جامعة الملك سعود

كلية العلوم

قسم الفيزياء والفلك

مذكرة المقرر 104 فيز

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الباب 24

محاضرة رقم 4 (صيفي)

أجزاء كبيرة من هذه المذكرة معتمدة على عروض الأستاذة نورة علي المنيف - قسم الفيزياء.

2019

Physics 104
Chapter 24

Gauss's Law

Lecture No. 04

Chapter 24 *Gauss's Law*

24.1 Electric Flux
24.2 Gauss's Law
24.3 Application of Gauss's Law to Various Charge Distributions
24.4 Conductors in Electrostatic Equilibrium

- ❖A Spherically Symmetric Charge Distribution
- ❖The Electric Field Due to a Point Charge
- ❖The Electric Field Due to a Thin Spherical Shell
- ❖The Electric Field Due to A Cylindrically Symmetric Charge Distribution
- ❖the electric field due to an infinite plane

24-2 Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\theta dA = \oint EdA$

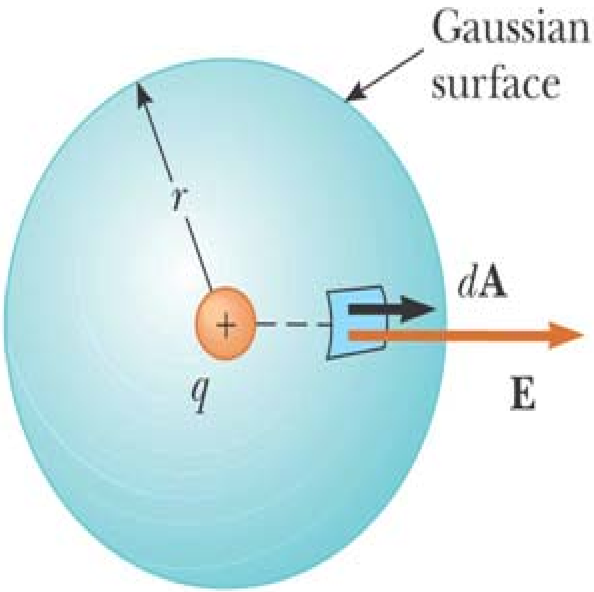
For a Point charge $E=kq/r^2$

$$\Phi = \oint EdA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2 (4\pi r^2)$$

