جامعة الملك سعود

كلية العلوم

قسم الفيزياء والفلك

مذكرة المقرر 104 فيز

تنسيق: أ.د. ناصر بن صالح الزايد

الباب 24

محاضرة رقم 3 (صيفي)

أجزاء كبيرة من هذه المذكرة معتمدة على عروض الأستاذة نورة علي المنيف - قسم الفيزياء.

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Physics 104
Chapter 24

Gauss's Law

Lecture No. 05

Chapter 24 *Gauss's Law*

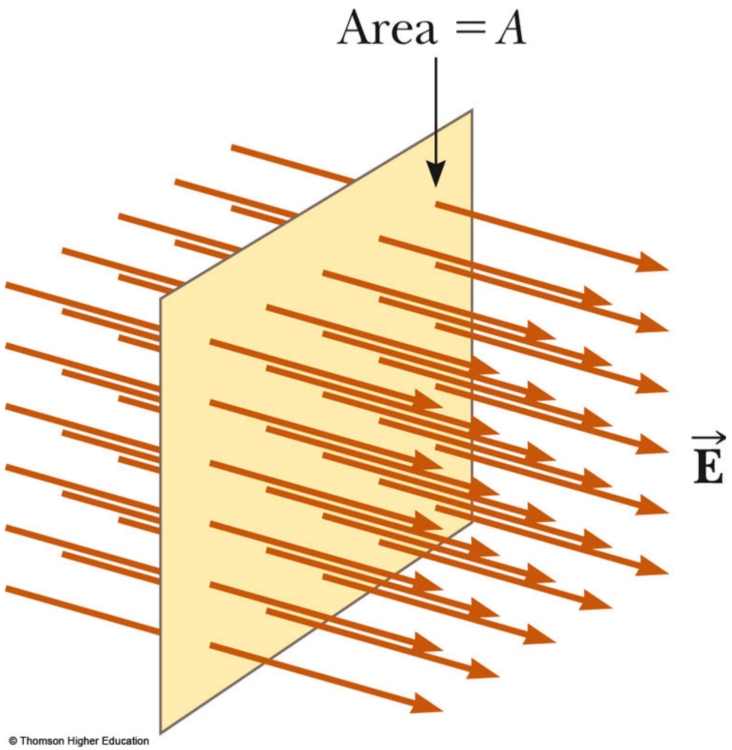
24.1 Electric Flux
24.2 Gauss's Law
24.3 Application of Gauss's Law to Various Charge Distributions
24.4 Conductors in Electrostatic Equilibrium

- ❖A Spherically Symmetric Charge Distribution
- ❖The Electric Field Due to a Point Charge
- ❖The Electric Field Due to a Thin Spherical Shell
- ❖The Electric Field Due to A Cylindrically Symmetric Charge Distribution
- ❖the electric field due to an infinite plane

24.1 Electric Flux

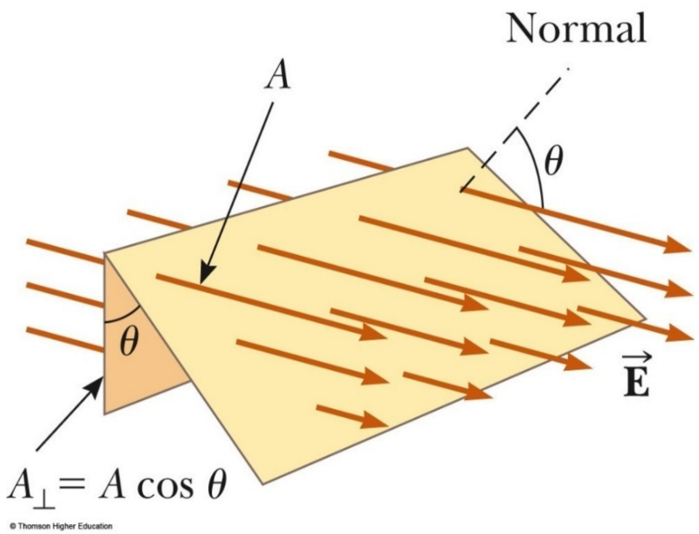
- Electric flux is the product of the magnitude of the electric field and the surface area, A, perpendicular to the field

