جامعة الملك سعود

كلية العلوم

قسم الفيزياء والفلك

مذكرة المقرر 104 فيز

تنسيق: أ.د. ناصر بن صالح الزايد

الباب 23

محاضرة رقم 2 (صيفي)

أجزاء كبيرة من هذه المذكرة معتمدة على عروض الأستاذة نورة علي المنيف - قسم الفيزياء.

2019

Physics 104
Chapter 23

Electric Field

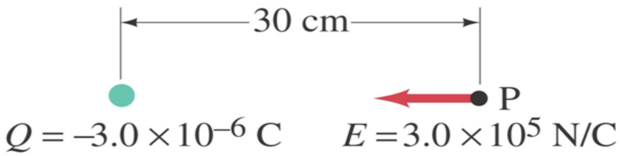
Lecture No. 02

The Electric Field

Example : Electric field of a single point charge.

(a) Calculate the magnitude and direction of the electric field at a point P which is 30 cm to the right of a point charge $Q = -3.0 \times 10^{-6}$ C.

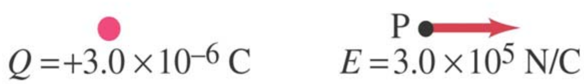
$$E = k\frac{|q|}{r^2}$$



(a)

$$E = \frac{(9.0 \times 10^9 \text{ N-m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})}{(0.30 \text{ m})^2}$$

$E = 3.05 \times 10^5$ N/C *towards* the charge



(b)

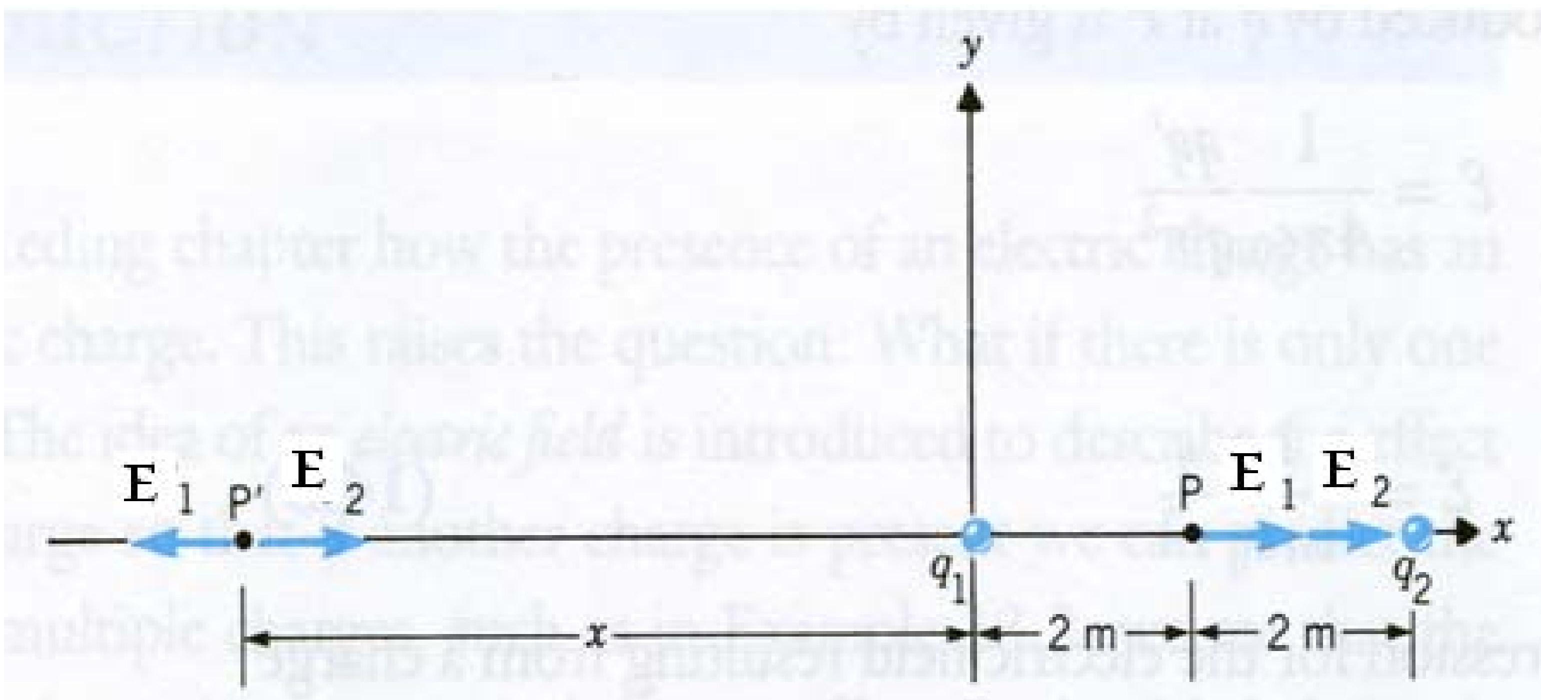
(b) due to a positive charge Q, each 30 cm from P.
Solution: Substitution gives $E = 3.0 \times 10^5$ N/C. The field points away from the positive charge and towards the negative one

Example

- A charge $q_1 = 3 \times 10^{-6}\ \mathrm{C}$ is located at the origin of the x axis. A second charge $q_2 = -5 \times 10^{-6}\ \mathrm{C}$ is also on the *x* axis 4 m from the origin in the positive *x* direction

(a) Calculate the electric field at the midpoint *P* of the line joining the two charges.

(b) At what point P' on that line is the resultant field zero?



