CHAPTER 7

WORK, ENERGY, AND POWER

Work & Energy

So far: **Motion analysis with *forces*.**

NOW: **An alternative analysis using the concepts of *Work & Energy*.**

Easier? My opinion is yes!

Conservation of Energy:

NOT **a new law! Just Newton's Laws in a different language.**

- One of the most important concepts in physics
  - Alternative approach to mechanics
- Many applications beyond mechanics
  - Thermodynamics (movement of heat)
  - Quantum mechanics...
- Very useful tools
  - You will learn new (sometimes much easier) ways to solve problems

Definition of Work:

Work, W, is energy transferred to or from an object by means of a force acting on the object.

The transfer is not a flow as in a liquid.

Consider it more like an electronic transfer between two bank accounts. The Riyals amount in one account increases while the Riyals amount in the second account decreases, but nothing material has actually passed between the two accounts.

We can find an expression for work by considering an object that moves due to a force applied to it….

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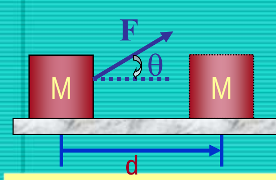
7.2 Work Done by Constant Force

- Work in physics is done only when a sum of forces exerted on an object made a motion to the object.
- For an object moving under a *Constant Force*, Work done (W) = product of magnitude of displacement (d) × component of force parallel to displacement ($F_{||}$):

Which force did the work? Force $\vec{F}$



How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d}$

$W = F_{//}d = Fd\cos\theta$

Scalar Quantity

What does this mean?

Physical work is done only by the component of of the force along the movement of the object.

Work is energy transfer!!

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Units of Work and Energy

Force x Distance = Work

Newton × Meter = Joule

$[M][L] / [T]^2$ $[L]$ $[M][L]^2 / [T]^2$

mks	cgs	other
N-m (Joule)	Dyne-cm (erg) $= 10^{-7}$ J	BTU = 1054 J calorie = 4.184 J foot-lb = 1.356 J eV $= 1.6\times10^{-19}$ J

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Work can be positive or negative

- Man does positive work lifting box
- Man does negative work lowering box

- Gravity does positive work when box lowers
- Gravity does negative work when box is raised

$$W = F_{||}d = Fd\cos\theta$$

- **Can exert a force & do no work!**

  Could have $d = 0 \Rightarrow W = 0$

  Could have $F \perp d$

  $\Rightarrow \theta = 90^o, \cos\theta = 0$

  $\Rightarrow W = 0$

  Example, walking at constant v with grocery bag:

FIGURE 6-2 Work done on the bag of groceries in this case is zero since F is perpendicular to the displacement d

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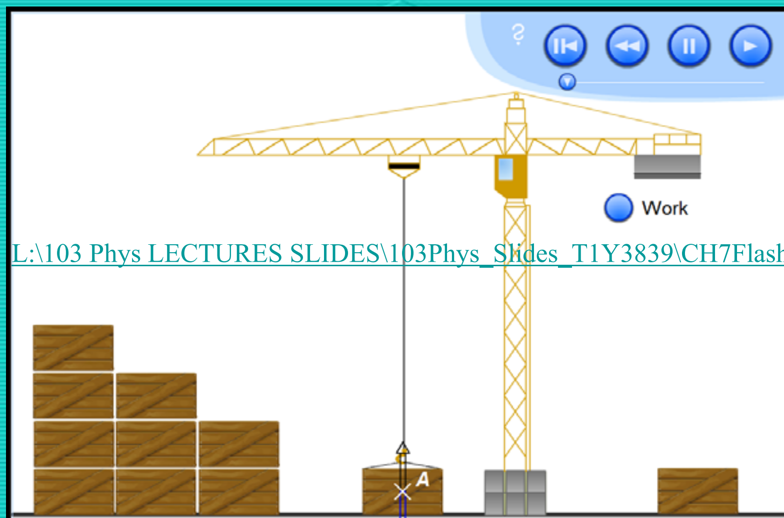

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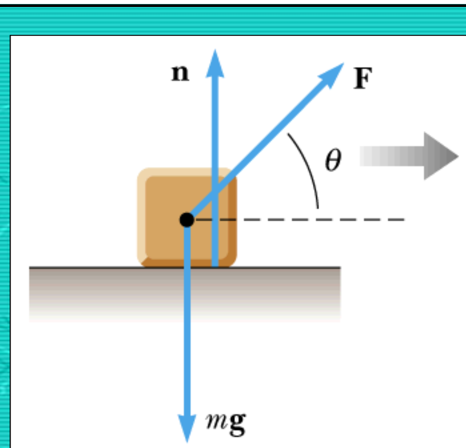
What is the work done by

