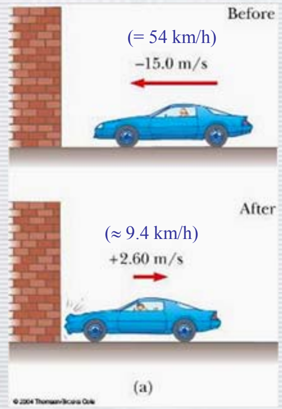
Examples 9.4

In a crash test, an automobile of mass 1500 kg collides with a wall. The initial and final velocities of the automobile are v_i=-15.0 m/s and v_f=2.60 m/s in the x-direction. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?



Let's assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From the problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0) = -22500\ kg \cdot m/s$$

$$\vec{p}_f = m\vec{v}_f = 1500 \times (2.60) = 3900\ kg \cdot m/s$$

Therefore the impulse on the automobile due to the collision is

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\ kg \cdot m/s$$
$$= 26400\ kg \cdot m/s = 2.64 \times 10^4 kg \cdot m/s$$

The average force exerted on the automobile during the collision is

$$\overline{\vec{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150} = 1.76 \times 10^5\ N$$

Read the rest of the example

Sample Problem

A pitched 140 g baseball, in horizontal flight with a speed v_i of 39.0 m/s, is struck by a bat. After leaving the bat, the ball travels in the opposite direction with speed v_f , also 39.0 m/s.

(a) What impulse I acts on the ball while it is in contact with the bat during the collision?

$$I = p_f - p_i = mv_f - mv_i$$
$$= (0.140\ kg)(39.0\ m/s) - (0.140\ kg)(-39.0\ m/s)$$
$$= 10.9\ kg \cdot m/s$$

Note that I is the impulse on the ball. The final direction of the ball is positive.

(b) The impact time Δt for the baseball-bat collision is 1.20 ms. What average force acts on the baseball?

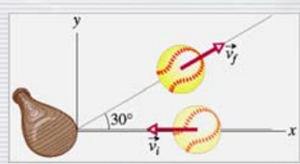
$$F_{avg} = \frac{I}{\Delta t} = \frac{10.9\ kg \cdot m/s}{0.00120\ s} = 9080\ N$$

Note that this average force is from the bat to the ball. The positive direction of the force is in the final velocity of the ball.

(c) Now suppose the collision is not head-on, and the ball leaves the bat with a speed v_f of 45.0 m/s at an upward angle of 30.0°. What now is the impulse on the ball?

The impulse on the ball is :

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i$$



$$I_x = p_{fx} - p_{ix} = m(v_{fx} - v_{ix})$$
$$= (0.140\ kg)[(45.0\ m/s)(\cos 30.0^\circ) - (-39.0\ m/s)] = 10.92\ kg \cdot m/s$$

$$I_y = p_{fy} - p_{iy} = m(v_{fy} - v_{iy})$$
$$= (0.140\ kg)[(45.0\ m/s)(\sin 30.0^\circ) - 0]$$
$$= 3.150\ kg \cdot m/s$$

$$\vec{I} = (10.9\,\hat{i} + 3.15\,\hat{j})\ kg \cdot m/s$$

$$I = \sqrt{I_x^2 + I_y^2} = 11.4\ kg \cdot m/s$$

$$\theta = \tan^{-1}\frac{I_y}{I_x} = 16^\circ$$

Sample Problem

A ballot box with mass m = 6.0 kg slides with speed v = 4.0 m/s across a frictionless floor in the positive direction of an x axis. It suddenly explodes into two pieces. One piece, with mass m_1 = 2.0 kg, moves in the positive direction of the x axis with speed v_1 = 8.0 m/s. What is the velocity of the second piece, with mass m_2 ?

$$\vec{P}_i = m\vec{v}$$

$$\vec{P}_{f\,1} = m_1\vec{v}_1 \quad and \quad \vec{P}_{f\,2} = m_2\vec{v}_2$$

$$\vec{P}_f = \vec{P}_{f\,1} + \vec{P}_{f\,2} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$P_i = P_f$$

$$mv = m_1v_1 + m_2v_2$$

$$(6.0\,kg)(4.0\,m/s) = (2.0\,kg)(8.0\,m/s) + (4.0\,kg)v_2$$

$$v_2 = 2.0\,m/s$$

MOMENTUM CONSERVATION

Consider two objects, 1 and 2, and assume that no external forces are acting on the system composed of these two particles.

