# CHAPTER 3
## Introduction to vectors

Physical quantities are classified as scalars, vectors, etc.

**Scalar** : described by a real number with units
*examples* - mass, charge, energy . . .

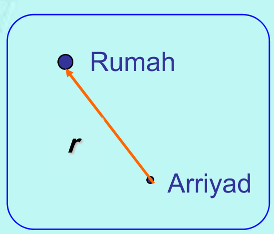
**Vector** : described by a scalar (the magnitude) and a direction in space
*examples* - displacement, velocity, force . . .



- In 1 dimension, we could specify direction with a + or - sign.
- In 2 or 3 dimensions, we need more than a sign to specify the direction of something:
- To illustrate this, consider the *position vector* ***r*** in 2 dimensions.

Example: Where is Arriyad?

- Choose origin at Arriyad
- Choose coordinates of distance (km), and direction (N,S,E,W)
- In this case ***r*** is a vector that points 100 km Northwest



## Vector-What is

Observe the following physical quantities:

1. Velocity, displacement, acceleration and force.
2. Mass, time, temperature

What is/are the distinct different between quantities in “1” and in “2” ?

The answer is:

1. Velocity, displacement, acceleration and force.
   All quantities that have direction associated with them, apart from magnitude.
2. Mass, time, temperature
   All quantities that have only magnitude.



Generally speaking,

All physical quantities can be divided into 2 categories:

1. Vector quantity

needs magnitude and direction to describe it.

Example: Position of the car is 11.2 km, east.

2. Scalar quantity

needs only magnitude to describe it.

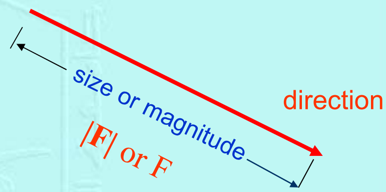
Example: Temperature of the copper bar is 223 $^{o}$C

Physical quantities that have both a size and direction are represented by vectors, which are drawn as arrows.

Notation: bold face **F**

underline <u>F</u>

over arrow $\vec{\mathbf{F}}$




The magnitude is a scalar: it has only a size, in some defined units.

Sometimes there are separate names for the magnitude of vectors alone: e.g. "speed" is the magnitude of the velocity vector.



## PROPERTIES OF VECTOR

### Vector-Addition/Subtraction

Basically, there are 3 methods of adding/subtracting vectors:

1. Graphical method
   - ⇨ Tail-to-tip method
   - ⇨ Parallelogram method
2. Components method (most important !)

Notes:

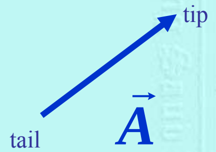
In vector, a negative sign means opposite direction

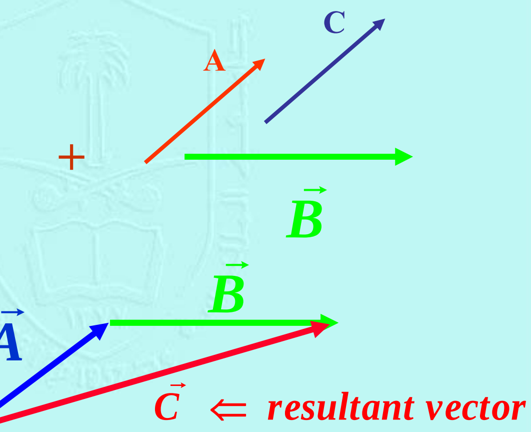
whereas in scalar quantity, a negative is used to denote "lost" (positive means "gain")

## Vector-Addition/Subtraction

**Equality**: vectors with the same magnitude and direction are equal, regardless of location. So both displacement and position are vectors.

⇨ Tail-to-tip Method:




The resultant vector **C = A + B** is drawn from the tail of **A** to the tip of **B**.