- the gravitational force **= 0**
- the normal force **= 0**
- the force F **= F cos θ**

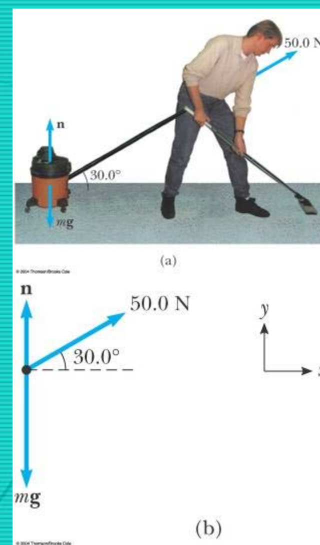
when the block is displaced along the horizontal.



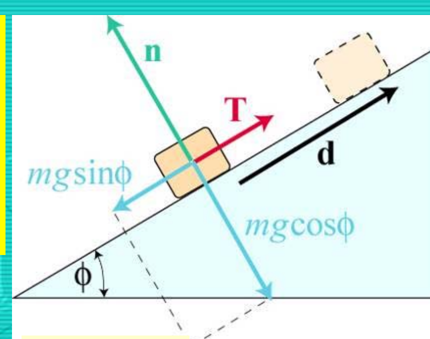
Example 7.1

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F = 50.0 N at an angle of 30º with the horizontal.

Calculate the work done by the force on the vacuum cleaner as its displaced 3.00 m to the right .



Example: On a frictionless plane, the block is subject to three forces: gravity, the normal force and tension T. If it moves up the incline through displacement d:



Work done by **T**: $W_{\mathbf{T}} = T \cdot d$ *"effective"*

Work done by **n**: $W_{\mathbf{n}} = 0 \cdot d = 0$ *"ineffective"*

Work done by $m\mathbf{g}$: $W_{m\mathbf{g}} = (-mg\sin\phi) \cdot d = -\,mgd\sin\phi$ *"effective", but...*

Net work $\Sigma W = W_{\mathbf{T}} + W_{\mathbf{n}} + W_{m\mathbf{g}} = Td - mgd\sin\phi$

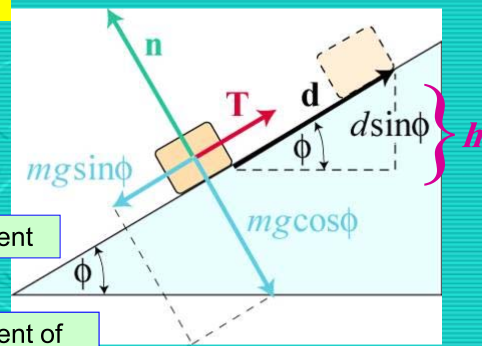
WORK DONE BY GRAVITY

$$W_T = T \cdot d; \quad W_n = 0$$

$$W_g = (-mg\sin\phi)\cdot d$$

Component of force in direction of displacement × Displacement

Force × Component of displacement in direction of force.



Box moves up the plane through displacement **d**

Alternatively,

$$W_g = (mg)\cdot(-d\sin\phi)$$
$$= -mgh$$

Work done by gravity = weight × height change

Negative for upwards displacement, positive for down

DEFINITION OF SCALAR(DOT) PRODUCT AND WORK

Work is the scalar product (or dot product) of the force F and the displacement d.

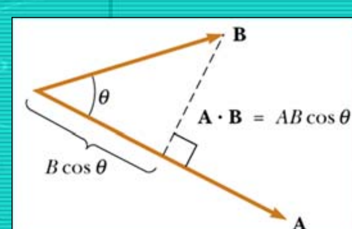
$$W = \vec{F} \bullet \vec{d} = |F||d|\cos\theta$$

F and **d** are vectors W is a scalar quantity

Definition:

Scalar product between vector **A** and **B**

$$\vec{A} \bullet \vec{B} \equiv AB \cdot \cos\theta$$



Scalar product is commutative:

$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$$

Distributive law of multiplication:

$$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$$

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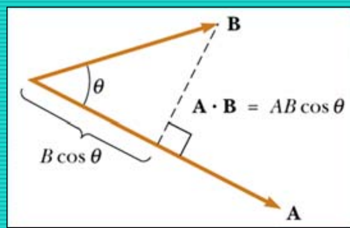
Scalar Product using unit vectors:



The dot product $\mathbf{A}\cdot\mathbf{B}$ equals the magnitude of **A** multiplied by $B\cos\theta$, which is the projection of **B** onto **A.** Or vice versa.