Impulse applied to object 1

$$\vec{F}_{21}\Delta t = m_1\vec{v}_1 - m_1\vec{u}_1$$

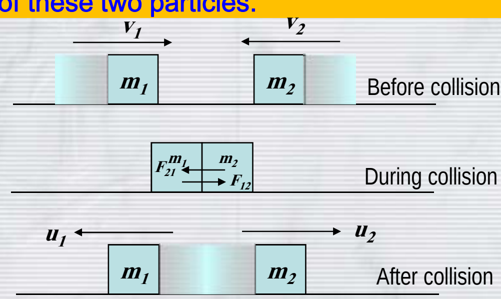
Impulse applied to object 2

$$\vec{F}_{12}\Delta t = m_2\vec{v}_2 - m_2\vec{u}_2$$

Apply Newton's Third Law

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$or \;\; \vec{F}_{12}\Delta t = -\vec{F}_{21}\Delta t$$



Total impulse applied to system $\quad 0 = m_1\vec{v}_1 - m_1\vec{u}_1 + m_2\vec{v}_2 - m_2\vec{u}_2$

or

$$\boxed{m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2}$$

➡ **Net momentum of the two objects before and after collision is the same**

➡ **THE TOTAL MOMENTUM OF THE SYSTEM IS CONSERVED**

➡ For conservation of momentum, the external forces must be zero

In one dimension in component form,

$$m_1 v_{1x} + m_2 v_{2x} = m_1 u_{1x} + m_2 u_{2x}$$
$$m_1 v_{1y} + m_2 v_{2y} = m_1 u_{1y} + m_2 u_{2y}$$

Examples 7.3

A neutron moving at 2700 m/s collides head on with a nitrogen nucleus at rest and is absorbed. The neutron and nitrogen masses are m = 1.67 × 10^{-27} kg - and M = 23.0 ×10^{-27} kg, respectively. What is the final velocity of the combined object?

Momentum before; $p_i = mv$

Momentum after; $p_f = (m+M)u$

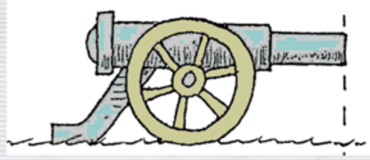
Net force = 0

➡ **mv = (m+M)u** ⇨ $u = \dfrac{mv}{(m+M)} = 183$ m/s

Whenever there is no net external force acting on a system. its momentum is conserved. Momentum is therefore always conserved for an isolated system, one subjected only to internal forces.

Examples 7.4

A cannon is mounted inside a railroad car, which is initially at rest but can move frictionlessly (Fig. 7.5). It fires a cannonball of mass m = 5 kg with a horizontal velocity v = 15 m/s relative to the ground at the opposite wall. The total mass of the cannon and railroad car is M = 15,000 kg. (Assume that the mass of the exhaust gases is negligible.) (a) What is the velocity V of the car while the cannonball is in flight? (b) If the cannonball becomes embedded in the wall, what is the velocity of the car and ball after impact?



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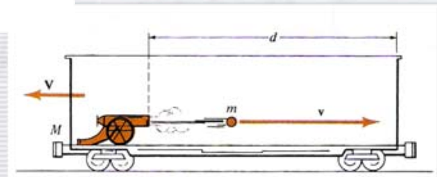


(a) Momentum before; $p_i = 0$

Momentum after; $p_f = mv+MV$

Net force = 0

➡ **0 = mv+MV** ⇨ $V = -\dfrac{m}{M}v = -5\times10^{-3}$ m/s

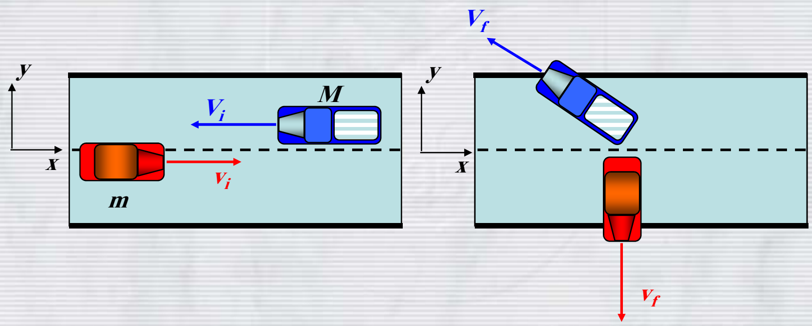
The recoil speed of the car and cannon is very small because of their large mass.