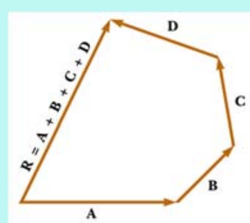


## Adding several vectors together.

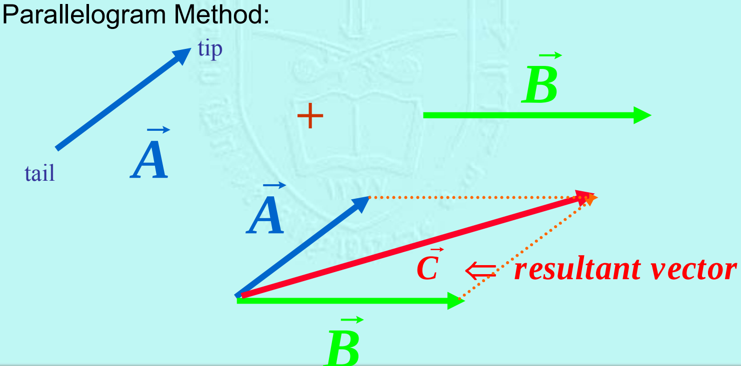
Resultant vector

**R=A+B+C+D**

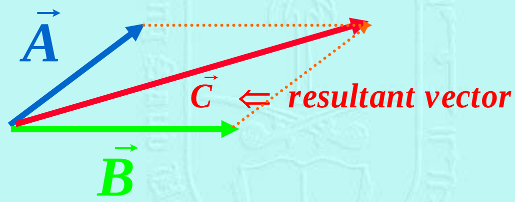
is drawn from the tail of the first vector to the tip of the last vector.




⇨ Parallelogram Method:



Is A + B = B + A

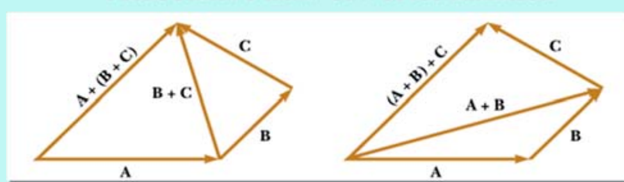



A + B = B + A YES

Commutative Law of vector addition



Associative Law of vector addition

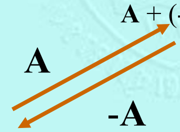



**A+(B+C) = (A+B)+C**

**<u>Negative of a vector.</u>**

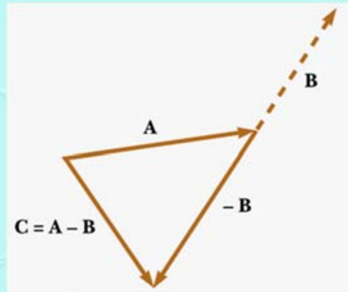
The vectors **A** and –**A** have the same magnitude but opposite directions.

**A** + (-**A**) = 0

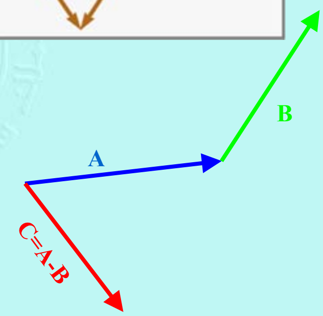


Subtracting vectors:

**A - B = A + (-B)**

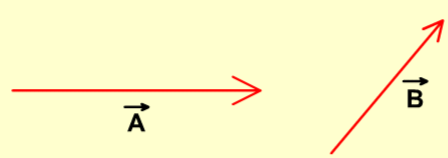



Subtraction: Head to head






## Subtracting 2 Vectors



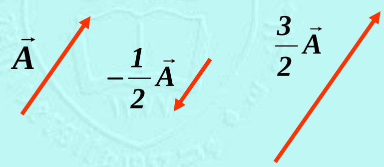
## Important Properties

$$\vec{a}+\vec{b}=\vec{b}+\vec{a} \qquad (commutative\ law).$$

$$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c}) \qquad (associative\ law).$$

$$\vec{a}=\vec{a}-\vec{b}=\vec{a}+(-\vec{b}) \qquad (vector\ substraction).$$

### Multiplication by a scalar:



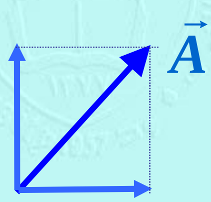

Other complicated combination rules, the **dot product** and the **vector product**, will come later



## COMPONENTS METHOD:

- graphical method is less accurate and not useful for vectors in 3D.
- Need a precise and powerful method ==> components method.

