CHAPTER 9

LINEAR MOMENTUM , IMPULSE AND COLLISIONS

9.1 Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

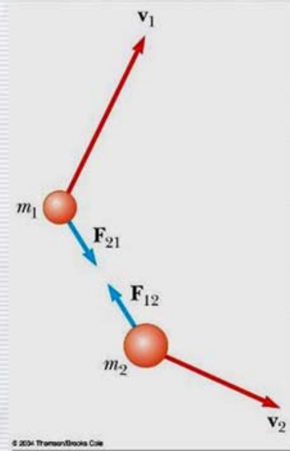
Linear momentum of an object whose mass is m and is moving at a velocity of v is defined as

$$\boxed{\vec{p} = m\vec{v}}$$

Consider two particles interact with each other

By Newton's 3rd law:

$$\vec{F}_{21} = -\vec{F}_{12}$$
$$\vec{F}_{21} + \vec{F}_{12} = 0$$
$$m_1\vec{a}_1 + m_2\vec{a}_2 = 0$$
$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0$$
$$\frac{d(m_1\vec{v}_1)}{dt} + \frac{d(m_2\vec{v}_2)}{dt} = 0$$
$$\frac{d(m_1\vec{v}_1 + m_2\vec{v}_2)}{dt} = 0$$



Linear momentum $\vec{p} \equiv m\vec{v}$

$$\frac{d(\mathbf{p}_1 + \mathbf{p}_2)}{dt} = 0$$

i.e., Total momentum $\vec{p}_{tot} = \sum \vec{p} = \vec{p}_1 + \vec{p}_2$ remains constant

What can you tell from this definition about momentum?

- Momentum is a vector quantity.
- The heavier the object the higher the momentum
- The higher the velocity the higher the momentum
- Its unit is ***kg.m/s***

What else can use see from the definition? Do you see force?

$$\vec{p} = m\vec{v} \qquad (\text{kg·m/sec.})$$

VECTOR ⇒ $p_x = mv_x \;,\; p_y = mv_y \;,\; p_z = mv_z$

Relationship to force:

$$\frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\cancel{\frac{dm}{dt}}$$
$$= m\mathbf{a}$$
$$= \sum \mathbf{F}$$

Mass usually doesn't change with time

Improved form of 2nd Law: $\sum \vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$

Probably was Newton's original form of 2nd Law.
Works even when mass does change, *e.g.*, rockets.

***Momentum is conserved** in the absence of a **net external** force.*

$\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$: *Constant* ⇒ *Conservation of Total Momentum*

$$\therefore \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Momentum Conservation

$$\vec{F}_{EXT} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow \boxed{\frac{\Delta\vec{P}}{\Delta t} = 0} \Leftarrow \vec{F}_{EXT} = 0$$

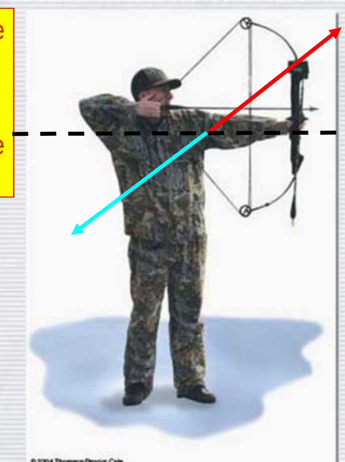
- The concept of momentum conservation is one of the most fundamental principles in physics.
- This is a component (vector) equation.
  - We can apply it to any direction in which there is no external force applied.
- You will see that we often have momentum conservation even when energy is not conserved.

Example 9.1

An archer standing at rest on frictionless ice. He fires an arrow horizontally.

m_1 = 60 kg , m_2 = 0.5 kg , v_2 = 50 m/s

What is the velocity of the archer after firing the arrow?



$$m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = 0$$

$$\vec{v}_{1f} = -\frac{m_2}{m_1}\vec{v}_{2f} = -\ 0.42\ m/s$$

If the arrow were shot at an angle θ with theh horizontal, what is the recoil velocity

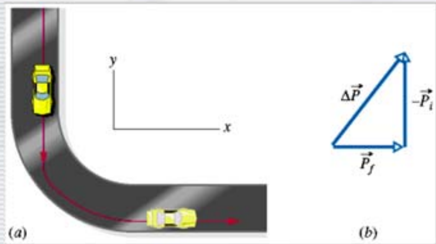
$$m_1 v_{1f} + m_2 v_{2f}\cos\theta = 0$$

$$v_{1f} = -\frac{m_2}{m_1} v_{2f}\cos\theta$$

READ Examples 9.2

Example:

Figure below shows a 2.0 kg toy race car before and after taking a turn on a track. Its speed is 0.50 m/s before the turn and 0.40 m/s after the turn. What is the change $\Delta \vec{P}$ in the linear momentum of the car due to the turn?



