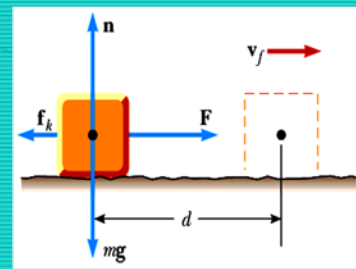
Work due to friction

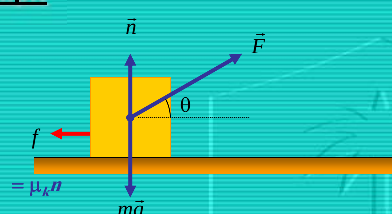
If friction is involved in moving objects, work has to be done against the kinetic frictional force.

This work is:



$$W_f = f_k \cdot d = f_k d \cos 180^\circ = -f_k d$$

Example



A block of mass m is pulled by a force F for a distance d on a rough surface as shown.
What is the total work don on the block?

$$\hat{y} : F_{net,y} = n + F \sin\theta - mg = 0$$

$$n = mg - F \sin\theta$$

$$\hat{x} : F_{net,x} = F \cos\theta - \mu_k n$$

$$= F \cos\theta - \mu_k (mg - F \sin\theta) = F(\cos\theta + \mu_k \sin\theta) - \mu_k mg$$

$$W_{net} = F_{net,x} \cdot d$$

N.B. find the work of each force and add the work, can you get the same answer? why

WORK IS SCALAR QUANTITY

Example

2.40×10^2 N force is pulling an 85.0-kg refrigerator across a horizontal surface. The force acts at an angle of 20.0˚ above the surface. The coefficient of kinetic friction is 0.200, and the refrigerator moves a distance of 8.00 m. Find

(a) the work done by the pulling force, and

(b) the work done by the kinetic frictional force.

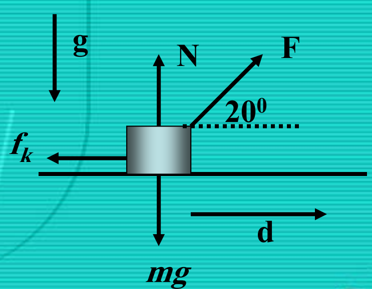
(a) $W = F \cos\theta \, d = 1.8 \times 10^3$ J

(b) $W_f = f_k d \cos\theta$

$f_k = \mu_k (mg - F \sin\theta)$

$= 1.5 \times 10^2$ N

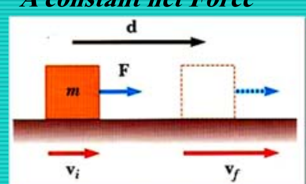
so $W_f = -1.2 \times 10^3$ J



Dr. Abdallah M.Azzeer 3

7.5 Work & Kinetic Energy...

A constant net Force



Work done by $\vec{F}$ ➡ $W_{net} = \vec{F} \cdot \vec{d} = Fd$

$\because \; F = ma,$

$$v_f^2 = v_i^2 + 2ad \Rightarrow d = \frac{1}{2a}\left(v_f^{\,2} - v_i^{\,2}\right)$$

$$\Rightarrow W_{net} = ma \cdot \frac{1}{2a}\left(v_f^{\,2} - v_i^{\,2}\right)$$

$W_{net} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$ $\quad K \equiv \frac{1}{2} m v^2$ $\quad W_{net} = K_f - K_i = \Delta K$

{***Net Work*** done on object}

=

{***change*** in ***kinetic energy*** of object}

Dr. Abdallah M.Azzeer 4

Kinetic Energy; Work-Energy Principle

- ***Energy*** ≡ The ability to do work
- ***Kinetic Energy*** ≡ The energy of motion
- **"Kinetic" ≡ Greek word for motion**
- **An object in motion has the ability to do work**
- **Net work an object = Change in KE.**

$$W_{net} = \Delta K$$

The Work-Energy Principle

- **Note:** W_{net} = work done by the net (total) force.
- W_{net} is a scalar.
- W_{net} can be positive or negative (because ΔKE can be both + & -)
- Units are Joules for both work & KE.

