2.5 FINDING THE MOTION OF AN OBJECT

Derivation of equations:

All equations that will be derived, are used to describe ONLY the motions with constant acceleration.

$$\bar{v} = \frac{x - x_o}{t} \qquad (1)$$

$$\bar{v} = \frac{v + v_o}{2} \qquad (2)$$

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{v - v_o}{t} \qquad (3)$$

From (3), we will get

$$a = \frac{v - v_o}{t}$$

$$\boxed{v = v_o + at \qquad (4)}$$

From (1),we will get

$$\bar{v} = \frac{x - x_o}{t} \Rightarrow x = x_o + \bar{v}t$$

Then, substitute the average velocity (above) with that from (2)

$$\Rightarrow \quad x = x_o + \left(\frac{v + v_o}{2}\right)t$$

$$\boxed{x - x_o = \frac{1}{2}(v + v_o)t \qquad (5)}$$

From (5), substitute the v with that from (4)

$$\Rightarrow \quad x = x_o + \left(\frac{(v_o + at) + v_o}{2}\right)t$$

$$\boxed{x - x_o = v_o t + \frac{1}{2}at^2 \qquad (6)}$$

From (5), substitute the t with that from (4)

$$\Rightarrow \quad x = x_o + \frac{1}{2}(v + v_o)\left(\frac{v - v_o}{a}\right)$$

$$\Rightarrow \quad x - x_o = \frac{1}{2a}(v + v_o)(v - v_o)$$

$$\Rightarrow \quad v^2 - v_o^2 = 2a(x - x_o)$$

$$\boxed{v^2 = v_o^2 + 2a(x - x_o) \qquad (7)}$$

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Summary; Equations of motion with constant acceleration

$$(x, v, a, t)$$

$v = v_o + at$	(v, a, t)	without the "x"
$x - x_o = v_o t + \frac{1}{2}at^2$	(x, a, t)	without the "v"
$x - x_o = \frac{1}{2}(v + v_o)t$	(x, v, t)	without the "a"
$v^2 = v_o^2 + 2a(x - x_o)$	(x, v, a)	without the "t"

Sign convention for quantity "x", "v" and "a" is VERY IMPORTANT!

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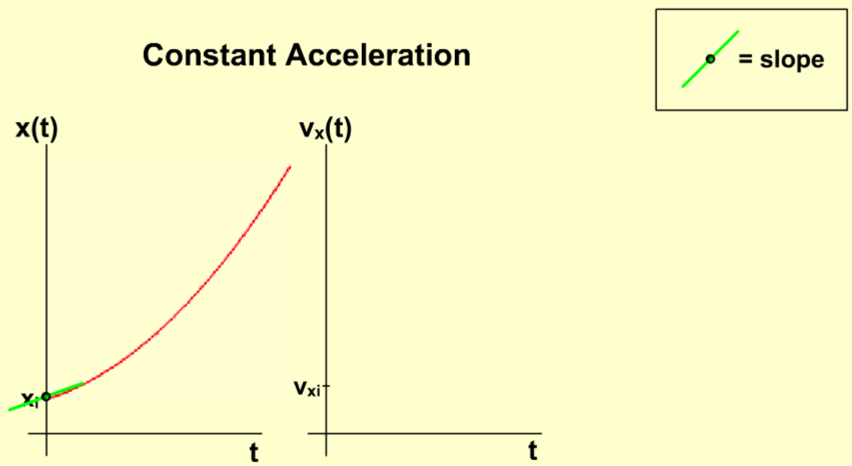
Constant Acceleration



$x(t) = x_i + v_{xi}\, t + 1/2\, a_x\, t^2$

Playing
Paused

Sign convention :

$$v = v_o + (-a)\, t \qquad\qquad v = v_o + at$$

Both "*v*" and "*a*" have opposite sign !

Both "*v*" and "*a*" have same sign !