Example - Solution

(a) Since q_1 is positive and q_2 is negative, at any point between them, both electric fields produced by them are the same direction which is toward to q_2. Thus,

$$\mathbf{E}_1 = 9 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2} \frac{3 \times 10^{-6}\ \text{C}}{(2\ \text{m})^2} = 6.75 \times 10^3\ \text{N/C}$$

$$\mathbf{E}_2 = 9 \times 10^9 \frac{\text{N-m}^2}{\text{C}^2} \frac{5 \times 10^{-6}\ \text{C}}{(2\ \text{m})^2} = 11.25 \times 10^3\ \text{N/C}$$

The resultant electric field E at P is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = 18 \times 10^3\ \text{N/C}$$

(b) It is clear that the resultant E can not be zero at any point between q_1 and q_2 because both E_1 and E_2 are in the same direction. Similarly E can not be zero to the right of q_2 because the magnitude of q_2 is greater then q_1 and the distance r is smaller for q_2 than q_1. Thus, can only be zero to the left of q_1 at some point P' to be found. Let the distance from P' to q_1 be x.

$$k\frac{q_1}{x^2} = k\frac{q_2}{(x+4)^2}$$

$$3(x+4)^2 = 5x^2$$

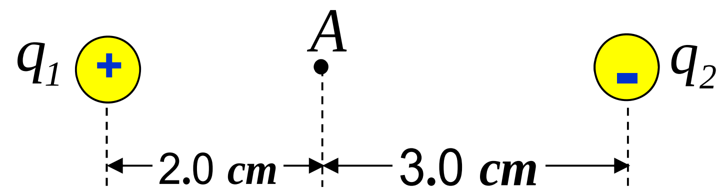
$$2x^2 - 24x - 48 = 0$$

$$x = 13.75m \quad , \quad x = -1.75m$$

Apparently, we need to take *x* which is negative.

Example

Two point charges, q_1=1.0 C and q_2=-4.0 C, are placed 2.0 cm and 3.0 cm from the point A respectively as shown in the figure.

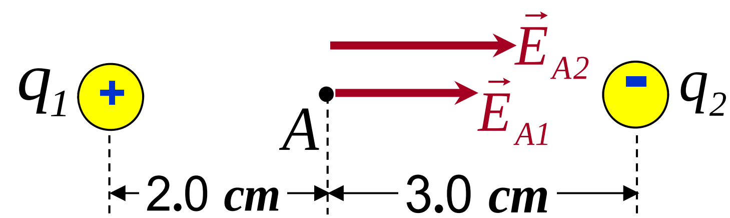


Find

a) the magnitude and direction of the electric field intensity at point A.

b) the resultant electric force exerted on q =4.0 C if it is placed at point A.

(Given Coulomb's constant, k = 9.0 x 10^9 N m^2 C^{-2})



a)

$$E_{A1} = \frac{kq_1}{r_1^2} = \frac{(9.0\times10^9)(1.0)}{(2.0\times10^{-2})^2} = 2.3\times10^{13} \text{ N C}^{-1} \rightarrow$$

$$E_{A2} = \frac{kq_2}{r_2^2} = \frac{(9.0\times10^9)(4.0)}{(3.0\times10^{-2})^2} = 4.0\times10^{13} \text{ N C}^{-1} \rightarrow$$

$$\vec{E}_A = \vec{E}_{A1} + \vec{E}_{A2}$$

$$\left|\vec{E}_A\right| = 2.3\times10^{13} + 4\times10^{13} = 6.3\times10^{13}\,N/C$$

Direction : to the right

b)

$$E_A = \frac{F_A}{q}$$

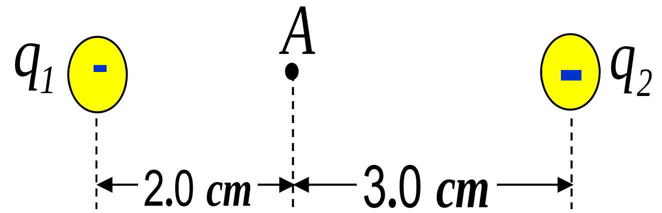
$$F_A = qE_A$$

$$F_A = 4\times6.3\times10^{13} = 24.2N\times10^{13}$$

Direction : to the right

Example

Two point charges, q_1=-1.0 C and q_2=-4.0 C, are placed 2.0 cm and 3.0 cm from the point A respectively as shown in the figure.



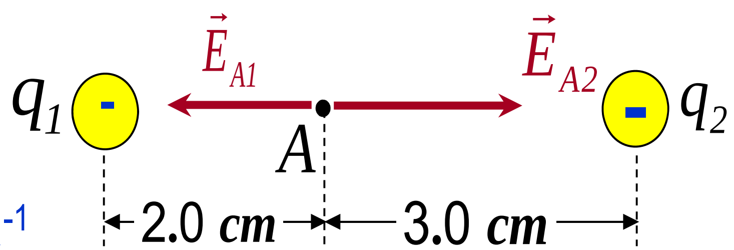
Find

a) the magnitude and direction of the electric field intensity at point A.

b) the resultant electric force exerted on q =4.0 C if it is placed at point A.