$$\Phi = 4\pi kq$$

$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12} \frac{C^2}{Nm^2}$$



$$\boxed{\Phi_{net} = \frac{q_{in}}{\varepsilon_0}}$$

Gauss' Law

Gauss' Law

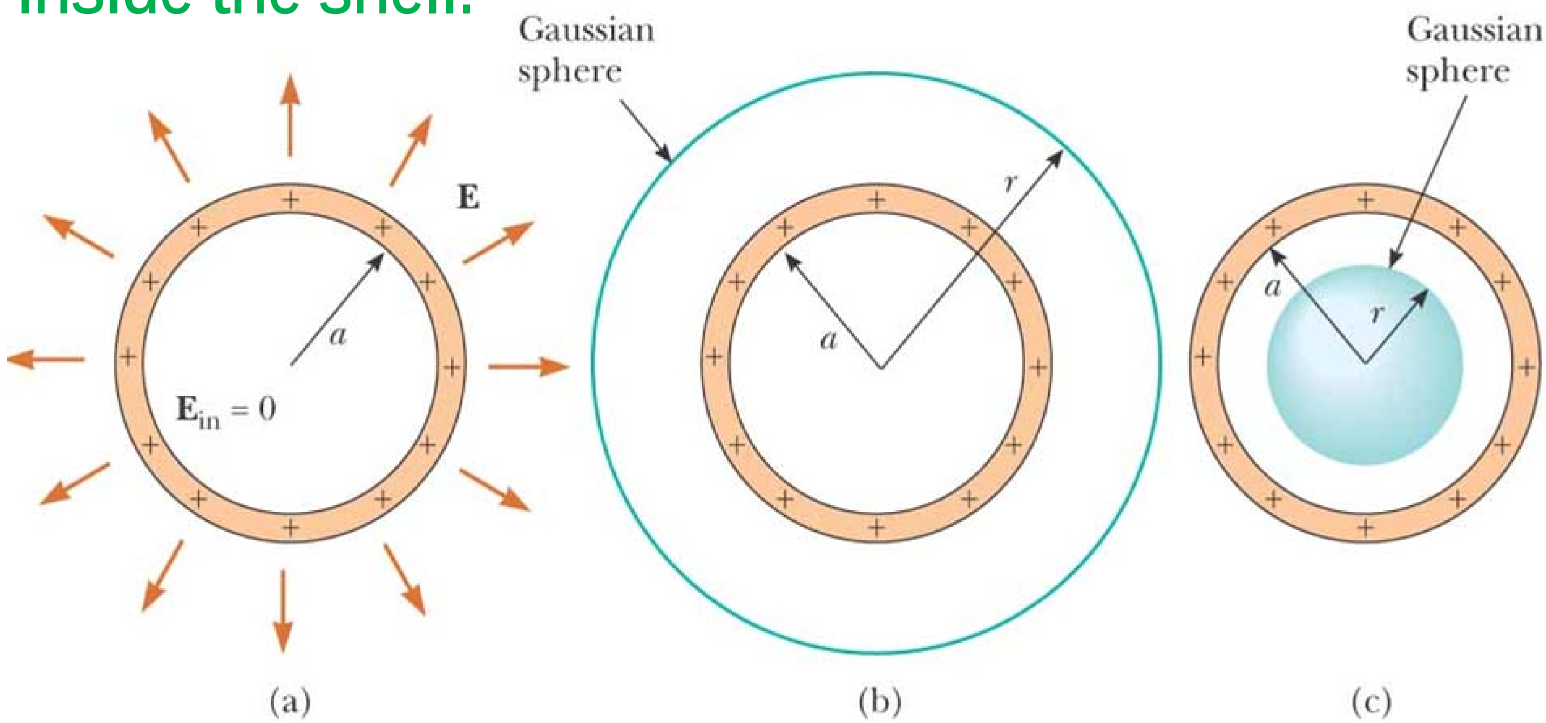
The flux of electric field through a closed surface is proportional to the charge enclosed.

Example 24.6 **The Electric Field Due to a Thin Spherical Shell**

A thin spherical shell of radius *a has a total charge Q distributed* uniformly over its surface Find the electric field at points

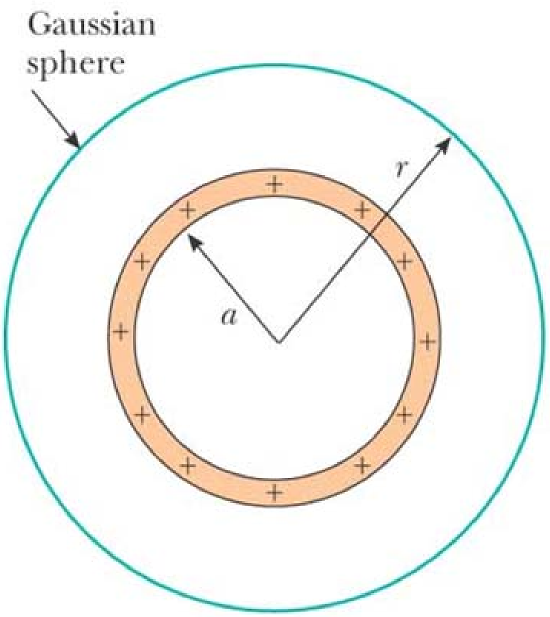
(A) outside and

(B) inside the shell.