READ EXAMPLES 2.6 to 2.8 in the Textbook

EXAMPLES

A motorist traveling at 90 km/h applied the brakes for 5 s. If the deceleration was 3 m/s², then his final speed will be:

(a) 35 m/s
(b) 25 m/s
(c) 15 m/s
(d) 10 m/s

$$v = v_o + at = \frac{90\times 1000}{3600} - 3\times 5 = 10\ m$$

A jet plane accelerates on a runway from rest at 4 m/s2, the distance and the velocity of the jet after 5 sec is:

(a) 30 m, 20 m/s
(b) 40 m, 10 m/s
(c) 50 m, 20 m/s
(d) 100 m, 10 m/s

$$x = v_o t + \frac{1}{2}at^2 = 50\ m$$

$$v = v_o + at = 20\ m/s$$

A car travels north at 40 m/s for 1 h. It stops for 50 minutes and return south traveling 10 km for 20 minutes. Its average velocity and speed respectively are:

(a) 10.5, 10 m/s
(b) 12.1, 12.5 m/s
(c) 17.2, 19.7 m/s
(d) 14.2, 16 m/s

Displacement=40×3600-10000=134000 m
Distant=40×3600+10000=154000 m
Total time=3600+50×60+20×60=7800 sec
Av. Velocity=134000/7800=17.2 m/s
Av. Speed =154000/7800=19.7 m/s

A car is traveling 80 km/h in a school zone. A police car starts from rest just as the speeder car passes it and accelerates at a constant rate of 8 km/h.s.

(a) When does the police car catch the speeder car?

(b) How fast the police car traveling when it catches the speeder?

(a) Since the car is traveling with constant velocity, its position x_c is given by

$$x - x_o = v_o t + \frac{1}{2}at^2 \qquad (6)$$

, with $x_o = 0$ and a = 0:

$$x_c = v_0 t = (80\ km/h)\ t$$

The position of x_p of the police car ,is given by

$$x_p = \frac{1}{2}at^2 = \frac{1}{2}(8\ km/h.s)\ t^2$$

We find the time when the two cars are at the same position by setting $x_c = x_p$ and solving for t:

$$(80\ km/h)\ t = \frac{1}{2}(8\ km/h.s)\ t^2$$

OR

$$t\,(4\,t\ km/h.s - 80\ km/h) = 0$$

The two solutions are

$t = 0$ *corresponding to the initial conditions*

and $t = \dfrac{80\ km/h}{4\ km/h.s} = 20\ s$

The police car thus catches the speeder at time $t = 20$ s

(b) The velocity of the police car is given by $v = v_o + at$, **with** $v_0 = 0$:

$$v_p = at = (8\ km/h.s)\ t$$

At t = 20 s, the velocity of the police car is;

$$v_p = (8\ km/h.s)(20\ s) = 160\ km/h$$

At this time, the speed of the police car is twice that of the speeder. This must be true because the average velocity of the police car is half its final velocity, and since both cars cover the same distance in the same time, they must have equal average velocities.

EXAMPLES

تحتاج طائرة إلى 20 s ومدرج طوله 400 m حتى تقلع من السكون ، لذا تكون سرعتها عندما تترك عجلاتها الأرض هي

(a) 20 m/s (b) 32 m/s (c) 40 m/s (d) 80 m/s

تقطع حافلة مسافة 400 m بين نقطتي توقف .إذا بدأت من السكون وتسارعت بمقدار $1.5\ m/s^2$ حتى تصل سرعتها الى 9 m/s وبعد ذلك استمرت بنفس السرعة لفترة من الزمن ومن ثم تباطأت بمقدار $2\ m/s^2$ لكي تقف0 أحسب الزمن الكلي الذي استغرقته الحافلة لقطع المسافة بين نقطتي التوقف .

خلال رحلة الحافلة نجد أن هناك ثلاث مراحل مبينة في الشكل السابق:

المرحلة الأولى:(I)

$v_0 = 0$; $a = +1.5\ m/s^2$; $v_f = 9\ m/s$

$v_f = v0 + at$ → $t_I = (v_f - v_0)/a = 6\ s$

خلال هذا الزمن تكون الحافلة قطعت مسافة

$x_I = (v_{2f} - v_{20})/2a = 27\ m$

المرحلة الثالثة:(II)