$$\Phi_E = EA$$



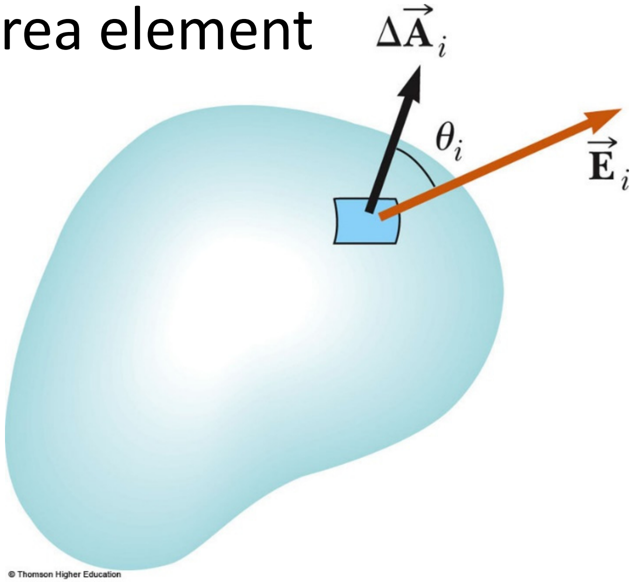
24.1 Electric Flux

- The electric flux is proportional to the number of electric field lines penetrating some surface
- The field lines may make some angle θ with the perpendicular to the surface: $\Phi_E = EA\cos\theta$
- The flux is a *maximum* when the surface is perpendicular to the field
- The flux is a *minimum* (zero) when the surface is (parallel) to the field
- The *zero*<flux<*maximum* when the surface is at $0<\theta<90$



24.1 Electric Flux

- If the field varies over the surface, $\Phi = EA\cos\theta$ is valid for only a small element of the area
- In the more general case, look at a small area element
- In general, this becomes



$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum E_i \cdot \Delta A_i$$

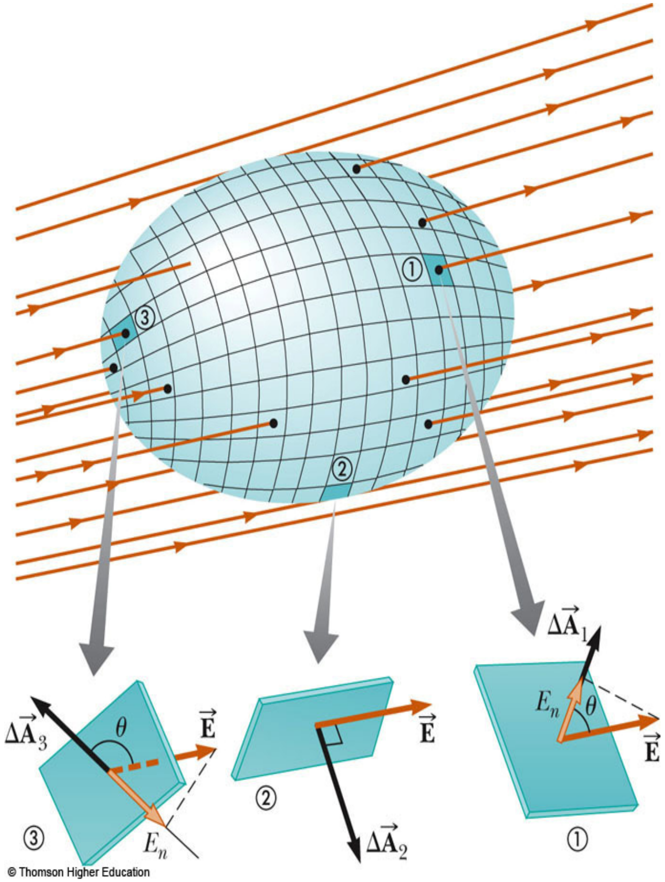
$$\Phi_E = \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

