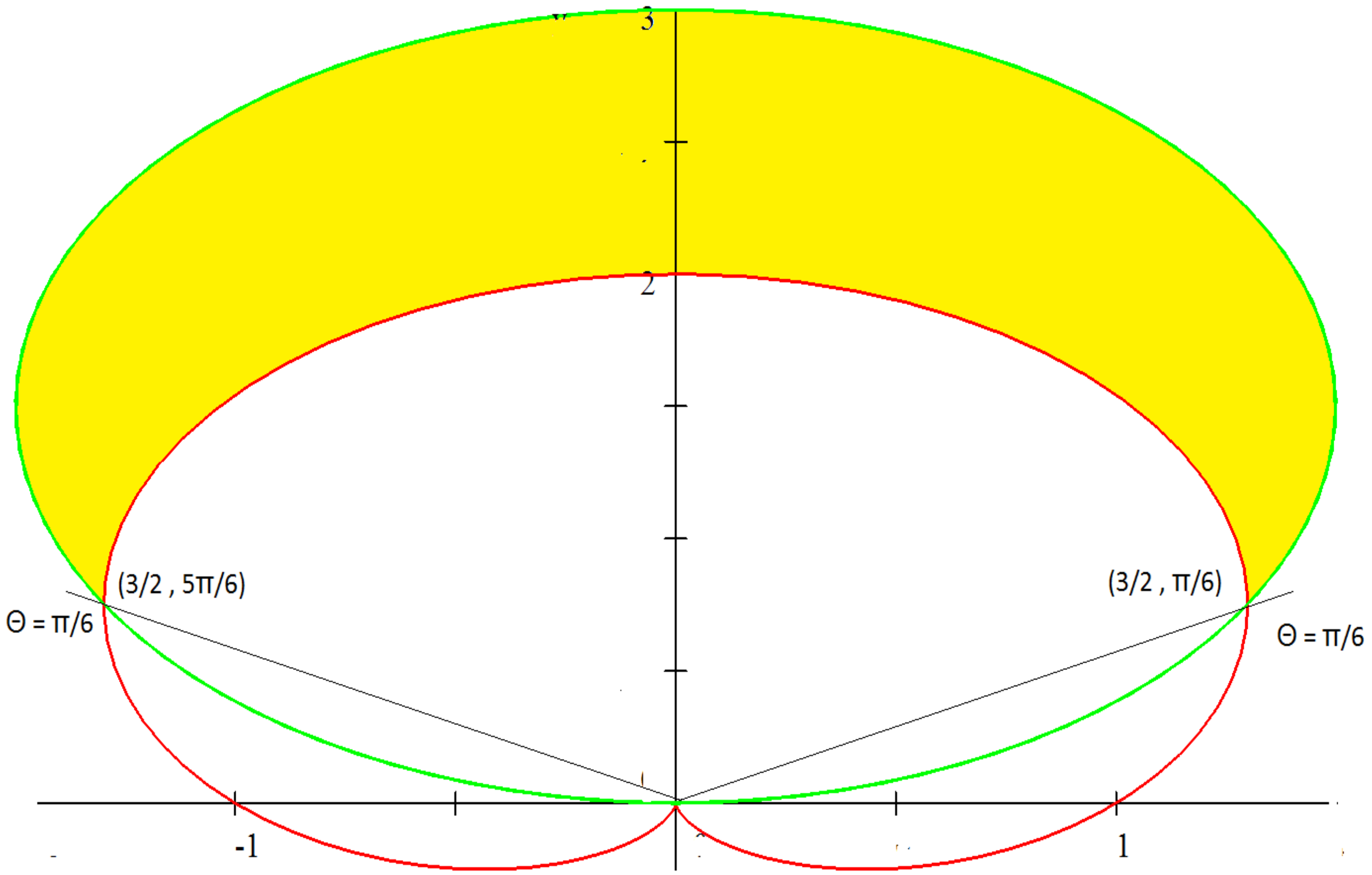
$$A = 8\int_0^{\frac{\pi}{4}} \tfrac{1}{2}\cos^2 2\theta \, d\theta$$

$$= 4\int_0^{\frac{\pi}{4}} \tfrac{1}{2}(1+\cos 4\theta) \, d\theta$$

$$= 2\left(\theta + \frac{\sin 4\theta}{4}\right)$$

$$= 2\left[\left(\frac{\pi}{4}+0\right) - (0-0)\right] = \frac{\pi}{2}$$

Example (3) The area of the region inside r = 3sinθ, but outside r= 1 + sinθ



- The area A is the difference of the area A_1 enclosed by r = 3sinθ and the lines θ=π/6 and θ =5π/6 and the area A_2 enclosed by r = 1 + sinθ and the same lines.
- How do we know that? Solve: 3sinθ= 1 + sinθ
- Thus,