Any vector that lies in a particular plane (2D) can be resolved into 2 perpendicular components.



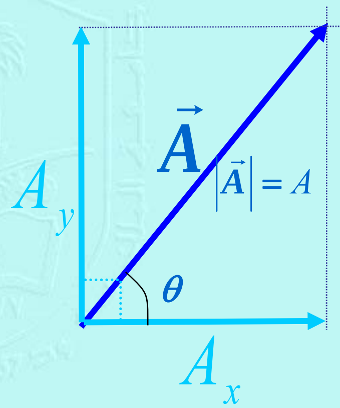
## Components of a vector

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

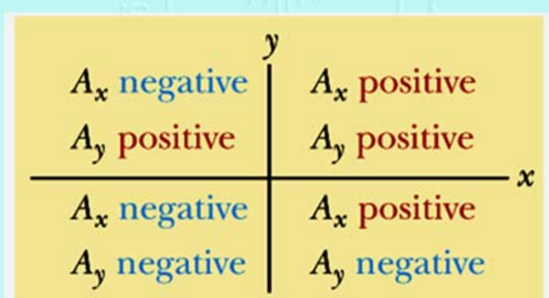
$$\tan\theta = \frac{A_y}{A_x}$$






**The signs of the components $A_x$ and $A_y$ depend on the angle θ and they can be positive or negative.**

**(Examples)**



## Components Method:

1. The vector in component form:

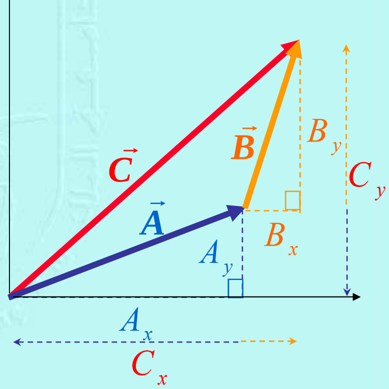
$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$

2. The vector in magnitude-angle form:

$$|\vec{C}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$
$$= \sqrt{C_x^2 + C_y^2}$$

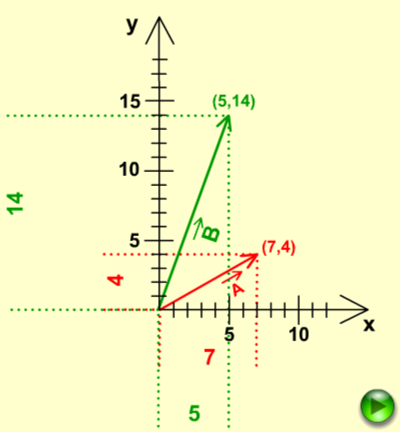
$$\tan\theta = \frac{A_y + B_y}{A_x + B_x} = \frac{C_y}{C_x}$$

$$\vec{C} = \vec{A} + \vec{B}$$






## To Add 2 Vectors Numerically ...

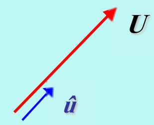


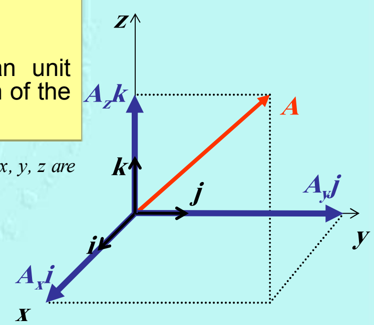
## Unit Vectors:

- A **Unit Vector** is a vector having length 1 and no units.
- It is used to specify a direction.
- Unit vector $u$ points in the direction of $U$.
  Often denoted with a "hat": $u = \hat{u}$

$$|\hat{u}| = 1$$

- Useful examples are the Cartesian unit vectors [ $i, j, k$ ] point in the direction of the $x$, $y$ and $z$ axes.