(Given Coulomb's constant, k = 9.0 x 10^9 N m^2 C^{-2})

a)



$$E_{A1} = \frac{kq_1}{r_1^2} = \frac{(9.0\times10^9)(1.0)}{(2.0\times10^{-2})^2} = 2.3\times10^{13}\ \text{N C}^{-1}$$

Direction : to the left

$$E_{A2} = \frac{kq_2}{r_2^2} = \frac{(9.0\times10^9)(4.0)}{(3.0\times10^{-2})^2} = 4.0\times10^{13}\ \text{N C}^{-1}$$

Direction : to the right

The electric field strength at point A due to the charges is given by

$$\vec{E}_A = \vec{E}_{A1} + \vec{E}_{A2}$$

$$\left|\vec{\boldsymbol{E}}_{\boldsymbol{A}}\right| = -2.3\times10^{13} + 4.0\times10^{13}$$

$$\left|\vec{\boldsymbol{E}}_{\boldsymbol{A}}\right| = 1.7\times10^{13} \text{ N C}^{-1}$$

Direction : to the right

b)

$$E_A = \frac{F_A}{q}$$

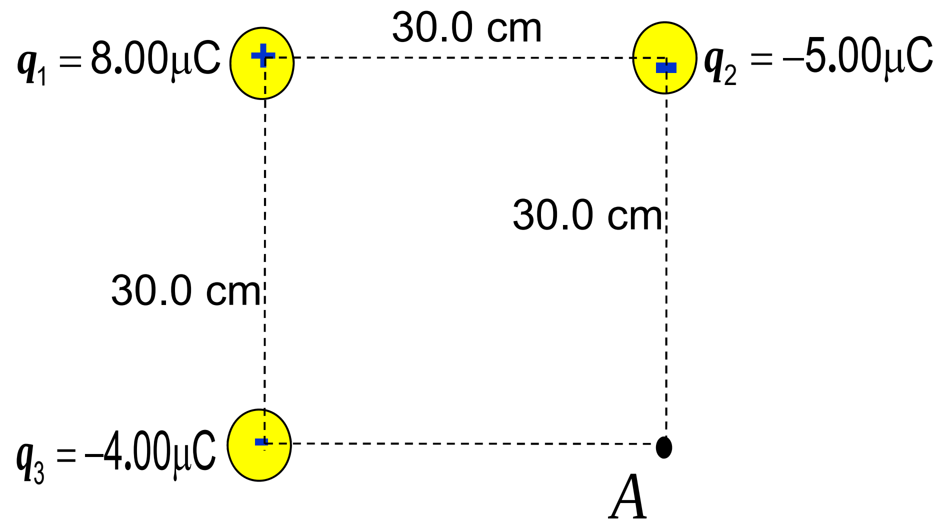
$$F_A = qE_A$$

$$\boldsymbol{F}_{\boldsymbol{A}} = (4.0)(1.7\times10^{13}) = 6.8\times10^{13} \text{ N}$$

Direction : to the right

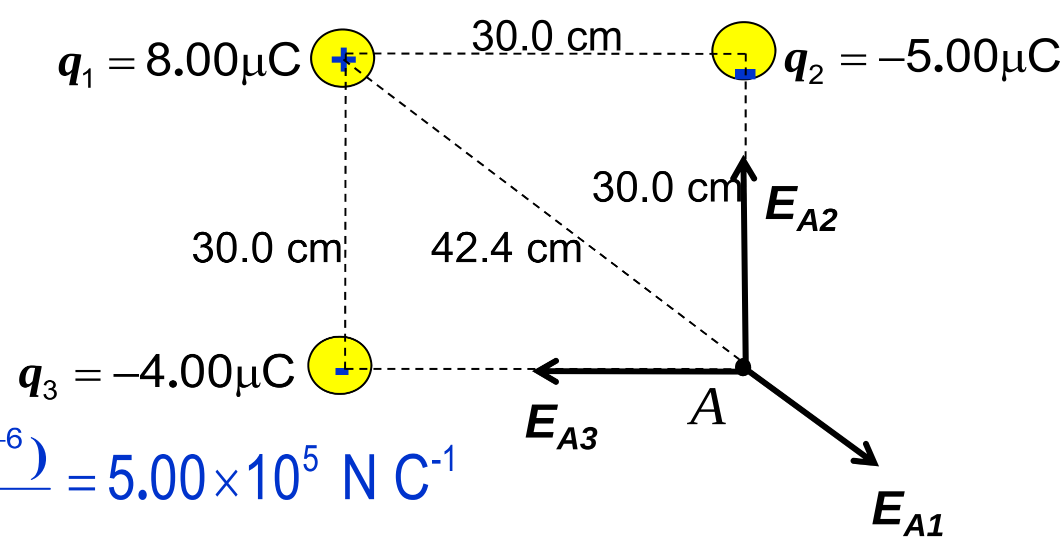
Example

Three charges are placed on three corners of a square, as shown above. Each side of the square is 30.0 cm. Calculate the electric field strength at point A. What would be the force on a 6.00 μC charge placed at the point A?



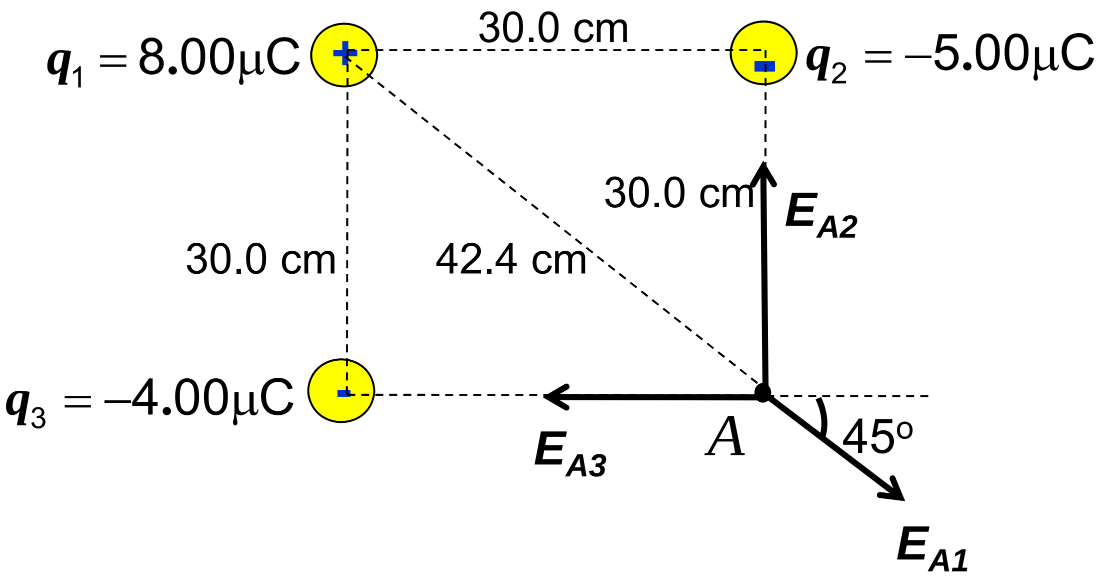
$$E_{A1} = \frac{kq_1}{r_1^2} = \frac{(9.0\times10^9)(8.00\times10^{-6})}{(42.4\times10^{-2})^2}$$