$v_0 = 9\ m/s$; $v_f = 0$; $a = -2\ m/s2$

$t_{III} = (v_f - v_0)/a = 4.5\ s$

خلال هذا الزمن تكون الحافلة قطعت مسافة

$x_{III} = (v_{2f} - v_{20})/2a = 20.25\ m$

وبناء علية تكون الحافلة قد قطعت مسافة في المرحلة الثانية (II)

$x_{II} = 400 - (x_I + x_{III}) = 352.75\ m$

و حيث أن a=0 في المرحلة الثانية (II) فإن

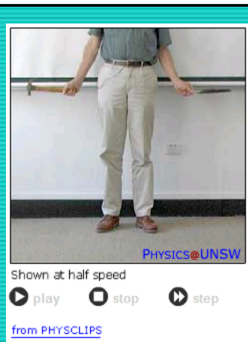
$x_{II} = v_0 t_{II}$ → $t_{II} = x_{II}/v_0 = 39.2\ s$

الزمن الكلي للرحلة

$T = t_I + t_{II} + t_{III} = 50\ s$

2.6 Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80 m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is ALWAYS toward the center of the earth, which we normally call (-y) ; where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80 m/s^2$



⦿ Free falling Objects

In the absence of air resistance, all objects fall towards the earth with the same constant acceleration ($a = -g = -9.8\ m/s^2$), due to gravity

$\vec{a} = \vec{g}$; free falling acceleration

gravitational acceleration $g = 9.80\ m/s^2$

y

$\vec{a} = -g$

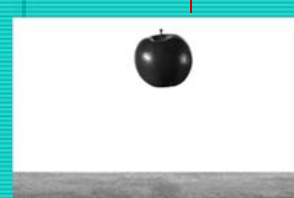
Free fall motions: Summary

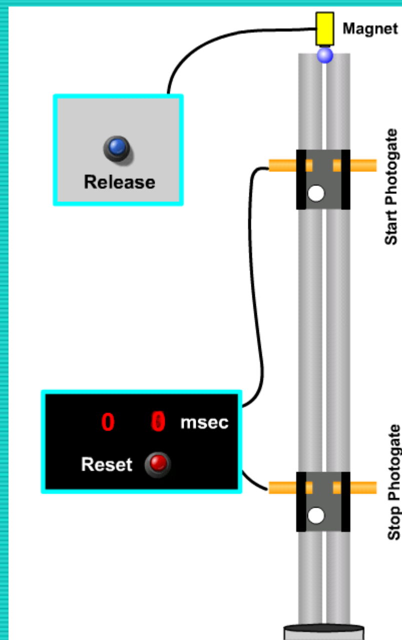
$$v = v_o - gt$$

$$y - y_o = v_o t - \frac{1}{2} g t^2$$

$$y - y_o = \frac{1}{2}(v + v_o)t$$

$$v^2 = v_o^2 - 2g(y - y_o)$$





Example

A man throws a brick upward from the top of a building. TRUE OR FALSE. (Assume the coordinate system is defined with positive being upward)

a) At ‘A’ the acceleration is positive → F

b) At ‘B’ the velocity is zero → T

c) At ‘B’ the acceleration is zero → F

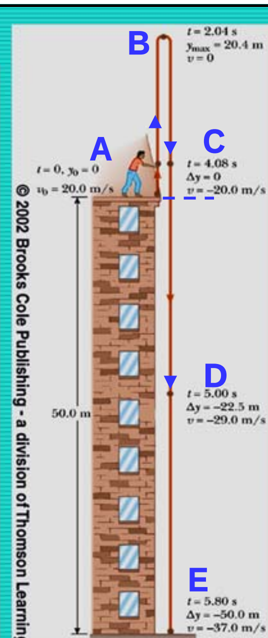
d) At ‘C’ the velocity is negative → T

e) At ‘C’ the acceleration is negative → T

f) The speed at ‘C’ and at ‘A’ are equal → T

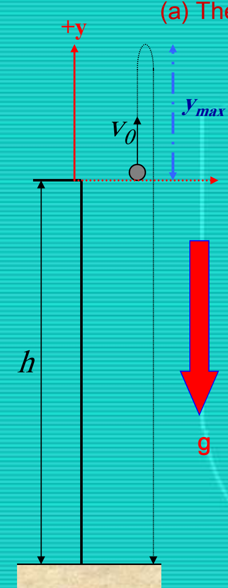
g) The velocity at ‘C’ and at ‘A’ are equal F

h) The speed is greatest at ‘E’ → T



A ball is thrown up from the top of a building 40 m high with initial velocity of 10.0 m/s, determine:

