TORQUE AND ANGULAR ACCELERATION

$\Sigma \boldsymbol{F} \Rightarrow a$

$\Sigma \boldsymbol{\tau} \Rightarrow \alpha$

$$\Sigma \boldsymbol{F} = ma$$

$$\Sigma \boldsymbol{\tau} = I\alpha$$

Two forms of Newton's second law

$$K = \frac{1}{2} m v^2$$

$$K_R = \frac{1}{2} I \omega^2$$

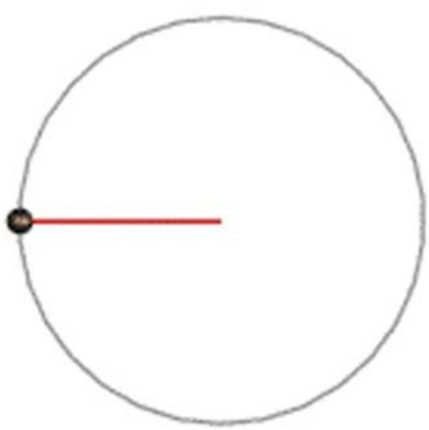
$\Delta s \Leftrightarrow \Delta\theta$

$v \Leftrightarrow \omega$

$a \Leftrightarrow \alpha$

$t \Leftrightarrow t$

$m \Leftrightarrow I$



10-8. Work ,Power and Energy in Rotational Motion

Let's consider a motion of a rigid body with a single external force **F** exerting on the point P, moving the object by *ds*.

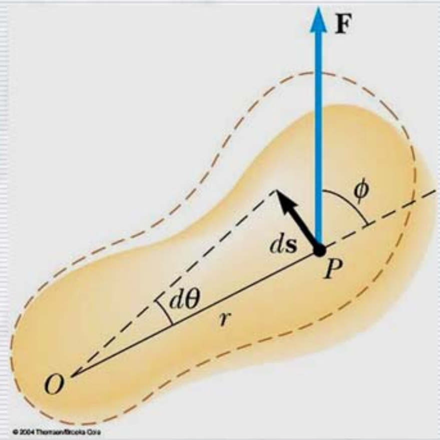
The work done by the force **F** as the object rotates through the infinitesimal distance $ds = rd\theta$ is

$$dW = \vec{F} \cdot d\vec{s} = (F \sin\phi) r d\theta$$

$$dW = (rF\sin\phi) d\theta = \tau d\theta$$

$$P = \frac{dW}{dt} = \frac{\tau d\theta}{dt} = \tau\omega$$

$$\boxed{P = \tau\omega}$$



$$\sum \tau = I\alpha = I\left(\frac{d\omega}{dt}\right) = I\left(\frac{d\omega}{d\theta}\right)\left(\frac{d\theta}{dt}\right) = I\omega\left(\frac{d\omega}{d\theta}\right)$$

$$dW = \sum \tau d\theta = I\omega d\omega$$

$$W = \int_{\theta_i}^{\theta_f} \sum \tau d\theta = \int_{\omega_i}^{\omega_f} I\omega d\omega = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2$$

The rotational work done by an external force equals the change in rotational energy.

$$\boxed{W = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2}$$

Table 10.3

Useful Equations in Rotational and Linear Motion

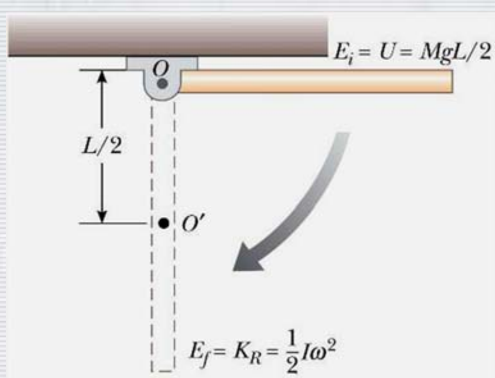
Rotational Motion About a Fixed Axis	Linear Motion
Angular speed $\omega = d\theta/dt$	Linear speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Linear acceleration $a = dv/dt$
Net torque $\Sigma\tau = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha_t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$	Work $W = \int_{x_i}^{x_f} F_x \, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

example 10.14

A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end (Fig 10.24). The rod is released from rest in the horizontal position.