$$= 4.00\times10^5 \text{ N C}^{-1}$$



$$E_{A2} = \frac{kq_2}{r_2^2} = \frac{(9.0\times10^9)(5.00\times10^{-6})}{(30.0\times10^{-2})^2} = 5.00\times10^5 \text{ N C}^{-1}$$

$$E_{A3} = \frac{kq_3}{r_3^2} = \frac{(9.0\times10^{9})(4.00\times10^{-6})}{(30.0\times10^{-2})^2} = 4.00\times10^{5}\ \text{N C}^{-1}$$

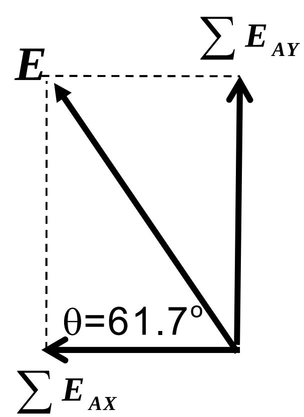


$$\sum E_{AX} = E_{A1}\cos 45^o - E_{A3}$$
$$= -1.17\times10^{5}\ \text{N/C}$$

$$\sum E_{AY} = E_{A2} - E_{A1}\sin 45^o$$
$$= 2.17\times10^{5}\ \text{N/C}$$

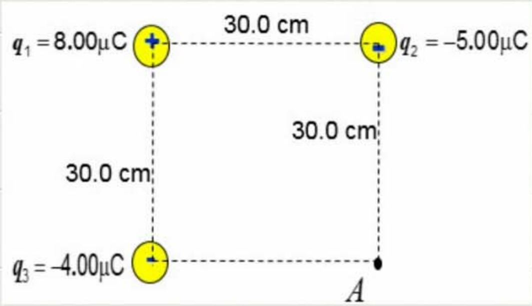
$$E = \sqrt{\sum E_{AX}^{\ 2} + \sum E_{AY}^{\ 2}}$$
$$E = 2.46\times10^{5}\ \text{N/C}$$

$$\tan\theta = \frac{\sum E_{AY}}{\sum E_{AX}}$$
$$\theta = 61.7^o$$



Example: solution

We need to find E at A:



x component: need q_{1x}, q_{2x}, q_{3x}

① q_{1x}: $E_{1x} = k\frac{q_1}{(\sqrt{2}\times 0.3)^2}\cos 45 = 9\times10^9 \times \frac{8\times10^{-6}}{2\times0.3^2}\times 0.71 = 2.83\times10^5$ N/C

② $q_{2x}: 0$ ③ q_{3x}: $E_{3x} = -9\times10^9 \frac{4\times10^{-6}}{0.3^2} = -4\times10^5$ N/C

$\therefore E_{Ax} = E_{1x} + E_{3x} = 2.83\times10^5 - 4\times10^5 = -1.17\times10^5$ N/C ←

y compo: need q_{1y}, q_{2y}, q_{3y}:

① q_{1y}: $E_{1y} = 9\times10^9 \frac{\times 8\times10^{-6}}{2\times0.3^2}\times 0.71 = -2.83\times10^5$ N/C ↓ سالب بسبب الاتجاه لأسفل

② q_{2y}: $E_{2y} = +9\times10^9 \frac{5\times10^{-6}}{0.3^2} = 5\times10^5$ N/C ↑ + لأعلى

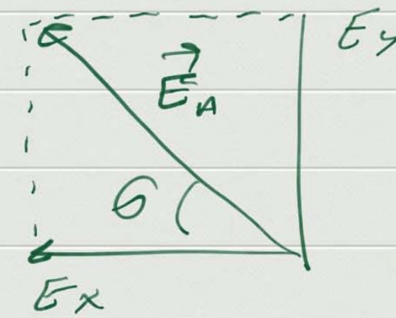
$\therefore E_{Ay} = 5\times10^5 - 2.83\times10^5 = +2.17\times10^5$ N/C ↑

$\therefore E_A = \sqrt{E_x^2 + E_y^2} = \sqrt{1.17\times10^{5^2} + 2.17\times10^{5^2}} = 2.46\times10^5$ N/C #

Example: solution

To find direction of E at A:

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{2.17\times10^5}{-1.17\times10^6}\right) = -61.7^{\circ}$$ [illegible]



b) to find force on $q = 6\times10^{-6}$ C at A.