(a) The time at which the ball reaches its maximum height.
(b) The maximum height.
(c) The time at which the ball returns to the position from which it was thrown.
(d) The velocity of the ball at this instant.
(e) The time at which the ball reach the ground.
(f) The velocity of the ball when its reach the ground.
(g) If the ball thrown downward with the same velocity (10 m/s), what the velocity of the ball when its reach the ground?



(a) The time at which the ball reaches its maximum height.

At max. height $v = 0$

$$v = v_o - gt = 0$$

$$\Rightarrow \therefore t = \frac{v_0}{g} = \frac{+10}{9.8} = 1.02\ s$$

(b) The maximum height.

$$v^2 = v_o^2 - 2g\,y = 0$$

$$\Rightarrow y_{max} = \frac{v_o^2}{2g} = \frac{(10)^2}{2(9.8)} = 5.1\ m$$

(c) & (d) left for you to try.

(e) The time at which the ball reach the ground.

$$y = v_o t - \frac{1}{2}gt^2$$

$$y = h = -40 = (+10)t - \frac{1}{2}(9.8)t^2$$

$4.9\,t^2 - 10\,t - 40 = 0$

$$a\,x^2 + b\,x + c = 0$$

solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

t = 4.05 s

(f) The velocity of the ball when its reach the ground.

$$v = v_o - gt$$

$$= +10 - (9.8)(4.05) = -29.69\ m/s$$

Try using $v^2 = v_o^2 - 2g\,y$

(g) left for you to try.

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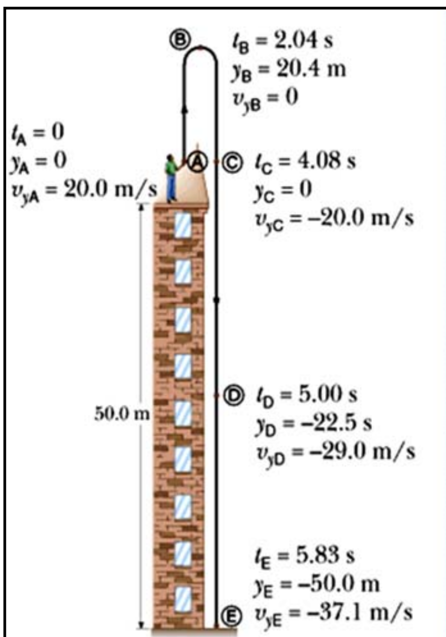

Example

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50 m high. Using $t_A = 0$ as the time the stone leaves the throwers hand at position A, determine:

(a) The time at which the stone reaches its maximum height.

(b) The maximum height.

(c) The time at which the stone returns to the position from which it was thrown.

(d) The velocity of the stone at this instant

(e) The velocity and and position of the stone at t = 5.00 s.

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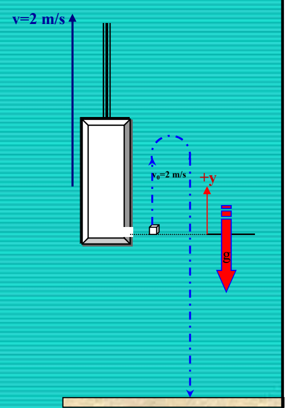
A box falls from an elevator that is ascending with a velocity of 2 m/s. It strikes the ground in 3 sec.

(a) How long will it take the box to reach its maximum height?

(b) How far from the ground was the box when it fell off the elevator?

(c) What is the height of the elevator when the box is at its highest point?

When the box falls from the elevator, its initial velocity will be $v_0 = 2$ m/s and $a = -g$.