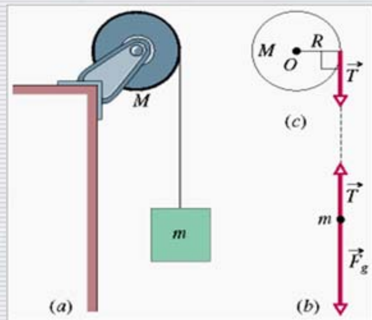
(A) What is its angular speed when it reaches its lowest position?

READ EXAMPLE 10.14 & try to solve



Sample Problem

Figure 11-17*a* shows a uniform disk, with mass M = 2.5 kg and radius R = 20 cm, mounted on a fixed horizontal axle. A block with mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.



SOLUTION:

$$TR = I\alpha$$ Both clockwise

$$T = \frac{I\alpha}{R}$$

$$a = \alpha R$$

Therefore,

$$T = \frac{Ia}{R^2} = \frac{(\frac{1}{2}MR^2)a}{R^2} = \frac{1}{2}Ma^2$$ Positive a is down.

$$mg - T = ma$$ Positive direction is down.

$$mg = ma + T = ma + \frac{1}{2}Ma = (2m + M)\frac{a}{2}$$

$$a = g\frac{2m}{M+2m} = (9.8\,\mathrm{m/s^2})\frac{(2)(1.2\,\mathrm{kg})}{2.5\,\mathrm{kg}+(2)(1.2\,\mathrm{kg})}$$
$$= 4.8\,\mathrm{m/s^2}$$

- Positive direction of a is down.

$$T = \tfrac{1}{2}Ma = \tfrac{1}{2}(2.\,\mathrm{kg})(4.8\,\mathrm{m/s^2})$$
$$= 6.0\,\mathrm{N}$$

$$\alpha = \frac{a}{R} = \frac{4.8\,\mathrm{m/s^2}}{0.20\mathrm{m}} = 24\,\mathrm{rad/s^2}$$

- The angular acceleration is clockwise.

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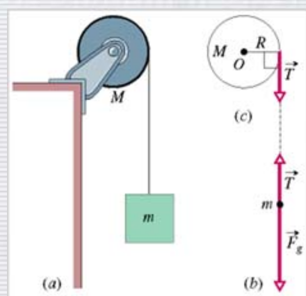
Sample Problem

Let the disk in the previous Sample Problem start from rest at time $t = 0$. What is its rotational kinetic energy K at $t = 2.5$ s?



$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t$$

Note that alpha is clockwise.

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(\tfrac{1}{2}MR^2)(\alpha t)^2 = \tfrac{1}{4}M(R\alpha t)^2$$
$$= \tfrac{1}{4}(2.5\,\mathrm{kg})[(0.20\,\mathrm{m})(24\,\mathrm{rad/s^2})(2.5\,\mathrm{s})]^2$$
$$= 90\,\mathrm{J}$$

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An alternative way :

$$K = K_i + W = 0 + W = W$$

$$W = \tau\,(\theta_f - \theta_i) = T\,R\,(\theta_f - \theta_i)$$

Note that the torque is clockwise

$$\theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\alpha t^2 \quad \text{Clockwise}$$

$$K = W = T\,R\,(\theta_f - \theta_i) = TR(\tfrac{1}{2}\alpha t^2) = \tfrac{1}{2} T R \alpha t^2$$

$$= \tfrac{1}{2}(6.0\ \text{N})(0.20\ \text{m})(24\ \text{rad/s}^2)(2.5\ \text{s})^2$$

$$= 90\ \text{J}$$

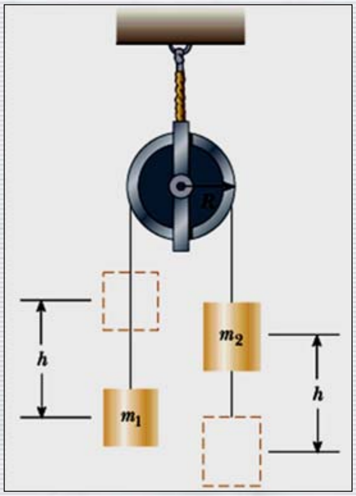
example 10.15

Two masses m_1 (5 kg) and m_2 (10 kg) are hanging from a pulley of mass M (3 kg) and radius R (0.1 m), as shown. There is no slip between the rope and the pulleys.

(a) What will happen when the masses are released?

(b) Find the velocity of the masses after they have fallen a distance of 0.5 m.