$\because F = Eq$

$= 2.46\times10^{5} \times 6\times10^{-6}$

$= 1.48$ N #

23-7 Motion of a Charged Particle in a Uniform Electric Field

Consider a particle with charge ***q*** and mass ***m***, moving in a region of space where the electric field **E** is **constant.**

As always, the force on a charge ***q*** is $\vec{F} = q\vec{E} = m\vec{a}$.

Constant acceleration,

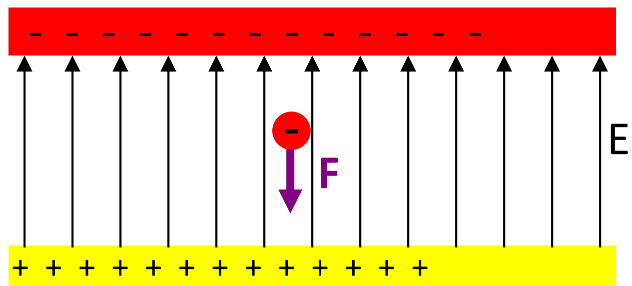
$\vec{a} = \vec{F}/m = (q\vec{E}/m)$

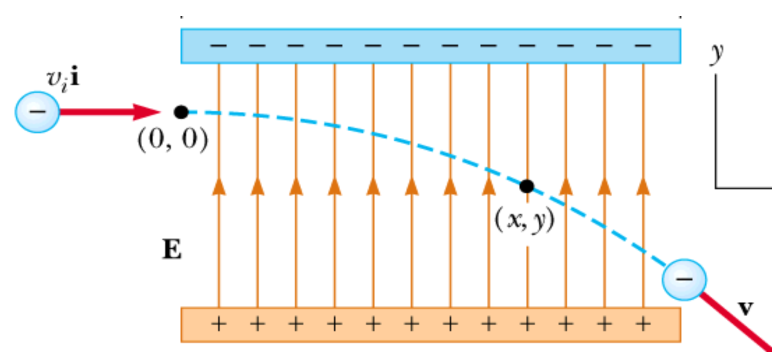
and will move in a **parabola**.

•If E is uniform (that is, constant in magnitude and direction), then the acceleration is constant.

• If the particle has a positive charge, then its acceleration is in the direction of the electric field.

•If the particle has a negative charge, then its acceleration is in the direction opposite the electric field.





EXAMPLE 23.10 **An Accelerating Positive Charge**

A positive point charge *q* of mass *m* is released from rest in a uniform electric field ***E*** directed along the *x* axis. Describe its motion.

we can apply the equations of kinematics in one dimension

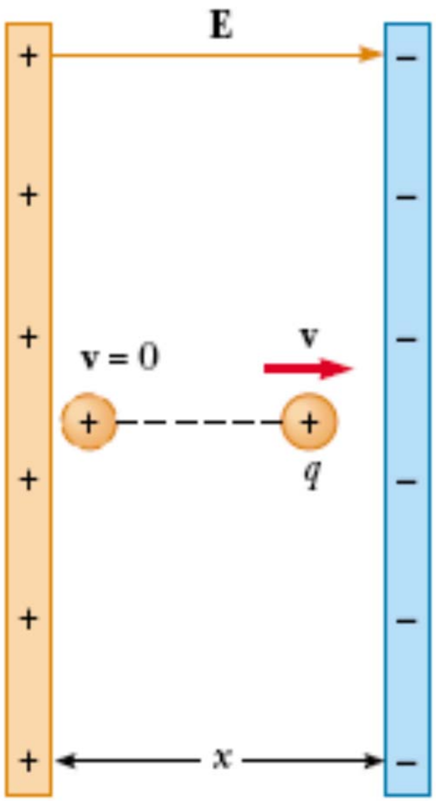
$$x_f = x_i + v_i + \frac{1}{2}at^2$$
$$v_f = v_i + at$$
$$v_f^2 = v_i^2 + 2a(x_f\text{-}x_i)$$

Taking $x_i = 0$ and $v_{xi} = 0$

$$x_f = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2 \qquad (1)$$

$$v_f = at = \frac{qE}{m}t \qquad (2)$$

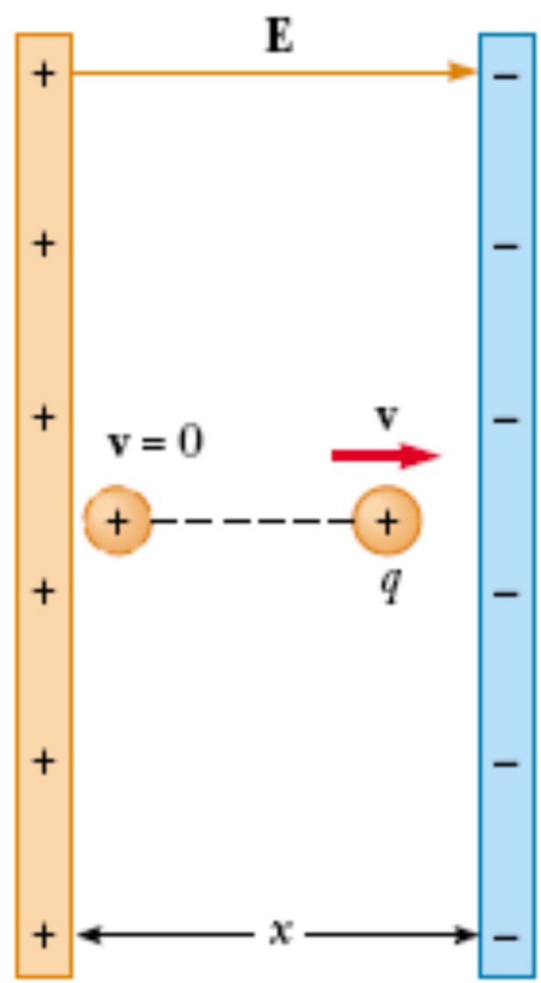
$$v_f^2 = 2ax_f = 2\frac{qE}{m}x_f \qquad (3)$$