(a) At maximum height, the box velocity is $v = 0$

$$v = v_0 - gt$$
$$0 = v_0 - gt$$
$$\therefore\ t = v_0 / g = 0.204\ s$$

(b) t = 3 sec ; $v_0 = 2\ m/s$

$$y = v_0 t - \frac{1}{2} g t^2$$

$$\therefore y = (2)(3) - \frac{1}{2}(9.8)(3)^2 = -38.1\ m$$

(c) *The box will reach the maximum height in* t = 0.204 sec. During this time, the elevator will move up with constant velocity ($v = 2$ m/s) and acceleration a = 0.

$$y = v_0 t + \frac{1}{2} a t^2$$

$$\therefore y = (2)(0.204) - 0 = 0.408\ m$$

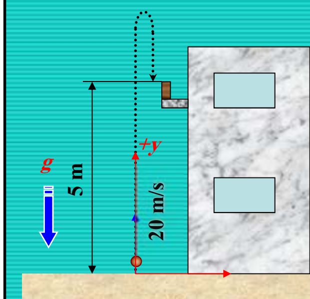
Therefore the height of the elevator, from the ground, when the box at its highest point = 38.1+0.408= 38.51 m

قذف حجر إلى أعلى بسرعة ابتدائية مقدارها 20 m/s ثم مُسِك خلال عودته إلى اسفل بواسطة شخص يقف على ارتفاع 5 m من نقطة القذف.

(أ) احسب سرعة الحجر عند مَسكه.

(ب) احسب الزمن الذي استغرقه الحجر في الهواء.

(ج) احسب أقصي ارتفاع يصل إليه الحجر.



(a) $v^2 = v_0^2 - 2gy$

$$v = \sqrt{v_0^2 - 2gy} = \pm 17.4 = -17.4\ m/s$$

(b) $v = v_0 - gt \quad \rightarrow \quad t = \frac{v - v_0}{-g} = \frac{(-17.4)-(20)}{-9.8} = 3.82\ s$

(c) at max. height v = 0

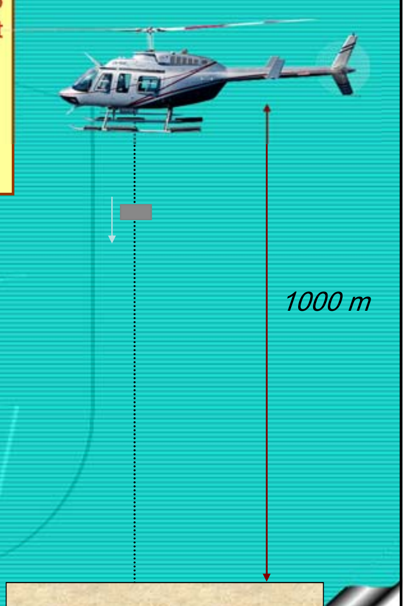
$$v^2 = v_0^2 - 2gy$$

$$y = \frac{v^2 - v_0^2}{-2g} = \frac{0-(20)^2}{-2(9.8)} = 20.4\ m \quad \text{(from ground)}$$

The pilot of a hovering helicopter drops a lead brick from a height of 1000 m. How long does it take to reach the ground and how fast is it moving when it gets there? (neglect air resistance)

14.3 s

-140 m/s



Previous Exam Questions

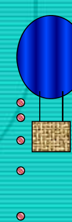
A sandbag that is dropped from a balloon strikes the ground after 20 s. If the balloon is moving vertically upward with a velocity of 20 m/s then the height of the balloon when the sandbag is dropped is:

(a) 2360 m

(b) 1960 m

(c) 400 m

(d) 1560 m



A man ascending at 7 m/s in a balloon 20 m above the ground accidentally drops a box. The velocity of the box just before touching the ground is:

(a) 14 m/s

(b) 18 m/s

(c) 21 m/s

(d) 58 m/s

$$v^2 = v_0^2 - 2gy$$
$$= (7)^2 - 2(9.8)(-20) = 441$$
$$v = 21 \ m/s$$

A ball is thrown vertically upward from the ground with a speed of 29.4 m/s. The time it takes the ball to arrive at a height of 19.6 m on its way back is:

(a) 5.235 s

(b) 1.345 s

(c) 0.652 s

(d) 0.052 s

READ EXAMPLES 2.9 & 2.12 in the Textbook

To get A+ : Study and Solve Problems As MUCH As you CAN