Table 7.1

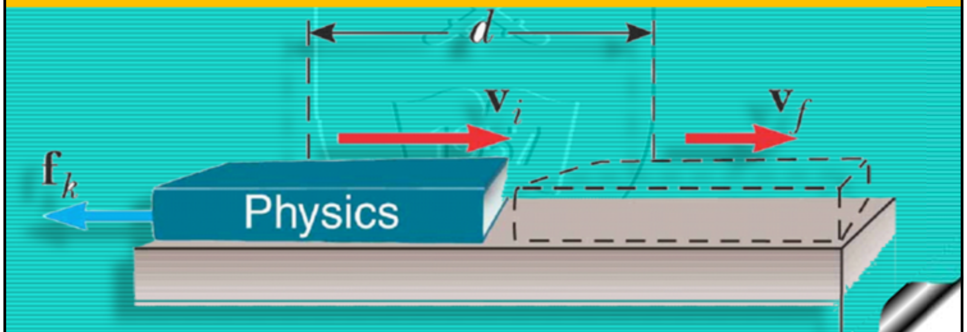
Kinetic Energies for Various Objects

Object	Mass (kg)	Speed (m/s)	Kinetic Energy (J)
Earth orbiting the Sun	5.98×10^{24}	2.98×10^{4}	2.65×10^{33}
Moon orbiting the Earth	7.35×10^{22}	1.02×10^{3}	3.82×10^{28}
Rocket moving at escape speed[a]	500	1.12×10^{4}	3.14×10^{10}
Automobile at 65 mi/h	2 000	29	8.4×10^{5}
Running athlete	70	10	3 500
Stone dropped from 10 m	1.0	14	98
Golf ball at terminal speed	0.046	44	45
Raindrop at terminal speed	3.5×10^{-5}	9.0	1.4×10^{-3}
Oxygen molecule in air	5.3×10^{-26}	500	6.6×10^{-21}

[a] Escape speed is the minimum speed an object must reach near the Earth's surface in order to move infinitely far away from the Earth.

7.6 the nonisolated system
COSERVATION OF ENERGY

A book sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left. The initial velocity of the book is v_i, and its final velocity is v_f. The normal force and the gravitational force are not included in the diagram



Dr. Abdallah M.Azzeer 7

Energy transfer mechanisms. (a) Energy is transferred to the block by *work*;

(b) energy leaves the radio from the speaker by *mechanical waves*;

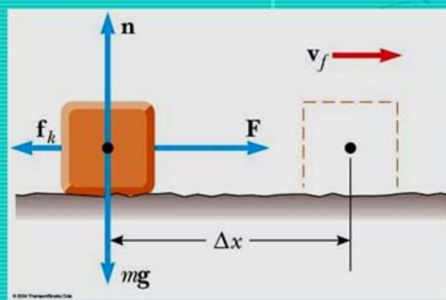
(c) energy transfers up the handle of the spoon by *heat*; (d) energy enters the automobile gas tank by *matter transfer*;

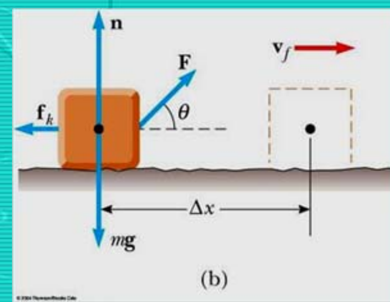
(e) energy enters the hair dryer by *electrical transmission*; and (f) energy leaves the lightbulb by *electromagnetic radiation*.

Dr. Abdallah M.Azzeer 8

We will be back to this point later

Example 7.9,7.10 &7.11





A Question

- A box is pulled up a rough ($\mu > 0$) incline by a rope-pulley-weight arrangement as shown below.
  How many forces are doing work on the box?