- Let's start with the Gaussian surface outside the sphere of charge, r > a
- We know from symmetry arguments that the electric field will be radial outside the charged sphere
  - If we rotate the sphere, the electric field cannot change
    - Spherical symmetry
- Thus we can apply Gauss' Law and get

$$\text{Flux} = \oint E \cdot dA = E \times \left(4\pi r^2\right) = q/\varepsilon_0 \qquad \text{(Gauss)}$$



- … so the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}$$

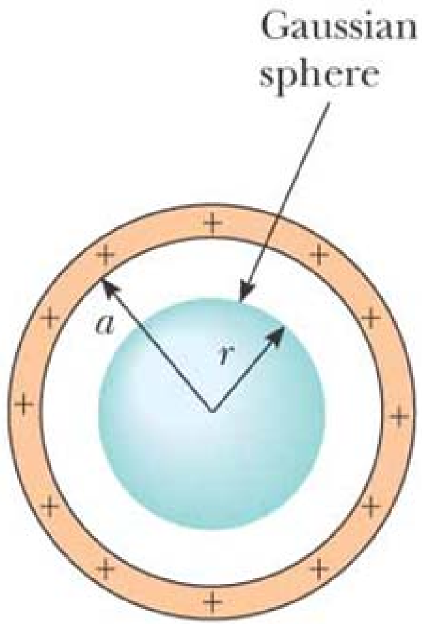
- Let’s let’s take the Gaussian surface inside the sphere of charge, $r < a$
- We know that the enclosed charge is zero so

$$\text{Flux} = \Phi_E = EA = 0$$

- We find that the electric field is zero everywhere inside spherical shell of charge

$$E = 0$$

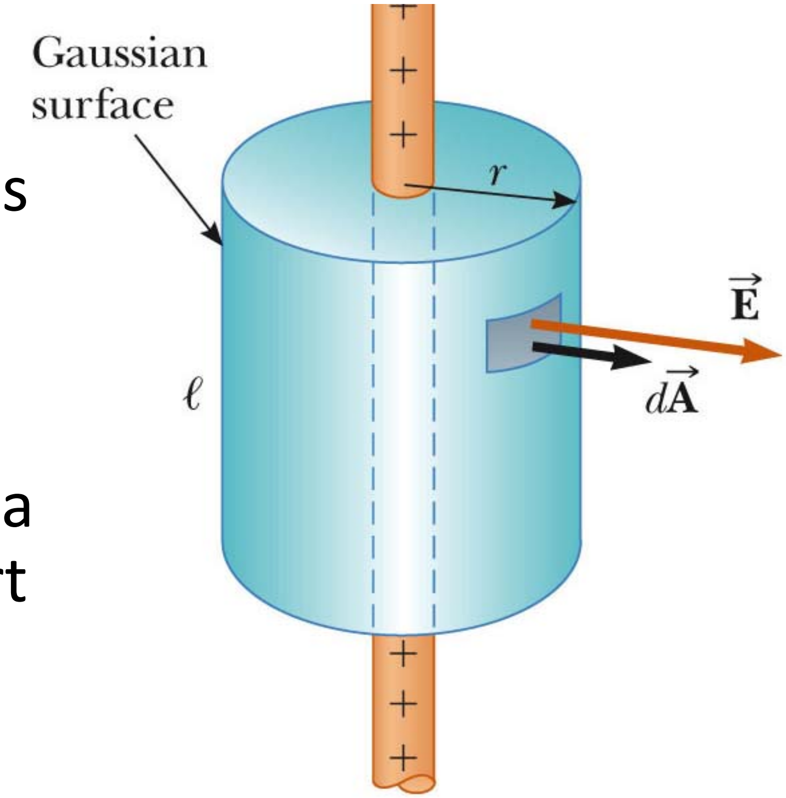
- Thus we obtain two results
  - The electric field outside a spherical shell of charge is the same as that of a point charge.
  - The electric field inside a spherical shell of charge is zero.



24-6 The Electric Field Due to A Cylindrically Symmetric Charge Distribution

Find the E-field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.