*In your textbook the notation for the unit vector along x, y, z are* $\hat{x}$, $\hat{y}$, $\hat{z}$

$$A = A_x i + A_y j + A_z k$$



**$A_x$, $A_y$, and $A_z$ are scalars multiplying $i, j, k$ They are called "components".**

The components of a vector can be found from trigonometry.

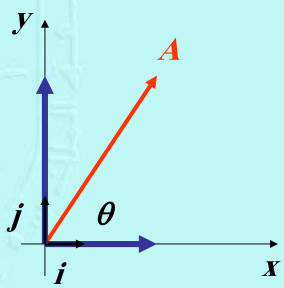
$A_x = |\mathbf{A}| \cos\theta$

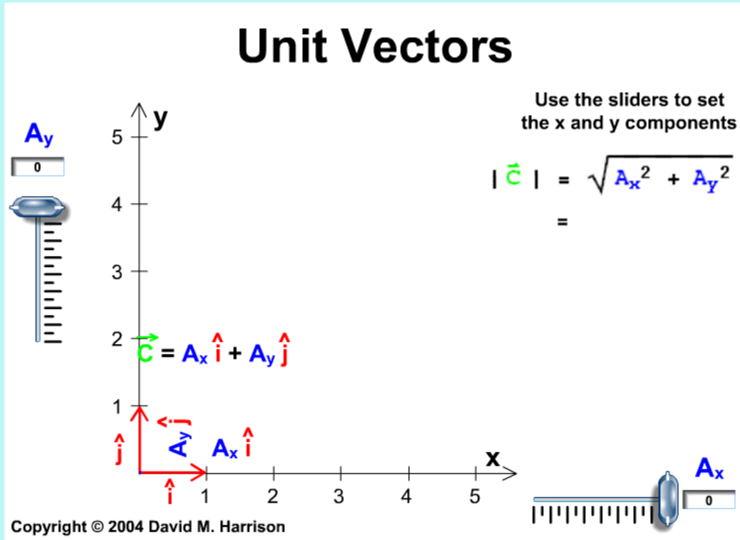
$A_y = |\mathbf{A}| \sin\theta$

Going backwards,

$\tan\theta = A_y / A_x$

$|\mathbf{A}| = \sqrt{A_x^{\,2} + A_y^{\,2}}$



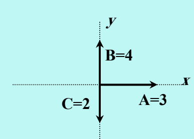





## *Problems from past EXAMS*

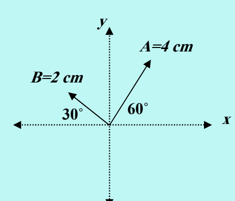
The magnitude of of the vectors shown in the figure is;

1. 3.6
2. 5.1
3. 6.7
4. 9.7




The direction of the vector $\vec{C} = \vec{A} - \vec{B}$ with respect to the *+x-axis* is:

1. **33°**
2. 39°
3. 46°
4. 90°



- Vector **A** = {0,2,1}
- Vector **B** = {3,0,2}
- Vector **C** = {1,-4,2}

What is the resultant vector, **D**, fro adding **A**+**B**+**C**?

(a) {3,5,-1}   (b) {4,-2,5}   (c) {5,-2,4}

$\mathbf{D} = (A_X\boldsymbol{i} + A_Y\boldsymbol{j} + A_Z\boldsymbol{k}) + (B_X\boldsymbol{i} + B_Y\boldsymbol{j} + B_Z\boldsymbol{k}) + (C_X\boldsymbol{i} + C_Y\boldsymbol{j} + C_Z\boldsymbol{k})$

$= (A_X + B_X + C_X)\boldsymbol{i} + (A_Y + B_Y + C_Y)\boldsymbol{j} + (A_Z + B_Z + C_Z)\boldsymbol{k}$