EXAMPLE 23.10 An Accelerating Positive Charge

The kinetic energy of the charge after it has moved a distance $x = x_f - x_i$, is

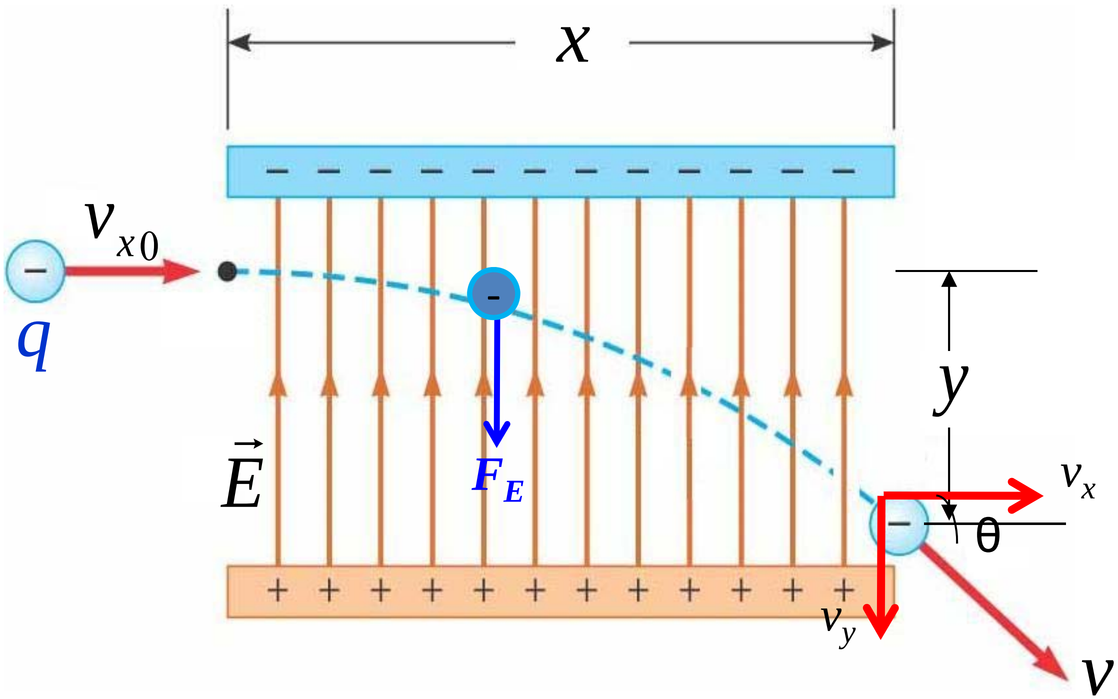
$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{2qE}{m}x = qEx$$



We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is $F_e x = q\mathbf{E}x$ and $W = \Delta K$

Charge moving perpendicularly to the field

Figure below shows the path of an electron *q* which enters the uniform electric field between two parallel metal plates with a velocity v_o .



- When the electron enters the electric field , E the only force that acts on the electron is **electrostatic force** , $F_E = qE$.

- This causes the electron of mass m to accelerate downwards
- with an acceleration a.

$$a_y = \frac{qE}{m} \quad (\pm)$$

- Since the horizontal component of the velocity of the electron remains unchanged as v_{xo} , the path of the electron in the uniform electric field is a parabola.
- The time taken for the electron to transverse the electric field is given by

$$\because x = v_{x0} t \qquad \therefore t = \frac{x}{v_{x0}}$$

- The vertical component of the velocity v_y , when the electron emerges from the electric field is given by

$$v_y = v_{y0} + a_y t$$

$$v_y = 0 + \left(\frac{qE}{m} \right) \left(\frac{x}{v_{x0}} \right)$$

$$v_y = \frac{qEx}{mv_{x0}}$$

- After emerging from the electric field, the electron travels with constant velocity v, where

$$\boxed{\boldsymbol{v} = \sqrt{\boldsymbol{v}_x^{\ 2} + \boldsymbol{v}_y^{\ 2}}}$$

- The direction of the velocity v is at an angle

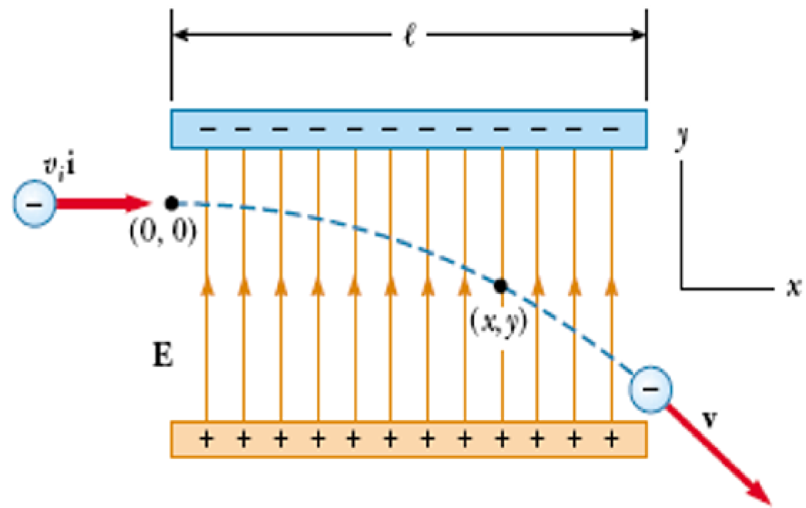
$$\theta = \mathbf{tan}^{-1}\left(\frac{\boldsymbol{v}_y}{\boldsymbol{v}_x}\right)$$ to the horizontal.