(b) As the ball becomes embedded in the wall, it exerts a force on the wall to the right in Fig. 7.5. The wall, in turn, exerts a force to the left on the ball. The ball and car both stop moving when this happens, since the net momentum is still zero. Meanwhile the car will have rolled to the left as the ball travelled to the right.

It is important to realize that momentum is a vector quantity, and that if the total momentum of a system is constant, each component must be constant.

Examples 7.6

A car of mass m = 1000 kg moving at 30 m/s collides with a car of mass M = 2000 kg travelling at 20 m/s in the opposite direction. Immediately after the collision, the 1000-kg car moves at right angles to its original direction at 15 m/s. Find the velocity of the 2000-kg car right after the collision



Conservation of momentum:

$$\boxed{m\vec{v}_i + M\vec{V}_i = m\vec{v}_f + M\vec{V}_f \quad (1)}$$

Split into components:

$$mv_{ix} + MV_{ix} = mv_{fx} + MV_{fx} \quad (2)$$

$$mv_{iy} + MV_{iy} = mv_{fy} + MV_{fy} \quad (3)$$

Where ; v_{ix} = +30 m/s, v_{fx} = 0, V_{ix} = -20 m/s

v_{iy} = 0, V_{iy} = 0, v_{fy} = - 15 m/s, V_{fy} = ?

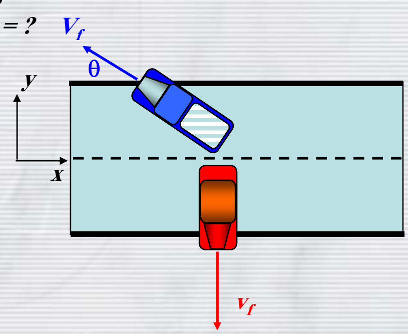
From (2) ➡ $V_{fx} = \dfrac{mv_{ix} + MV_{ix}}{M} = -5$ m/s

From (3) ➡ $V_{fy} = -\dfrac{m}{M} v_{fy} = +7.5$ m/s

MORE

$$|V_f| = \sqrt{V_{fx}^2 + V_{fy}^2} = 9 \text{ m/s}$$

$$\theta = \tan^{-1}(V_{fy}/V_{fx}) = 56.3^\circ$$

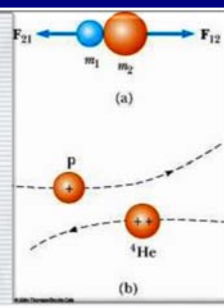


9.3 Collisions in 1D

Elastic and Inelastic COLLISIONS

Collisions involve forces internal to colliding bodies.

- Perfectly elastic collisions - conserve energy and momentum
- Inelastic collisions - conserve momentum
  - Totally inelastic collisions - conserve momentum and objects stick together



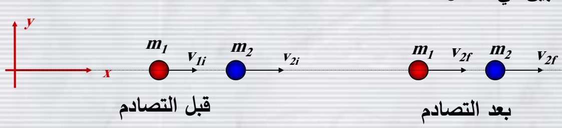
التصادم المرن (Elastic collision)

فى هذا النوع تكون كمية الحركة *momentum* و الطاقة الحركية *kinetic energy* قبل التصادم محافظة *conserved*

أي أن

كمية الحركة قبل التصادم = كمية الحركة بعد التصادم
الطاقة الحركية قبل التصادم = الطاقة الحركية بعد التصادم

افترض جسمان يتحركان بسرعة v_{1i} و v_{2i} في اتجاه x الموجب وبعد التصادم تكون سرعتهما v_{1f} و v_{2f} على التوالي ، كما هو مبين في الشكل:



مع ملاحظة أن كل سرعة موجبة ($v > 0$) تعني أن الجسيم يتحرك باتجاه محور x الموجب وكل سرعة سالبه ($v < 0$) تعني أن الجسيم يتحرك باتجاه محور x السالب أو الى اليسار.