$$\vec{v}_i = -(0.50\ m/s)\hat{j} \quad and \quad \vec{v}_f = (0.40\ m/s)\hat{i}$$

$$\vec{P}_i = M\vec{v}_i = (2.0\ kg)(-0.50\ m/s)\hat{j} = (-1\ kg\cdot m/s)\hat{j}$$

$$\vec{P}_f = M\vec{v}_f = (2.0\ kg)(0.40\ m/s)\hat{i} = (0.80\ kg\cdot m/s)\hat{i}$$

$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

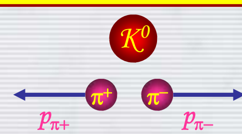
$$\Delta\vec{P} = (0.80\ kg\cdot m/s)\hat{i} - (-1.0 kg\cdot m/s)\hat{j}$$
$$= (0.8\,\hat{i} + 1.0\,\hat{j})\ kg\cdot m/s$$

Example 9.2

A type of particle, neutral kaon (K^0) decays (breaks up) into a pair of particles called pions (π^+ and π^-) that are oppositely charged but equal mass. Assuming K^0 is initially produced at rest, prove that the two pions must have momenta that are equal in magnitude and opposite in direction.

This reaction can be written as

$$\boxed{K^0 \rightarrow \pi^+ + \pi^-}$$



Since this system consists of a K^0 in the initial state which results in two pions in the final state, the momentum must be conserved. So we can write

$$\vec{p}_{K^0} = \vec{p}_{\pi^+} + \vec{p}_{\pi^-}$$

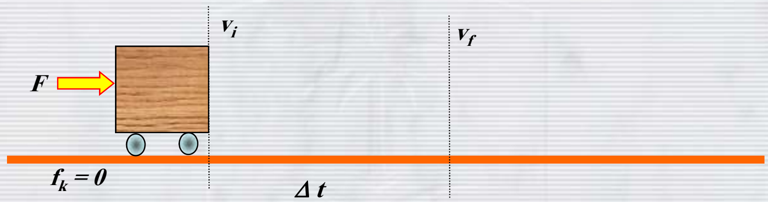
Since K^0 is produced at rest its momentum is 0.

$$\vec{p}_{K^0} = \vec{p}_{\pi^+} + \vec{p}_{\pi^-} = 0$$
$$\vec{p}_{\pi^+} = -\vec{p}_{\pi^-}$$

Therefore, the two pions from this kaon decay have the momenta with same magnitude but in opposite direction.

9.2 Impulse and Momentum

The change of momentum in a given time interval



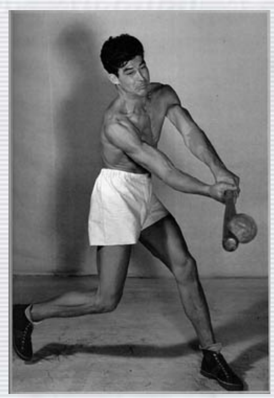
$$\bar{a} = \frac{v_f - v_i}{\Delta t}, \quad \because \bar{F} = m\bar{a} = m\left(\frac{v_f - v_i}{\Delta t}\right)$$

$$\therefore \quad \bar{F}\Delta t = m v_f - m v_i = p_f - p_i = \Delta p$$

Impulse = force times time=change in momentum $\boxed{\vec{I} = \vec{F}\Delta t = \Delta\vec{p}}$

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m(\vec{v} - \vec{v}_0)}{\Delta t} = m\frac{\Delta \vec{v}}{\Delta t} = m\vec{a} = \sum \vec{F}$$

Impulse and Momentum



A classic photograph taken by Arthur Edgerton at MIT in 1935 showing the moment of impact between bat and softball. The huge force exerted by the bat on the ball causes severe distortion of the ball as it is hit.

Impulse and Momentum

- Newton's 2nd Law:

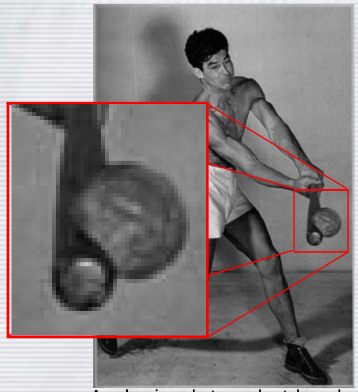
$$\vec{F} = \frac{\Delta}{\Delta t}\,(\vec{p})$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{F}\,\Delta t$$

Impulse $\vec{I} = \vec{F}\,\Delta t$

→Impulse-momentum theorem



A classic photograph taken by Arthur Edgerton at MIT in 1935 showing the moment of impact between bat and softball. The huge force exerted by the bat on the ball causes severe distortion of the ball as it is hit.