$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \tfrac{1}{2}(3\sin\theta)^2\,d\theta - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \tfrac{1}{2}(1+\sin\theta)^2\,d\theta$$

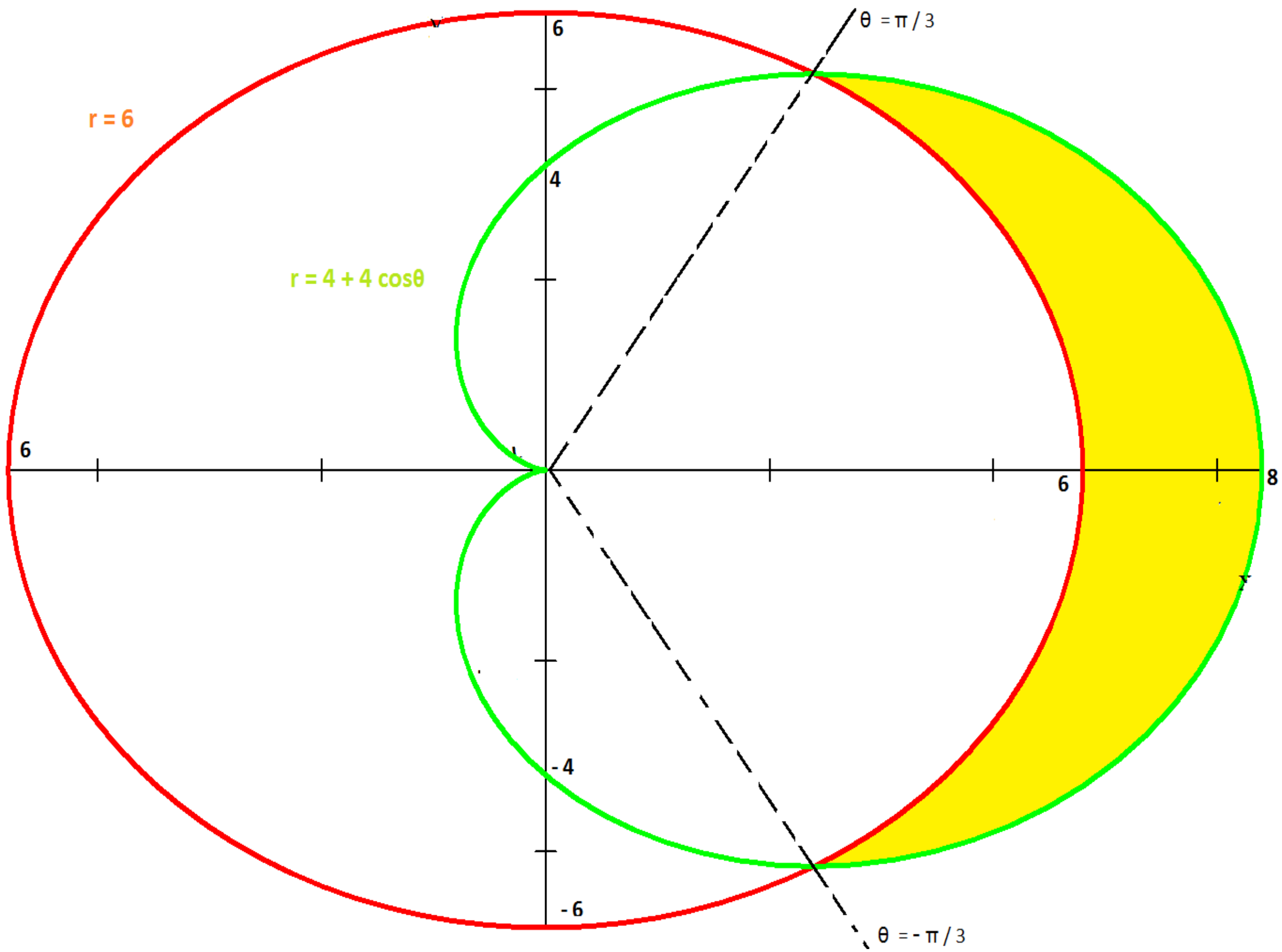
$$= \pi \quad Why?$$

See the details on page 687 (*Example* 2) *of the text book*

Question

The area enclosed by r = 4 +4 cosθ,

but outside the circle r = 6



Solution

- The area A is the difference of the area A_1 enclosed by r = 4 + 4cosθ and the lines θ=-π/3 and θ =π/3 and the area A_2 enclosed by r = 6 and the same lines.
- How do we know that? Solve: 6 = 4 + 4cosθ
- Thus,

$$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tfrac{1}{2}(4 + 4\cos\theta)^2 \, d\theta - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tfrac{1}{2}(6)^2 \, d\theta$$