- The surface integral means the integral must be evaluated over the surface in question
- SI units: $N \cdot m^2/C$

Electric Flux, Closed Surface

- For a closed surface, by convention, the ***A*** vectors are perpendicular to the surface at each point and point outward
- (1) $\theta < 90^o$, $\Phi > 0$
- (2) $\theta = 90^o$, $\Phi = 0$
- (3) $180^o > \theta > 90^o$, $\Phi < 0$

The net flux through the surface is proportional to the number of lines leaving the surface minus the number entering the surface



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} \qquad = \oint E \, dA$$

Example 24.1 Electric Flux Through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of 1.00 μC at its center?

$$\because \Phi_E = EA \quad ---(1)$$

$$\because E = K\frac{q}{r^2} = 9\times10^{9}\,\frac{1\times10^{-6}}{1^2}$$

$$= 9\times10^{3}\ N/C$$

$$\because A = 4\pi r^2 = 4\pi\times1^2 = 12.6\ m^2$$

$$\therefore \Phi_E = 9\times10^{3}\times12.6 = 1.13\times10^{5}\ N\cdot m^2/C$$

Quick Quiz 24.1

Quick Quiz 24.1 Suppose the radius of the sphere in Example 24.1 is changed to 0.500 m. What happens to the flux through the sphere and the magnitude of the electric field at the surface of the sphere? (a) The flux and field both increase. (b) The flux and field both decrease. (c) The flux increases and the field decreases. (d) The flux decreases and the field increases. (e) The flux remains the same and the field increases. (f) The flux decreases and the field remains the same.

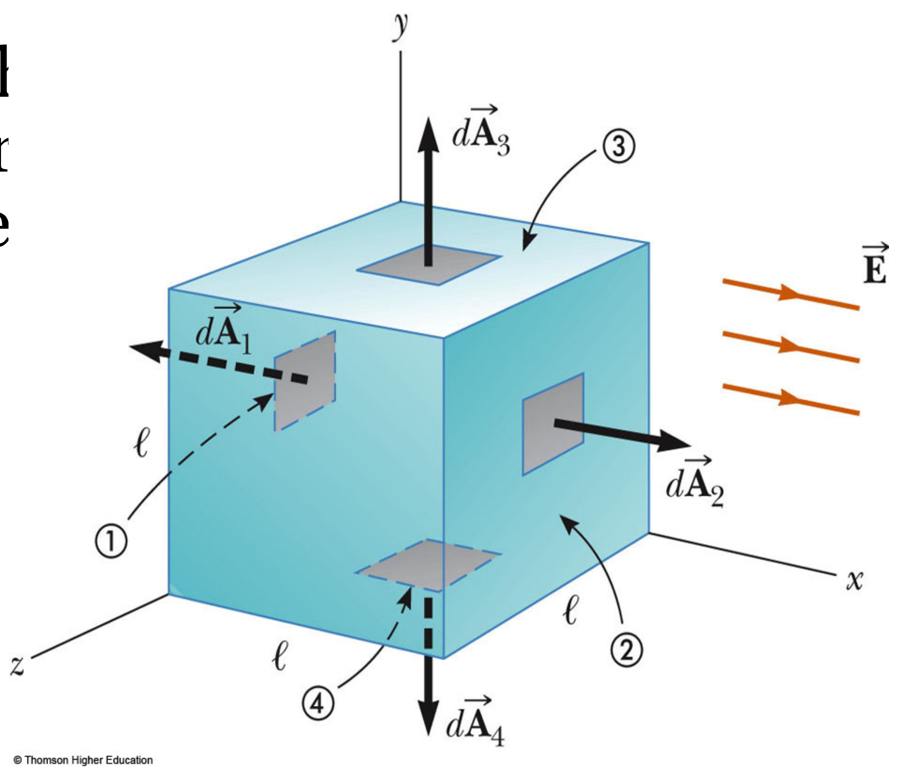
Quick Quiz 24.2

Quick Quiz 24.2 In a charge-free region of space, a closed container is placed in an electric field. A requirement for the total electric flux through the surface of the container to be zero is that (a) the field must be uniform, (b) the container must be symmetric, (c) the container must be oriented in a certain way, or (d) the requirement does not exist—the total electric flux is zero no matter what.