$= (0 + 3 + 1)\boldsymbol{i} + (2 + 0 - 4)\boldsymbol{j} + (1 + 2 + 2)\boldsymbol{k}$

$= \{4,-2,5\}$



The following three vectors:

$$\vec{a} = (4.2\ m)\,\hat{i} - (1.5\ m)\,\hat{j}\ ,$$
$$\vec{b} = (-1.6\ m)\,\hat{i} + (2.9\ m)\,\hat{j}\ ,$$
$$\text{and}\ \ \vec{c} = (-3.7\ m)\,\hat{j}\ .$$

Find their vector sum $\vec{r}$ ?

Solution

$$r_x = a_x + b_x + c_x = 4.2\ m - 1.6\ m + 0 = 2.6\ m$$
$$r_y = a_y + b_y + c_y = -1.5\ m + 2.9\ m - 3.7\ m = -2.3\ m$$
$$\vec{r} = (2.6\ m)\,\hat{i} - (2.3\ m)\,\hat{j}\ ,$$

The magnitude is $r = \sqrt{(2.6\ m)^2 + (-2.3\ m)^2} \approx 3.5\ m$

The angle from the positive x-direction is

$$\theta = \tan^{-1}\left(\frac{-2.3\ m}{2.6\ m}\right) = -41^\circ$$

If **A** = 10 **i** - 5 **j** and **B** = -6 **i** + 3 **j** what is **A** - **B** = ?

1. 16 **i** - 8 **j**
2. 4 **i** - 2 **j**
3. 16 **i** - 2 **j**
4. 4 **i** - 8 **j**
5. None of the above

$$\begin{array}{rcl} \mathbf{A} & = & 10\,\mathbf{i} \;-5\,\mathbf{j} \\ -\mathbf{B} & = & +6\,\mathbf{i} \;-3\,\mathbf{j} \\ \hline \mathbf{A}-\mathbf{B} & = & 16\,\mathbf{i} \;-8\,\mathbf{j} \end{array}$$