$A(B\cos\theta) = B(A\cos\theta)$

In terms of vector components:

$$
\begin{aligned}
\mathbf{A}\cdot\mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\
&= A_xB_x\,\mathbf{i}\cdot\mathbf{i} + \cancel{A_xB_y}\,\mathbf{i}\cdot\mathbf{j} + \cancel{A_xB_z}\,\mathbf{i}\cdot\mathbf{k} \\
&+ \cancel{A_yB_x}\,\mathbf{j}\cdot\mathbf{i} + A_yB_y\,\mathbf{j}\cdot\mathbf{j} + \cancel{A_yB_z}\,\mathbf{j}\cdot\mathbf{k} \\
&+ \cancel{A_zB_x}\,\mathbf{k}\cdot\mathbf{i} + \cancel{A_zB_y}\,\mathbf{k}\cdot\mathbf{j} + A_zB_z\,\mathbf{k}\cdot\mathbf{k} \\
&= A_xB_x + A_yB_y + A_zB_z
\end{aligned}
$$

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1$$
$$\mathbf{i}\cdot\mathbf{j} = \mathbf{i}\cdot\mathbf{k} = \mathbf{j}\cdot\mathbf{k} = 0$$

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Scalar Product using unit vectors:

We have the vectors **A** and **B**:

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k} \qquad \vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$$

Then: $\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$$\vec{A}\bullet\vec{A} = A_xA_x + A_yA_y + A_zA_z = A^2$$

READ EXAMPLES 7.2 and 7.3 in your textbook

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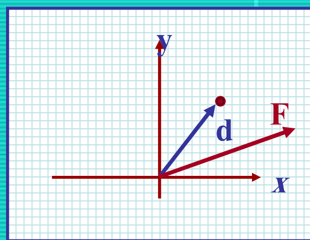
A particle moving in the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**) m as a constant force F=(5.0**i**+2.0**j**) N acts on the particle.



(a) Calculate the magnitude of the displacement and that of the force.

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6\,m$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4\,N$$

(b) Calculate the work done by the force F.

$$W = \vec{F}\cdot\vec{d} = \left(2.0\hat{i} + 3.0\hat{j}\right)\cdot\left(5.0\hat{i} + 2.0\hat{j}\right) = 2.0\times5.0\,\hat{i}\cdot\hat{i} + 3.0\times2.0\,\hat{j}\cdot\hat{j}$$

$$= 10 + 6 = 16\ (J)$$

Can you do this using the magnitudes and the angle between **d** and F?

$$\boxed{W = \vec{F}\cdot\vec{d} = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta}$$

$$\Rightarrow \cos\theta = \frac{\mathbf{F}\cdot\mathbf{d}}{Fd} = 0.823 \Rightarrow \theta = 34.61^\circ$$

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7.4 Work done by a varying Force

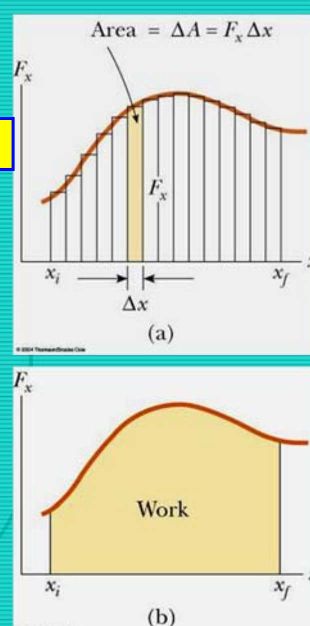
Work done by constant force = $Fd\cos\theta$

What about forces that are not constant?

$$W \approx \sum_{x_i}^{x_f} \Delta W \quad \text{where} \quad \Delta W = F_x \Delta x$$

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x\, dx$$

This ***integral*** means the area under the graph of ***f(x)*** between x_i and x_f.



Work = Area

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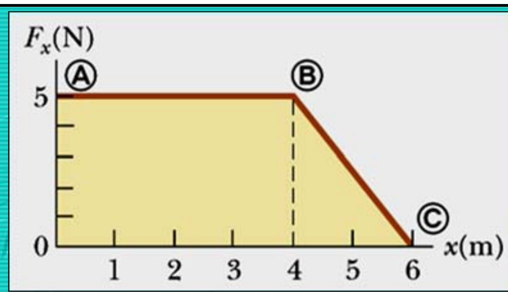
Example 7.4

A force acting on a particle varies as shown in the Figure.



What is the work done on the particle as it is moved from x = 0 to x = 6 m?

Hint: It is the area under the curve.