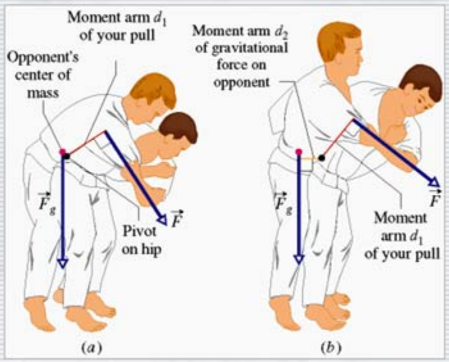
(c) What is the angular velocity of the pulley at that moment?



READ EXAMPLE 10.15 & try to solve

Sample Problem

To throw an 80 kg opponent with a basic judo hip throw, you intend to pull his uniform with a force $\vec{F}$ and a moment arm d_1 = 0.30 m from a pivot point (rotation axis) on your right hip (see Fig.). You wish to rotate him about the pivot point with an angular acceleration α of - 6.0 rad/s^2 - that is, with an angular acceleration that is *clockwise* in the figure. Assume that his rotational inertia *I* relative to the pivot point is 15 kg·m^2.



(a) What must the magnitude of $\vec{F}$ be if, before you throw him, you bend your opponent forward to bring his center of mass to your hip?

SOLUTION:

$$F = \frac{I\alpha}{d_1} = \frac{(15\ \mathrm{kg \cdot m^2})(6.0\ \mathrm{rad/s^2})}{0.30\ \mathrm{m}} = 300\ \mathrm{N}$$

- The angular acceleration and torque are both clockwise.

(b) What must the magnitude of $\vec{F}$ be if your opponent remains upright before you throw him, so that $\vec{F}_g$ has a moment arm *d*2 = 0.12 m from the pivot point?

$$-d_1F + d_2mg = -I\alpha$$

The clockwise torque is negative.
Note that alpha is + 6.0 rad/s ^{2}.

$$F = \frac{I\alpha}{d_1} + \frac{d_2mg}{d_1} = 300\ \mathrm{N} + \frac{(0.12\ \mathrm{m})(80\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{0.30\ \mathrm{m}} = 613.6\ \mathrm{N} \approx 610\ \mathrm{N}$$

To minimize the required force, you should bend your opponent to bring his center of mass to your hip.

Problem: Two bicycles roll down a hill that is 20m high. Both bicycles have a total mass of 12kg and 700 mm diameter wheels (r =0.350 m). The first bicycle has wheels mass of 0.6 kg each, and the second bicycle has wheels mass of 0.3 kg each. Neglecting air resistance, which bicycle has the faster speed at the bottom of the hill? (Consider the wheels to be thin hoops).

The only friction is static friction, so there are no non-conservative forces. (Static friction involves no motion and since work is defined as *W = Fd*, when there is no distance involved, there is no work/energy used).

Mechanical Energy is Conserved, so:

$\Sigma E_i = \Sigma E_f$

$KE_i + PE_i = KE_f + PE_f$

$(1/2)mv_i^2 + (1/2)I\omega_i^2 + mgh_i = (1/2)mv_f^2 + 2*(1/2)I\omega_f^2 + mgh_f$

$mgh_i = (1/2)mv_f^2 + 2(1/2)I\omega_f^2$ **(The two for two wheels).**

Now w is given by $\omega = v/r$ where v is the velocity of the rim of the wheel and is the same as the velocity of the bike. I is given by Mr^2 where M is the mass of the tire.

$mgh = (1/2)mv_f^2 + Mr^2v_f^2/r^2 = (1/2)mv_f^2 + Mv_f^2$

$$v_f = \sqrt{\frac{mgh}{\frac{1}{2}m + M}} \Rightarrow v_f = \sqrt{\frac{(12kg)(9.8\,m/s^2)(20m)}{\frac{1}{2}(12kg) + 0.3kg}}$$

So for the first bike $v_f = 19.3$ m/s

For the second bike, the wheels have a mass of 0.6 kg, and we get

$v_f = 18.9$ m/s

Why is this true? More of the potential energy goes into rotational kinetic energy and less into translational kinetic energy when the wheel has more mass.

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• ضرورة احضار البطاقة الجامعية وإلا سيمنع الطالب من حضور الامتحان.

شاكرين حسن تعاونكم ،،،