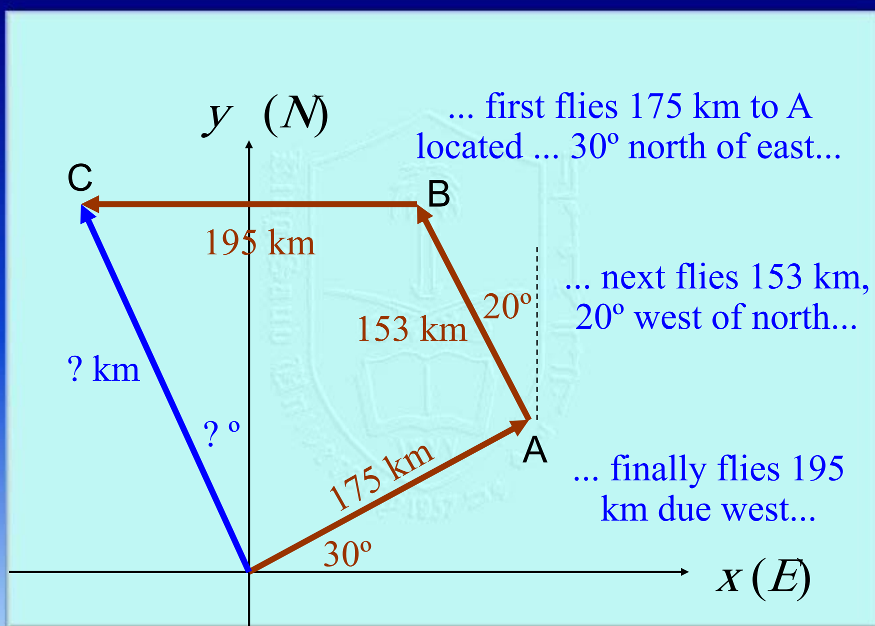
Example 3.6

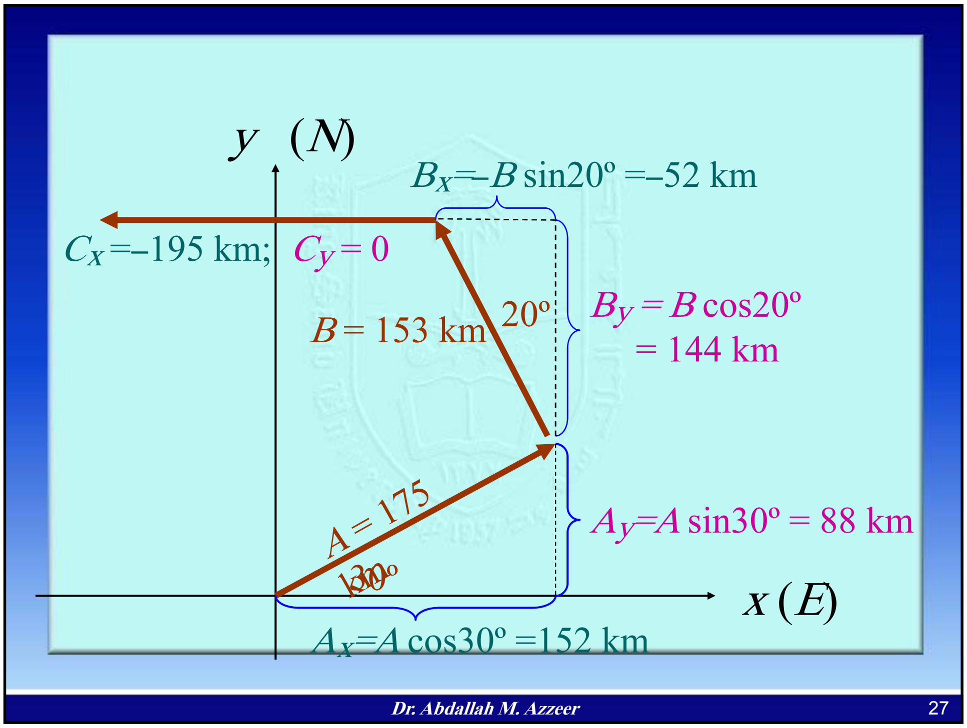
A commuter plane first flies 175 km to minor airport A located in the direction 30º north of east. Next it flies 153 km, 20º west of north to town B. Finally, it flies 195 km due west to city C. What is the location of city C relative to the starting point?

Strategy:

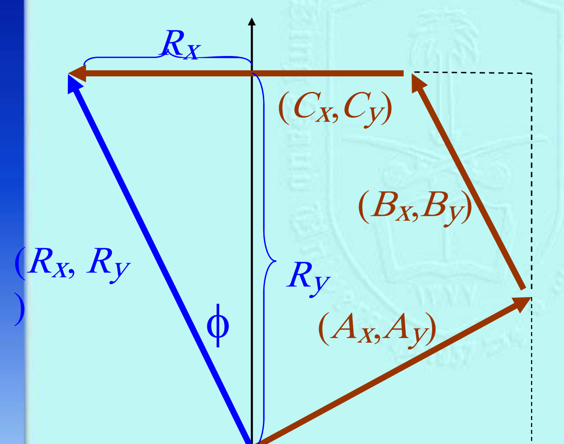
1. It's a vector problem; draw a diagram—a map.
2. Find *x*- (east) and *y*- (north) components of all vectors.
3. Add components ⇒ total displacement vector.
4. Determine magnitude and angle of total displacement.








$\mathbf{R} = (R_x, R_y) = (A_x, A_y) + (B_x, B_y) + (C_x, C_y) = (-95, 232)$ km




$$R = \sqrt{R_x^2 + R_y^2} = 251 \text{ km}$$

$$\tan\phi = \frac{|R_x|}{|R_y|} = \frac{95}{232} = 0.41$$

$$\phi = 22.3^\circ$$

West of north

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READ AND SOLVE THE EXAMPLES ON THIS CHAPTER also don't forget the assigned PROBLEMS