READ EXAMPLES 7.5 in your textbook

Work done by a Spring

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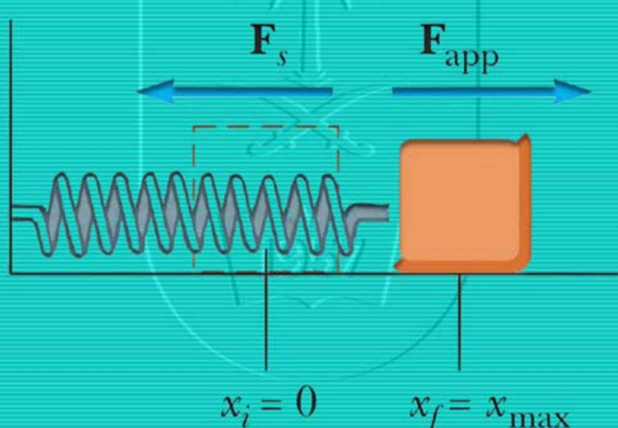


Work done by a Spring

Spring force

$F_s = -kx \quad \Rightarrow \vec{F}_S = -k\vec{x}$ **Hook's Law**

F_s : always directed opposite to the displacement

Work done by a spring

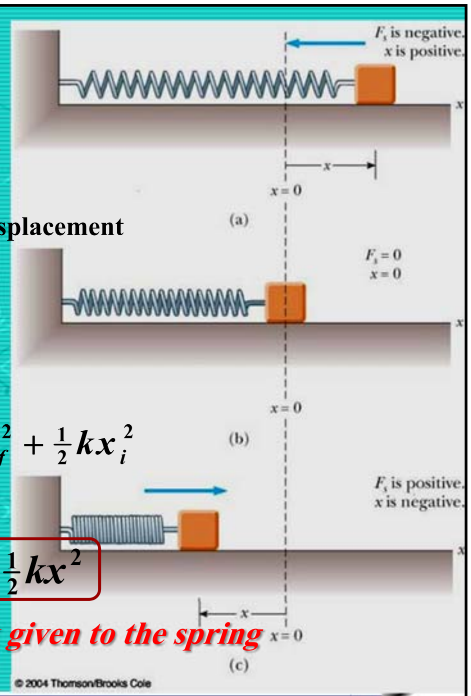
$$W_S = \int_{x_i}^{x_f} F_S \cdot dx$$

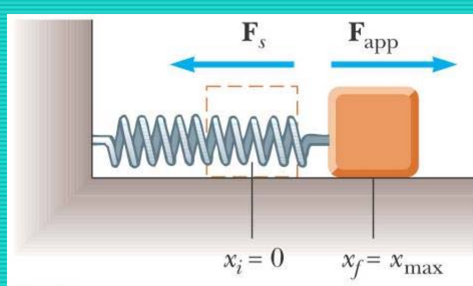
$$= \int_{x_i}^{x_f} -kx \cdot dx = -\tfrac{1}{2}kx_f^2 + \tfrac{1}{2}kx_i^2$$

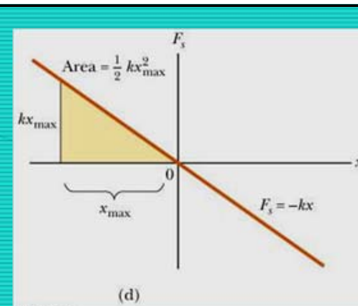
For $x_i = 0$, $x_f = x$

Work done by a spring : $\boxed{W_S = -\tfrac{1}{2}kx^2}$

cf) $\boxed{W_A = \tfrac{1}{2}kx^2}$: *Work given to the spring*







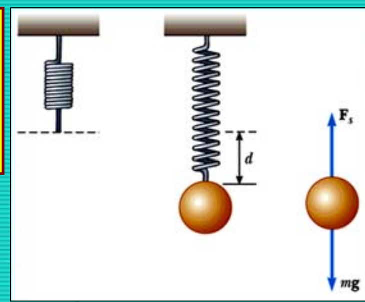
$$F_s = -k \cdot x$$

Work done by a spring

$$W = \frac{1}{2}k(x_i^2 - x_f^2)$$

Example 7.6

A 0.500 kg mass is hung from a spring extending the spring by a distance d = 0.2 m
(a) What is the spring constant of the spring?
(b) How much work was done on the spring?



(a) $F_s = - kx$ Hook's Law

$|F_s| = kd = mg$

$k = mg/d = 2.7 \times 10^2\ N/m$

(b)

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2)$$

$$= 0 - \frac{1}{2}kd^2 = -5.4 \times 10^{-2}\ J$$

Suppose this measurement is made on an elevator going up with ***a***. what is ***d & k ?***