- Symmetry $\Rightarrow$ E field must be $\perp$ to line and can only depend on distance from line
  - Select a cylinder as Gaussian surface. The cylinder has a radius r and a length of ℓ
  - $\vec{\mathbf{E}}$ is constant in magnitude and parallel to the surface (the direction of a surface is its norma at every point on the curved part the surface (the body of the cylinder.

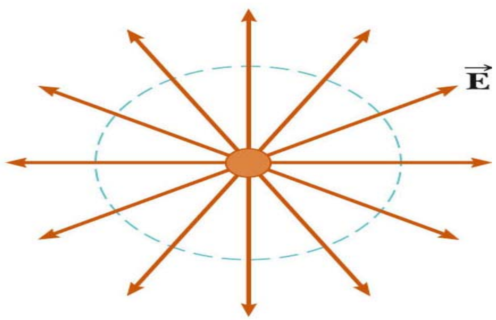


$$\frac{q}{L} = \lambda$$

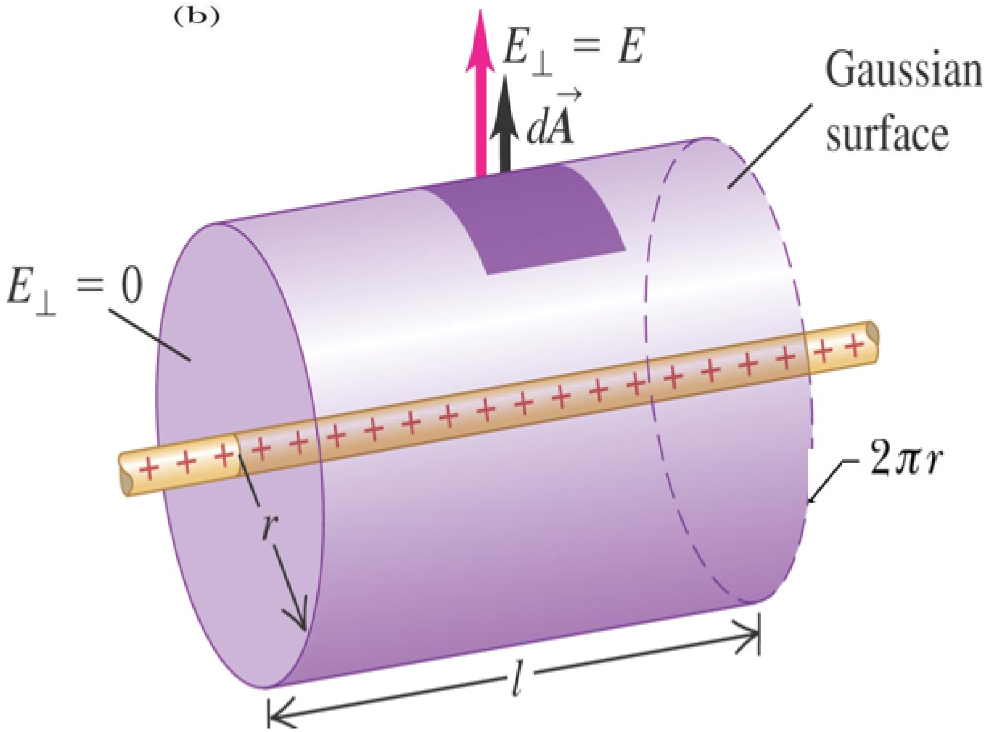
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \qquad \text{Gausss law}$$

$$\oint E \, dA = E \oint dA = 0 + E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$$



(b)