- Please note that:

  $V_{x} = v_{xo}$

EXAMPLE 23.11 An Accelerated Electron

An electron enters the region of a uniform electric field with v_i=3.00x10^6 m/s and E= 200 N/C. The horizontal length of the plates is ℓ = 100 cm.



(A) Find the acceleration of the electron while it is in the electric field.

$$\mathbf{a} = -\frac{eE}{m_e}\hat{\mathbf{j}} = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\hat{\mathbf{j}}$$

$$= -3.51 \times 10^{13}\hat{\mathbf{j}}\ \text{m/s}^2$$

(B) If the electron enters the field at time $t = 0$, find the time at which it leaves the field.

Solution The horizontal distance across the field is ℓ = 0.100 m. Using Equation 23.16 with $x_f = \ell$, we find that the time at which the electron exits the electric field is

$$t = \frac{\ell}{v_i} = \frac{0.100\ \text{m}}{3.00 \times 10^{6}\ \text{m/s}} = 3.33 \times 10^{-8}\ \text{s}$$

EXAMPLE 23.11 An Accelerated Electron

(C) If the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

Solution Using Equation 23.17 and the results from parts (A) and (B), we find that

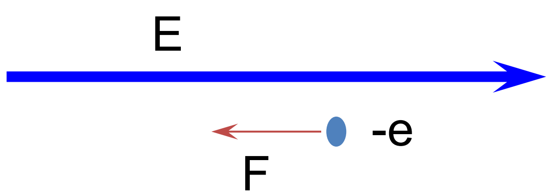
$$y_f = \frac{1}{2}a_y t^2 = -\frac{1}{2}(3.51 \times 10^{13}\ \mathrm{m/s^2})(3.33 \times 10^{-8}\ \mathrm{s})^2$$

$$= -0.019\,5\ \mathrm{m} = \boxed{-1.95\ \mathrm{cm}}$$

If the separation between the plates is less than this, the electron will strike the positive plate.

Example - Motion of a charged particle in an Electric Field

Determine the final velocity and kinetic energy of an electron released from rest in the presence of a uniform electric field of 300 N/C in the x direction after a period of 0.5 ms.

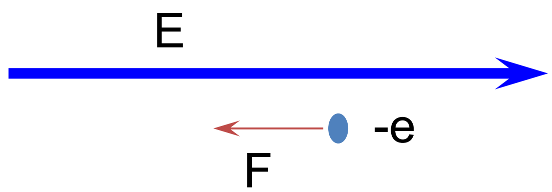


$$\vec{F} = q\vec{E} = -eE\,\hat{i}$$

$$\vec{F} = -\left(1.60\times10^{-19}\,C\right)\left(300\,\frac{N}{C}\right)\hat{i}$$

$$\vec{F} = -\left(4.80\times10^{-17}\,N\right)\hat{i}$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-\left(4.80\times10^{-17}\,N\right)\hat{i}}{9.11\times10^{-31}\,kg} = -\left(5.3\times10^{+13}\,\frac{m}{s^2}\right)\hat{i}$$



$$\vec{v} = \vec{v}_i + \vec{a}t$$

$$\vec{v} = 0 - \left(5.3 \times 10^{+13} \frac{m}{s^2}\hat{i}\right)\left(0.5 \times 10^{-3}\, s\right)$$

$$\vec{v} = -2.6 \times 10^{10} \frac{m}{s}\hat{i}$$

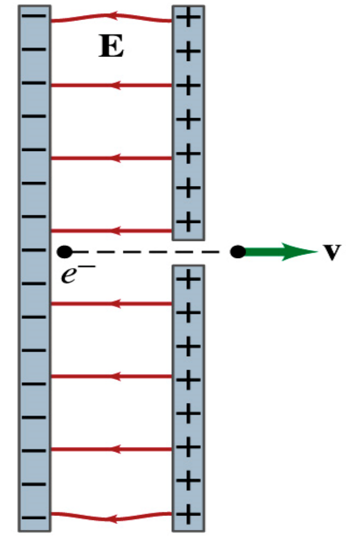
$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}\left(9.11 \times 10^{-31} kg\right)\left(-2.6 \times 10^{10} \frac{m}{s}\right)^2$$

$$K = 3.16 \times 10^{-10}\, J$$

Example

An electron (mass m = 9.1x10^{-31}kg) is accelerated in the uniform field E (E=2.0x10^{4}N/C) between two parallel charged plates. The separation of the plates is 1.5cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.



The magnitude of the force on the electron is F=qE and is directed to the right. The equation to solve this problem is

$$F = qE = ma$$

The magnitude of the electron's acceleration is

$$a = \frac{F}{m} = \frac{qE}{m}$$

Between the plates the field E is uniform, thus the electron undergoes a uniform acceleration

$$a = \frac{eE}{m_e} = \frac{\left(1.6\times10^{-19}\,C\right)\left(2.0\times10^{4}\,N/C\right)}{\left(9.1\times10^{-31}\,kg\right)} = 3.5\times10^{15}\,m/s^2$$

Example (Continued)

Since the travel distance is 1.5x10^{-2}m, using one of the kinetic eq. of motions,

$$v^2 = v_0^2 + 2ax \;\; \therefore v = \sqrt{2ax} = \sqrt{2 \cdot 3.5 \times 10^{15} \cdot 1.5 \times 10^{-2}} = 1.0 \times 10^7 \, m/s$$

Since there is no electric field outside the conductor, the electron continues moving with this speed after passing through the hole.