Impulse and Momentum

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}\,dt$$

In general, $\vec{F} = \vec{F}(t)$

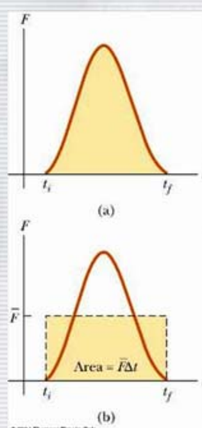
→Impulse = area under $\vec{F}, t$ curve

$$\bar{\vec{F}} = \frac{1}{\Delta t}\int_{t_i}^{t_f} \vec{F}\,dt$$

$$\vec{I} = \bar{\vec{F}}\,\Delta t$$

Simple case: constant Force

$$\vec{I} = \vec{F}\,\Delta t$$



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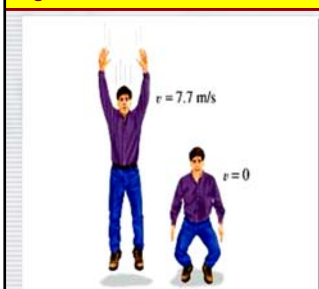
Example;

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7 m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$I = \overline{F}\Delta t = \Delta p = p_f - p_i = 0 - mv =$$

$$= -70kg \cdot 7.7m/s = -540N \cdot s$$

Example; cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $\bar{v} = \frac{0+v_i}{2} = \frac{7.7}{2} = 3.8m/s$

The time period the collision lasts is $\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6\times10^{-3}s$

Since the magnitude of impulse is $I = \bar{F}\Delta t = 540N\cdot s$

The average force on the feet during this landing is $\bar{F} = \frac{I}{\Delta t} = \frac{540}{2.6\times10^{-3}} = 2.1\times10^{5}N$

How large is this average force? $Weight = 70kg\cdot9.8m/s^2 = 6.9\times10^2N$

$$\bar{F} = 2.1\times10^5N = 304\times6.9\times10^2N = 304\times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

Example ; cont'd

What if the knees are bent in coming to rest? The body decelerates from 7.7 m/s to 0 m/s in a distance d=50 cm=0.5 m.

The average speed during this period is still the same $\bar{v} = \frac{0+v_i}{2} = \frac{7.7}{2} = 3.8m/s$

The time period the collision lasts changes to $\Delta t = \frac{d}{\bar{v}} = \frac{0.5m}{3.8m/s} = 1.3\times10^{-1}s$

Since the magnitude of impulse is $I = \bar{F}\Delta t = 540N\cdot s$

The average force on the feet during this landing is $\bar{F} = \frac{I}{\Delta t} = \frac{540}{1.3\times10^{-1}} = 4.1\times10^{3}N$

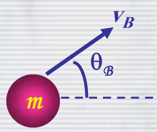
How large is this average force? $Weight = 70kg\cdot9.8m/s^2 = 6.9\times10^2N$

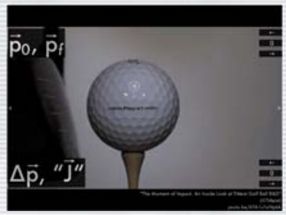
$$\bar{F} = 4.1\times10^3N = 5.9\times6.9\times10^2N = 5.9\times Weight$$

It's only 6 times the weight that the feet have to sustain! So by bending the knee you increase the time of collision, reducing the average force exerted on the knee, and will avoid injury!

Example 9.3

A golf ball of mass 50 g is struck by a club. The force exerted on the ball by the club varies from 0, at the instant before contact, up to some maximum value at which the ball is deformed and then back to 0 when the ball leaves the club. Assuming the ball travels 200m, estimate the magnitude of the impulse caused by the collision.





Note the deformation of the ball due to the large force from the club

The range R of a projectile is

$$R = \frac{v_B^2 \sin 2\theta_B}{g} = 200m$$

Let's assume that launch angle θ_i=45°.
Then the speed becomes:

$$v_B = \sqrt{200 \times g} = \sqrt{1960} = 44m/s$$

Considering the time interval for the collision, t_i and t_f, initial speed and the final speed are

$$v_i = 0 \text{ (immediately before the collision)}$$

$$v_f = 44\ m/s \text{(immediately after the collision)}$$

Therefore the magnitude of the impulse on the ball due to the force of the club is

$$\left|\vec{I}\right| = \left|\Delta\vec{p}\right| = m v_{Bf} - m v_{Bi} = 0 + 0.05 \times 44 = 2.2\ kg \cdot m/s$$

What is the average force on the ball during the collision with the club?

$$\overline{F} = \frac{I}{\Delta t} = 200 \text{ N}$$