❖Example 24-1: flux through a cube of a uniform electric field

Consider a uniform electric field **E** oriented in the *x* direction. Find the net electric flux through the surface of a cube of edge length *l*, oriented as shown in Figure

- The field lines pass through two surfaces perpendicular and are parallel to the other four surfaces
- For side 1, $\Phi_E = -El^2$
- For side 2, $\Phi_E = El^2$
- For the other sides, $\Phi_E = 0$
- Therefore, $\Phi_{total} = 0$

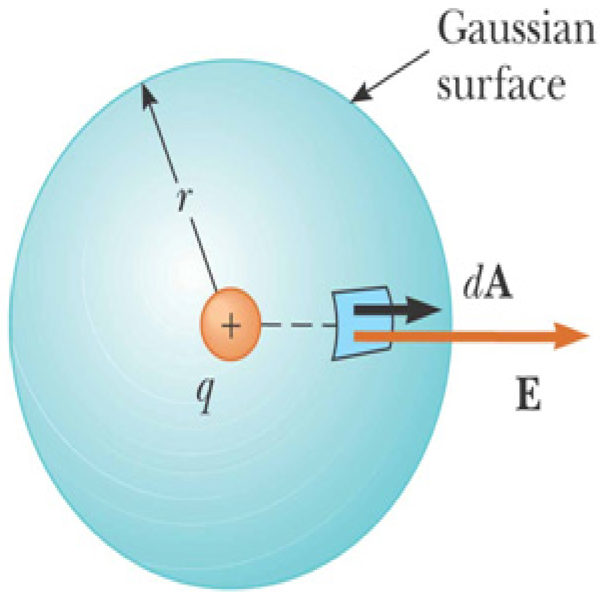


24-2 Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\theta dA = \oint EdA$

For a Point charge $E=kq/r^2$



$$\Phi = \oint EdA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2(4\pi r^2)$$

$$\Phi = 4\pi kq$$

$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12} \frac{C^2}{Nm^2}$$

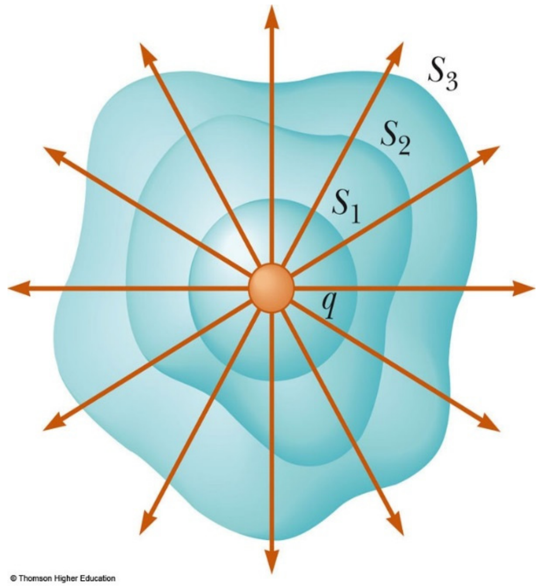
$$\boxed{\Phi_{net} = \frac{q_{in}}{\varepsilon_0}}$$