- (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the electric force on the electron is

$$F_e = qE = eE = \left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^4 N/C\right) = 3.2 \times 10^{-15} N$$

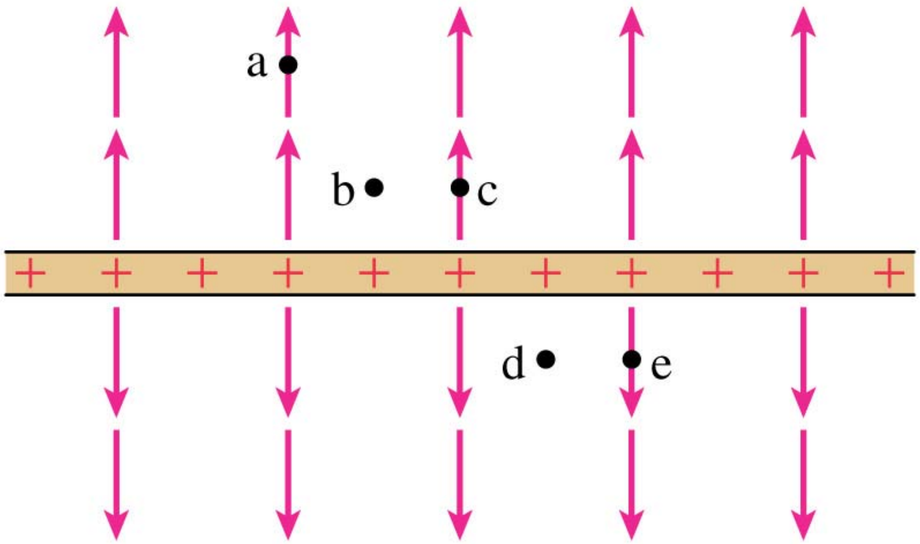
The magnitude of the gravitational force on the electron is

$$F_G = mg = 9.8 \, m/s^2 \times \left(9.1 \times 10^{-31} kg\right) = 8.9 \times 10^{-30} N$$

Thus the gravitational force on the electron is negligible compared to the electromagnetic force.

Quiz

Rank in order, from largest to smallest, the electric field strengths E_a to E_e at these five points near a plane of charge.



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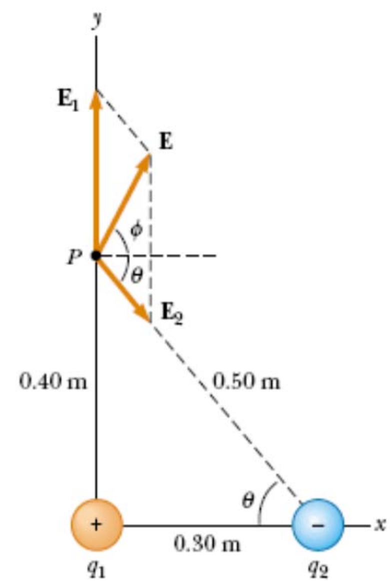
1. $E_a = E_b = E_c = E_d = E_e$
2. $E_a > E_c > E_b > E_e > E_d$
3. $E_b = E_c = E_d = E_e > E_a$
4. $E_a > E_b = E_c > E_d = E_e$
5. $E_e > E_d > E_c > E_b > E_a$

at any point P, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i$$

EXAMPLE 23.5 Electric Field Due to Two Charges

A charge $q_1 = 7.0\ \mu\text{C}$ is located at the origin, and a second charge $q_2 = -5.0\ \mu\text{C}$ is located on the x axis, 0.30 m from the origin (Fig. 23.13). Find the electric field at the point P, which has coordinates (0, 0.40) m.



$$E_1 = k_e \frac{|q_1|}{r_1^2} = \left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(7.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$
$$= 3.9 \times 10^5\ \text{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = \left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(5.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$
$$= 1.8 \times 10^5\ \text{N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \mathbf{j}\ \text{N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} - 1.4 \times 10^5 \mathbf{j})\ \text{N/C}$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} + 2.5 \times 10^5 \mathbf{j})\ \text{N/C}$$

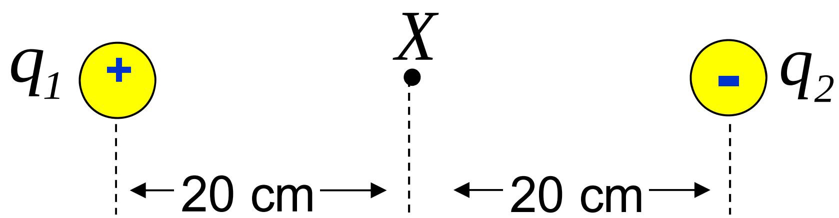
From this result, we find that $\mathbf{E}$ has a magnitude of 2.7×10^5 N/C and makes an angle ϕ of 66° with the positive x axis.

Exercise Find the electric force exerted on a charge of 2.0×10^{-8} C located at P.

Answer 5.4×10^{-3} N in the same direction as $\mathbf{E}$.

Exercise

1. Determine
   a) the electric field strength at a point X at a distance 20 cm from a point charge Q = + 6μC. **(1.4 x 10 6 N/C)**
   b) the electric force that acts on a point charge q= -0.20 μC placed at point X. **(0.28 N towards Q)**

2. 

Two point charges, q_1 = +2.0 C and q_2 = -3.0 C, are separated by a distance of 40 cm, as shown in figure above. Determine

a) The resultant electric field strength at point *X*.
   (1.13 x 10^3 kN C^{-1} towards q_2)
b) The electric force that acts on a point charge q = 0.50 μC placed at *X*.
   (0.57 N)