من قانون حفظ كمية الحركة :

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

وباعتبار أن التصادم مرن *elastic collision* فان الطاقة الحركية تكون محافظه، وعليه فان :

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

إذا كانت v_{1i} و v_{2i} و m_1 و m_2 معلومة مسبقا , فانه يمكن إيجاد v_{1f} و v_{2f} على النحو التالي:
يمكن كتابة المعادلة (1) على الصورة :

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

وكذلك المعادلة (2) على الصورة :

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

أو

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

بقسمة (4) على (3) مع افتراض أن $v_{1i} \neq v_{1f}$ و $v_{2i} \neq v_{2f}$ نحصل على :

$$v_{1i} + v_{1f} = v_{2f} + v_{2i} \ldots\ldots\ldots\ \ldots\ldots\ldots (5)$$

أو

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

يلاحظ من العلاقة السابقة أن السرعة النسبية للجسمين قبل التصادم هي نفسها بعد التصادم.
من العلاقة (5) نجد أن :

$$v_{2f} = v_{1i} - v_{2i} + v_{1f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ldots (6)$$

وبالتعويض عن v_{2f} من (6) فى المعادلة (1) نجد أن :

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{1i} - m_2v_{2i} - m_2v_{1f}$$

$$(m_1 - m_2)v_{1i} + 2m_2v_{2i} = (m_1 + m_2)v_{1f}$$

$$\therefore \quad v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \ldots\ldots\ldots\ldots (7)$$

وبالتعويض عن v_{1f} من (7) فى المعادلة (6) نجد أن (بعد التبسيط الرياضي) :

$$\therefore \quad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i} \ldots\ldots\ldots\ldots (8)$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

حالات خاصة:

(1) إذا كانت الكتلتين المتصادمة متساويتين ($m_1=m_2=m$) مثل تصادم كرتي بليارد ، فان:

$$v_{1f} = v_{2i}$$

$$v_{2f} = v_{1i}$$

أي أن الجسمان المتصادمان يتبادلان السرعة. وإذا كان الجسم الثاني ساكنا قبل التصادم ($v_{2i}=0$) فان:

$$v_{1f} = 0$$

$$v_{2f} = v_{1i}$$

(2) إذا كانت $m_1 >> m_2$ (مثل أن يكون الجسم الأول كرة بولنج والجسم الثاني كرة تنس طاولة)، والجسم الثاني ساكنا قبل التصادم ($v_2=0$) ففي هذه الحالة تكون :

$$v_{1f} \approx v_{1i}$$

$$v_{2f} \approx 2v_{1i}$$

أي أن سرعة كرة تنس الطاولة بعد التصادم تكون ضعف سرعة كرة البولنج قبل التصادم تقريبا ، وسرعة كرة البولنج بعد التصادم تقريبا لا تتغير عما هي عليه قبل التصادم.

(3) إذا كانت $m_1 << m_2$ (مثل أن يكون الجسم الأول كرة تنس طاولة والجسم الثاني كرة بولنج)، والجسم الثاني ساكنا قبل التصادم ($v_2=0$) ففي هذه الحالة تكون :

$$v_{1f} \approx -v_{1i}$$

$$v_{2f} \approx 0$$

أي أن الجسم الثاني ذو الكتلة الثقيلة (كرة البولنج) يظل تقريبا ثابت وسرعة بعد التصادم تساوي تقريبا الصفر في حين يرتد الجسم الأول ذو الكتلة الخفيفة (كرة تنس الطاولة) فى الاتجاه المعاكس و بنفس السرعة تقريبا قبل التصادم.

التصادم الغيرمرن *Inelastic collision*

فى هذا النوع تكون كمية الحركة ***momentum*** فقط محافظة ***conserved***

أي أن

كمية الحركة قبل التصادم = كمية الحركة بعد التصادم

وهناك فقد في الطاقة الحركية في هذا النوع من التصادم

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