Gauss' Law

Gauss' Law

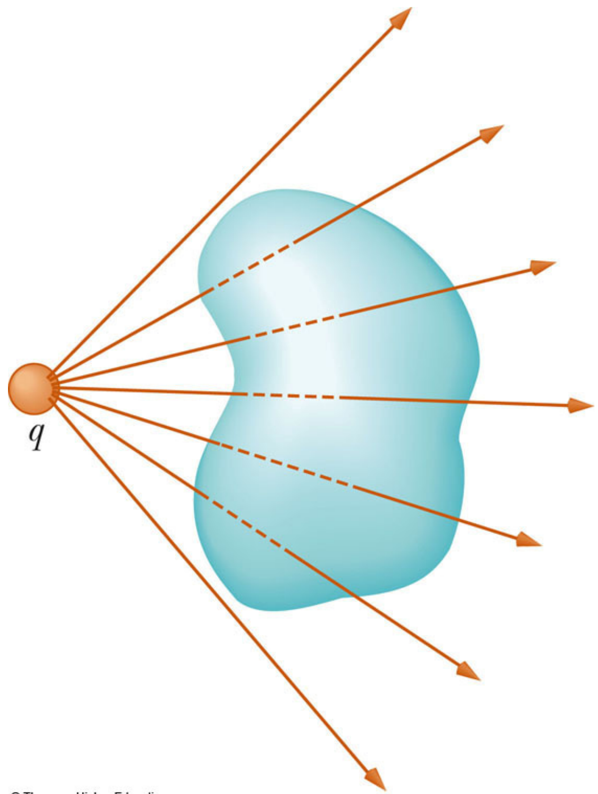
The flux of electric field through a closed surface is proportional to the charge enclosed.

- Gaussian surfaces of various shapes can surround the charge (only S_1 is spherical)
- The electric flux is proportional to the number of electric field lines penetrating these surfaces, and this number is the same
- Thus the net flux through any closed surface surrounding a point charge q is given by q/ε_o and is independent of the shape of the surface
- *the net flux through any closed surface surrounding a point charge q is given by q/ε_o and is independent of the shape of that surface.*



Gauss' Law

- If the charge is outside the closed surface of an arbitrary shape, then any field line entering the surface leaves at another point



- Thus the electric flux through a closed surface that surrounds no charge is zero

Gauss' Law

- Since the electric field due to many charges is the vector sum of the electric fields produced by the individual charges, the flux through any closed surface can be expressed as

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint \left(\vec{E}_1 + \vec{E}_2 + \ldots \right) \cdot d\vec{A}$$

- Although Gauss's law can, in theory, be solved to find Φ for any charge configuration, in practice it is limited to symmetric situations

- One should choose a Gaussian surface over which the surface integral can be simplified and the electric field determined

Problem 24-9

The following charges are located inside a submarine: 5.00 μC, –9.00 μC, 27.0 μC, and –84.0 μC.

(a) Calculate the net electric flux through the hull of the submarine.

(b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

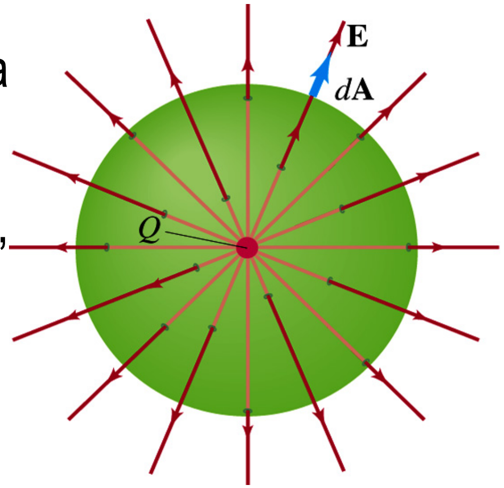
$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\left(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C}\right)}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -6.89\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C}^2$$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

Gauss' Law and Coulomb law

Example 24.4 **The Electric Field Due to a Point Charge**

Starting with Gauss's law, calculate the electric field due to an isolated point charge *q*.



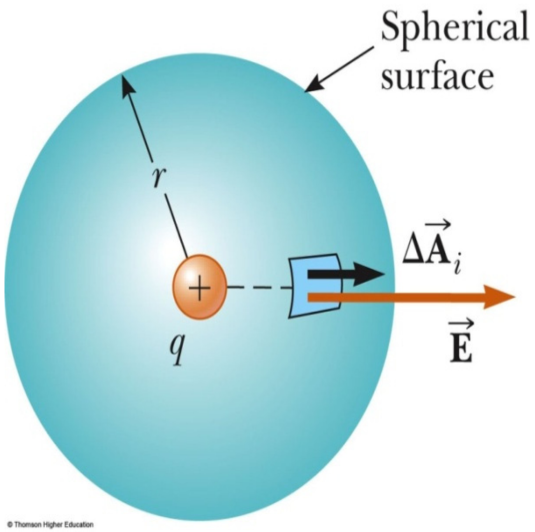
- The field lines are directed radially outwards and are perpendicula to the surface at every point, so