(a) *2*
(b) *3*
(c) *4*

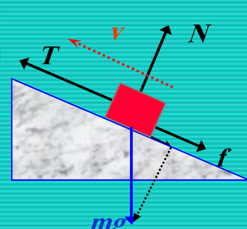
Solution

Draw FBD of box: Consider direction of motion of the box

Any force not perpendicular to the motion will do work:

N **does no work (⊥ v)**
T **does positive work**
f **does negative work**
mg **does negative work**

3 forces do work

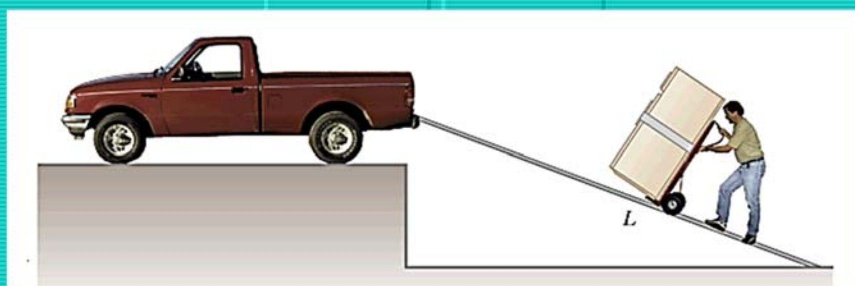


READ EXAMPLES 7.7 in your textbook

Example 7.8

A man loads a refrigerator onto a truck using a ramp. Ignore friction.

He claims he would be doing less work if the length of the ramp would be longer. Is this true?



$$W_{net} = W_{by\ man} + W_{by\ gravity} = 0$$

Dr. Abdallah M.Azzeer 11

Example

A rescue helicopter lifts a 79-kg person straight up by means of a cable. The person has an upward acceleration of 0.70 m/s^2 and is lifted from rest through a distance of 11 m.

(a) What is the tension in the cable?

(b) How much work is done by the tension in the cable

(c) How much work is done by the person's weight?

(d) Use the work-energy theorem and find the final speed of the person.

a) $T - mg = ma$, $T = 8.3 \times 10^2$ N

b) $W_T = Td = 9.13 \times 10^3$ J

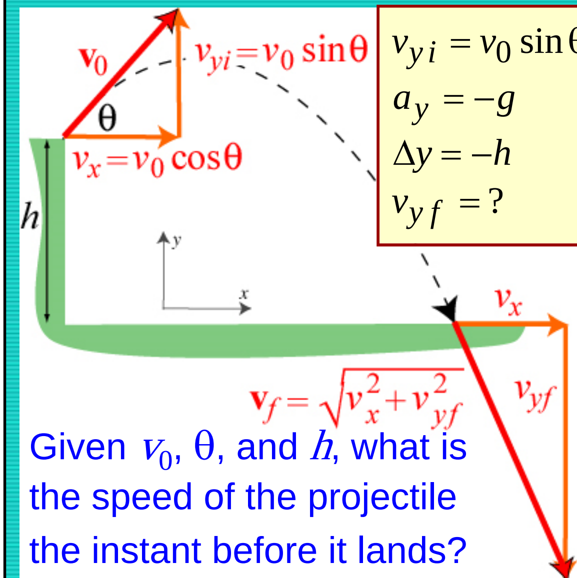
c) $W_w = -mgd = -8.5 \times 10^3$ J

d) $W_T + W_W = 1/2\ mv_f^2 - 1/2\ mv_o^2$

$v_f = 3.92$ m/s

Dr. Abdallah M.Azzeer 12

How would you attack this problem?



$$\left.\begin{aligned} v_{yi} &= v_0 \sin\theta \\ a_y &= -g \\ \Delta y &= -h \\ v_{yf} &= ? \end{aligned}\right\} v_{yf}^2 = v_{yi}^2 + 2a_y \cdot \Delta y$$

Eventually obtaining...