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

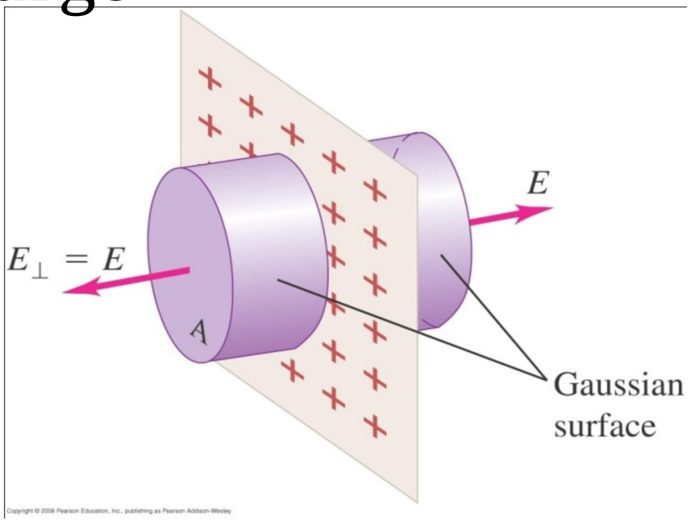
Line of Charge

$$\vec{E} = \frac{2k\lambda}{r}\hat{r}$$

Example 24.8 **A Plane of Charge**

•Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.

- Assume that we have a thin, infinite non-conducting sheet of positive charge



- The charge density in this case

is the charge per unit area, σ

From symmetry, we can see

- that the electric field will be

perpendicular to the surface of the sheet

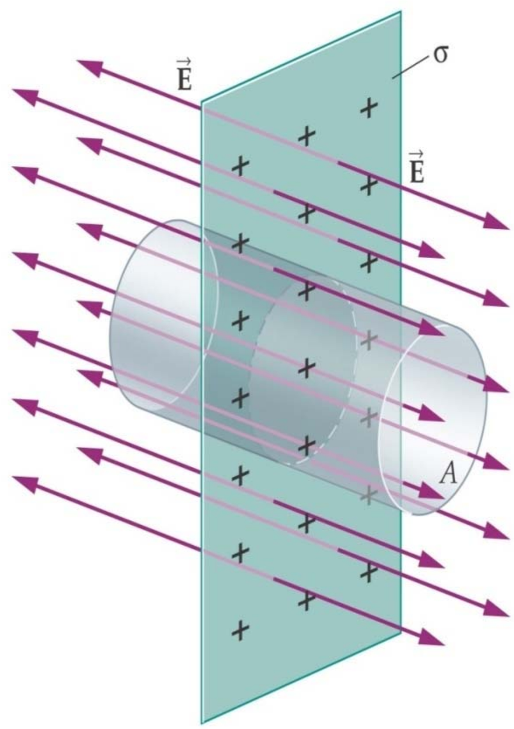
- To calculate the electric field using Gauss' Law, we assume a Gaussian surface in the form of a right cylinder with cross sectional area *A* and height 2*r*, chosen to cut through the plane perpendicularly.
- Because the electric field is perpendicular to the plane everywhere, the electric field will be parallel to the walls of the cylinder and perpendicular to the ends of the cylinder.
- Using Gauss' Law we get

$$\text{Flux} = \Phi_E = \oint E \cdot dA = q/\varepsilon_0 \qquad \text{(Gauss)}$$

$$EA + EA = \sigma A/\varepsilon_0$$

$$2EA = \sigma A/\varepsilon_0$$

$$E = \sigma/2\varepsilon_0$$



- the electric field from an infinite

non-conducting sheet with charge density σ

$$E = \sigma/2\varepsilon_0$$

24-4 Conductors in Electrostatic Equilibrium

By electrostatic equilibrium we mean a situation where there is no *net* motion of charge within the conductor

- The electric field is zero everywhere inside the conductor
- Any net charge resides on the conductor's surface
- The electric field just outside a charged conductor is perpendicular to the conductor's surface

•On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.