The angle θ between $\vec{E}$ and $d\vec{A}$ is zero at any point on the surface, we can re-write Gauss' Law as

$$\Phi_E \equiv \oint \vec{E}\cdot d\vec{A} = \oint E_n \, dA = \oint E dA = E\oint dA = E\cdot 4\pi r^2$$

$$\Phi_E = \oint \vec{E} \, . \, d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

E has the has same value at all points on the surface



$$\therefore \; E = \frac{Q}{4\pi\varepsilon_o r^2} = k\frac{Q}{r^2}$$

A spherical Gaussian surface centered on a point charge q

Example 24.5 **A Spherically Symmetric Charge Distribution**

An insulating solid sphere of radius *a* has a uniform volume charge density *ρ* and carries total charge *Q*.

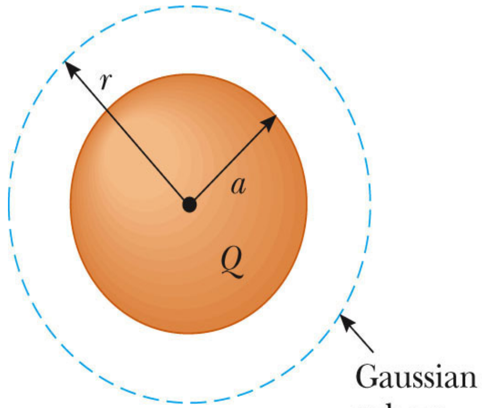
(A) Find the magnitude of the E-field at a point outside the sphere

(B) Find the magnitude of the E-field at a point inside the sphere

- For $r > a$

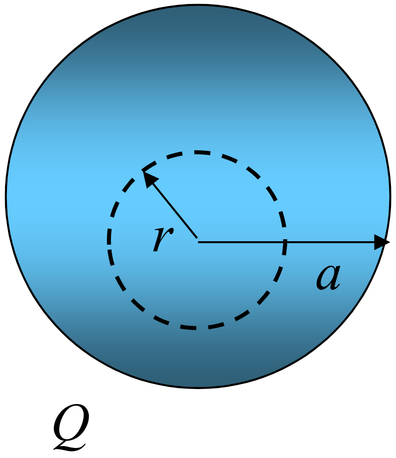
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = k_e \frac{Q}{r^2}$$



(a)

b--Find the magnitude of the E-field at a point inside the sphere r < a



Now we select a spherical Gaussian surface with radius *r* < *a*. Again the symmetry of the charge distribution allows us to simply evaluate the left side of Gauss's law just as before.

❶ $$\oint E\,dA = E\oint dA = E(4\pi r^2)$$

The charge inside the Gaussian sphere is no longer *Q*. If we call the Gaussian sphere volume *V'* then"V

Volume charge density": ρ = charge / unit volume is used to characterize the charge distribution

❷ $$\text{Right side: } Q_{in} = \rho V' = \rho \frac{4}{3}\pi r^3$$

❸ $$E\left(4\pi r^2\right) = \frac{Q_{in}}{\varepsilon_0} = \frac{4\rho\pi r^3}{3\varepsilon_0}$$

❹ $$E = \frac{4\rho\pi r^3}{3\varepsilon_0\left(4\pi r^2\right)} = \frac{\rho}{3\varepsilon_0}r \quad \text{but} \quad \rho = \frac{Q}{\frac{4}{3}\pi a^3} \quad \text{so} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3}r = k_e\frac{Q}{a^3}r$$

Field Due to a Spherically Symmetric Charge Distribution

- Inside the sphere, *E* varies linearly with *r* (*E* → 0 as *r* → 0)

- The field outside the sphere is equivalent to that of a point charge located at the center of the sphere