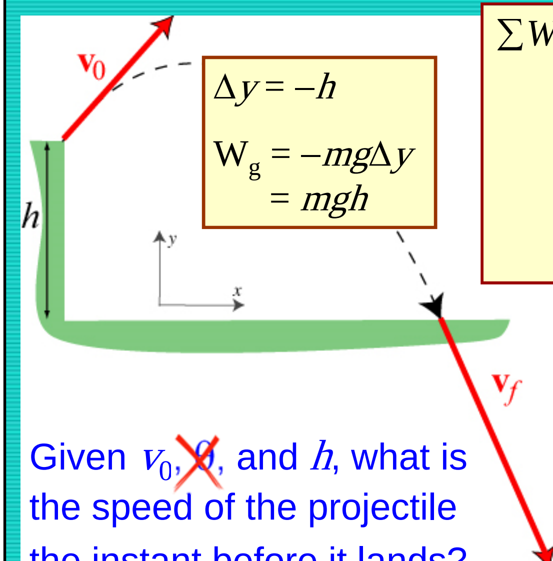
$$v_f = \sqrt{v_0^2 + 2gh}$$

Independent of θ!...

Given v_0, θ, and h, what is the speed of the projectile the instant before it lands?

OLD WAY

WORK – ENERGY THEOREM



$$\Delta y = -h$$

$$W_g = -mg\Delta y = mgh$$

$$\sum W = W_g = K_f - K_i$$

$$mgh = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2$$

$$v_f^2 = v_0^2 + 2gh$$

$$v_f = \sqrt{v_0^2 + 2gh}$$

Given v_0, ~~θ~~, and h, what is the speed of the projectile the instant before it lands?

NEW WAY

Example

Two blocks have masses m_1 and m_2, where $m_1 > m_2$. They are sliding on a frictionless floor and have the same kinetic energy when they encounter a long rough stretch (i.e. $\mu > 0$) which slows them down to a stop. Which one will go farther before stopping?

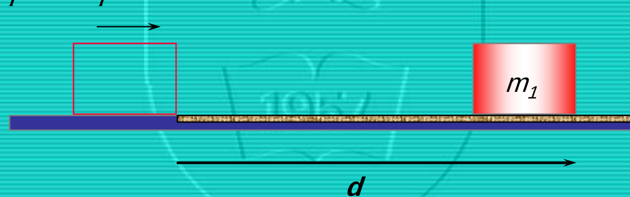
(a) m_1 (b) m_2 (c) *they will go the same distance*

Solution

for the first mass m_1

- **The work-energy theorem says that for any object $W = DK$**
- **In this example the only force that does work is friction (since both N and mg are perpendicular to the blocks motion).**
- **The net work done to stop the box is $-f\,d = -\mu\, mg\, d$.**

This work "removes" the kinetic energy that the box had:
$W = K_f - K_i = 0 - K_i$



for the second mass m_2

- **The net work done to stop a box is – $f d$ = - $\mu mg d$.**
  - **This work "removes" the kinetic energy that the box had:**
  - $W = K_f - K_i = 0 - K_i$
- **This is the same for both boxes (same starting kinetic energy).**

$$\mu m_2 g d_2 = \mu m_1 g d_1 \Rightarrow m_2 d_2 = m_1 d_1$$

Since $m_1 > m_2$ we can see that $d_2 > d_1$

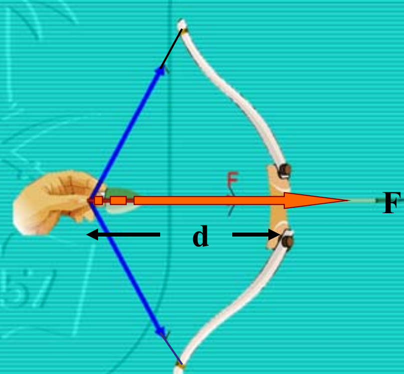
103 PHYS Dr. Abdallah M.Azzeer

Example

0.075-kg arrow is fired horizontally. The bowstring exerts an average force of 65 N on the arrow over a distance of 0.90 m. With what speed does the arrow leave the bow?

$$W_{net} = F \bullet d = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$v_f = 39$ m/s