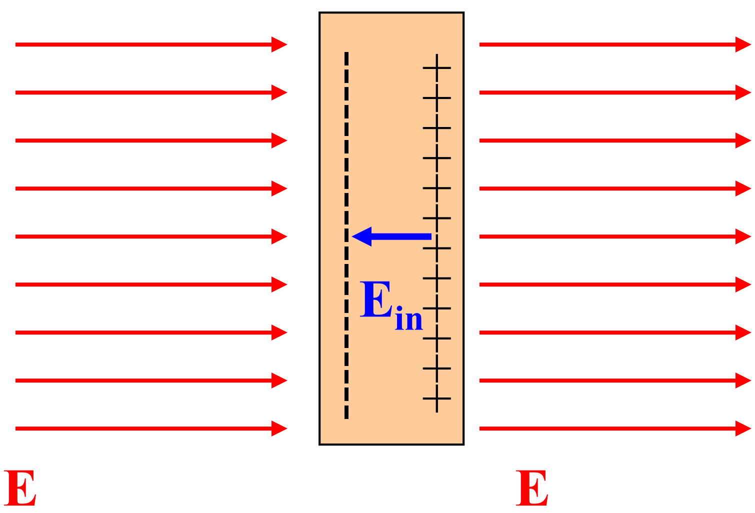
24-4 Conductors in Electrostatic Equilibrium

- The electric field is zero everywhere inside the conductor

Why is this so?

- Place a conducting slab in an external field, **E.**

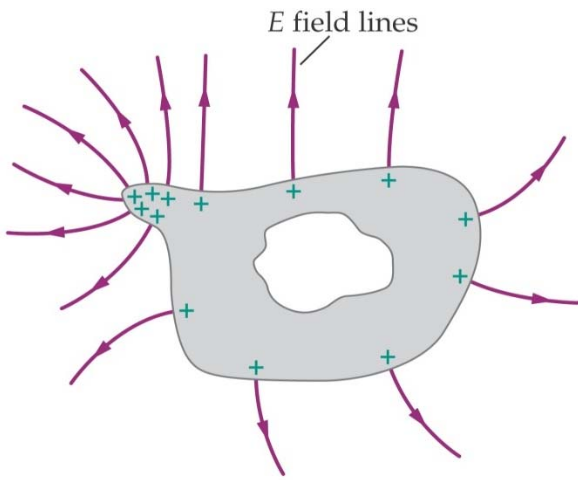
If there was a field in the conductor the charges would accelerate under the action of the field.



The charges in the conductor move creating an internal electric field that cancels the applied field on the inside of the conductor

When electric charges are at rest, the electric field within a conductor is zero.

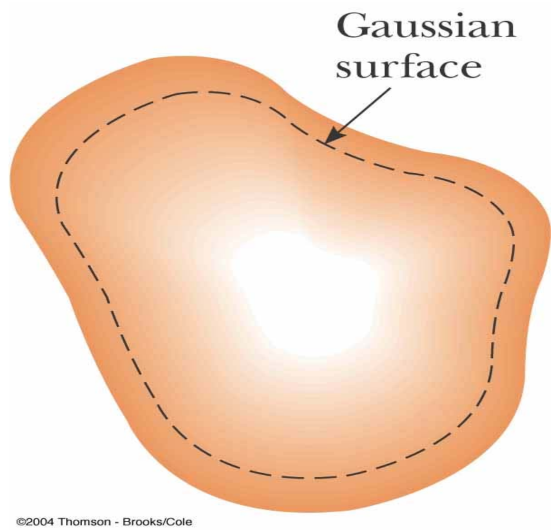
On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.



The electric field is stronger where the surface is more sharply curved.

•Charge Resides on the Surface (of a conductor)

- Choose a Gaussian surface inside the conductor but close to the actual surface
- The electric field inside the surface is zero
- There is no net flux through the gaussian surface
- Because the gaussian surface can be as close to the actual surface as desired, there can be no charge inside the surface



Since no net charge can be inside the surface, any net charge must reside ***on*** the surface
Gauss's law does not indicate the distribution of these charges, only that it must be on the surface of the conductor

Summary:

Gauss' Law

- *Gauss' Law* depends on the enclosed charge only

$$\Phi = \oint \vec{E} \bullet d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

1. If there is a positive net flux there is a net positive charge enclosed
2. If there is a negative net flux there is a net negative charge enclosed
3. If there is a zero net flux there is no net charge enclosed

- Gauss' Law works in cases of symmetry