Dr. Abdallah M.Azzeer 18

7.8 Power

Power is the rate at which work is done by a force

$P_{AVG} = W/\Delta t$ ***Average Power***

$P = dW/dt$ ***Instantaneous Power***

The unit of power is a Joule/second (J/s) which we define as a Watt (W)

1 W = 1 J/s = 1 kg.m²/s³

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\vec{F}\cdot\vec{x}\right)$$

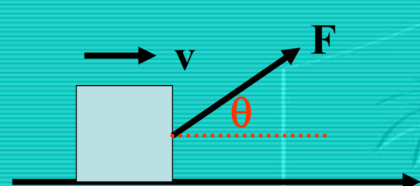
$$P = \vec{F}\cdot\frac{d\vec{x}}{dt} = \vec{F}\cdot\vec{v}$$

In general, power is defined for any type of energy transfer.

$P = dE/dt$

Where dE/dt is the rate at which energy is crossing the boundary of the system by a given transfer mechanism.

Dr. Abdallah M.Azzeer 19



$$\Delta W = F\Delta x\cos\theta$$

$$\Delta x = v\Delta t$$

$$P = \frac{\Delta W}{\Delta t} = \frac{F\Delta x\cos\theta}{\Delta t} = Fv\cos\theta$$

Note: power × time = work, so work can be measured in units of kW·h

(1 KWh = (10^3 Watt) × (3600 s) = 3.6×10^6 W s = 3.6×10^6 J =3.6 MJ.)

- **British units are hp (horse power)**
- **1 hp=550 ft.lb/s = 746 W**

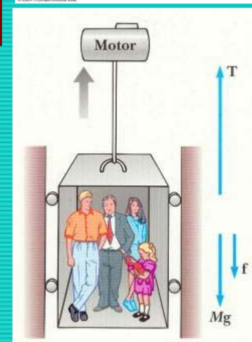
Dr. Abdallah M.Azzeer 20

Example 7.12

An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg . A constant friction forces of 4000 N retards its motion upward, as shown in the figure.

(a) What power deliver by the motor is required to lift the elevator car at a constant speed of 3 m/s?

(b) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of 1 m/s^2?



(a) M_{max} = 1800 kg, f = 4000 N

v = 3 m/s (constant) ➡ $a = 0$

$$\sum F = T - f - Mg = 0$$

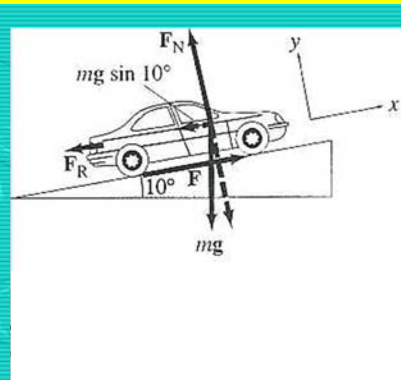
$$T = f + Mg = 4000\ N + 1800 \times 9.8\ N = 2.16 \times 10^4\ N$$

$$\textit{Power}: P = \vec{T} \cdot \vec{v} = Tv = \left(2.16\times10^4\right)\left(3\right) = 64.8\ kW$$

(b) Left for you to try

Example : Power Needs of Car

Calculate the power neded far the car (a) to climb a hill. (b) to pass another car.



(a)

$$\sum F_x = 0$$

$$F - F_R - mg\sin\theta = 0$$

$$P = Fv$$

(b)

Now, $\theta = 0$

$$\sum F_x = ma$$

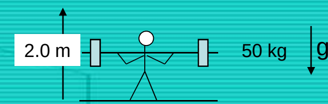
$$F - F_R = 0$$

$$v = v_0 + at$$

$$P = Fv$$

Problem: determine the power after an elapsed time of 3.0 s

$$\bar{P} = \frac{mgh}{\Delta t} = \frac{50\ \text{kg} \times 9.8\ \text{m/s} \times 2.0\,\text{m}}{3.0\,\text{s}} = 330\,\text{W}$$



Dr. Abdallah M.Azzeer

23