جامعة الملك سعود

كلية العلوم

قسم الفيزياء والفلك

مذكرة المقرر 104 فيز

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الباب 25

محاضرة رقم 5 (صيفي)

أجزاء كبيرة من هذه المذكرة معتمدة على عروض الأستاذة نورة علي المنيف ـ قسم الفيزياء.

2019

Physics 104
Chapter 25

Chapter 25 electric potential

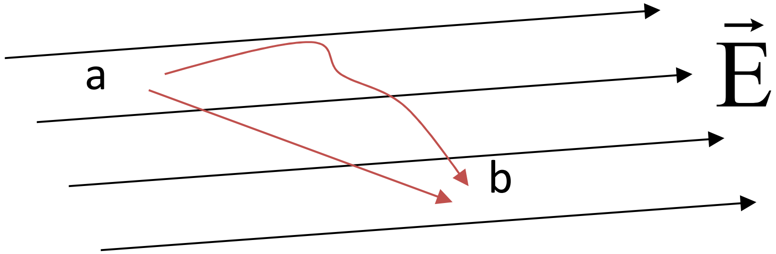
25-1 Potential difference and electric Potential
25-2 Potential Difference and electric field
25-3 Electric Potential and Potential energy due to point charges

Lecture No. 05

25.1 Electrical Potential and Potential Difference

- When a test charge is placed in an electric field, it experiences a force
  - $\vec{\mathbf{F}} = q\,\vec{\mathbf{E}}$
  - The force is conservative
- If the test charge is moved in the field by some external agent, the work done by the field is the negative of the work done by the external agent
- $d\vec{\mathbf{s}}$ is an infinitesimal displacement vector that is oriented tangent to a path through space

Work and Potential Energy



$$dW = \vec{F} \cdot d\vec{s}$$

$$dW = q\vec{E} \cdot d\vec{s}$$

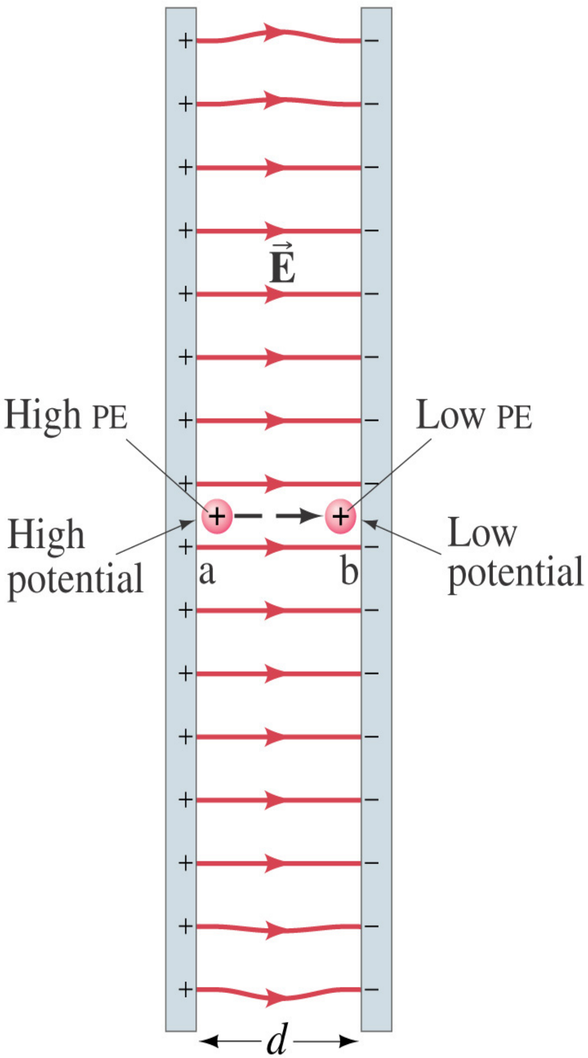
$$dW = -dU = q\vec{E} \cdot d\vec{s}$$

$$\int_a^b dU = -q\int_a^b \vec{E} \cdot d\vec{s}$$

$$\Delta U = -q\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

electric potential is defined $\Delta V = \dfrac{\Delta U}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

25-2 Potential Difference and electric field



When a force is "conservative" ie gravitational and the electrostatic force a potential energy can be defined

Change in electric potential energy is negative of work done by electric force:

$$V_{ba} = V_b - V_a \equiv \frac{U_b - U_a}{q} = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$PE_b - PE_a = -qEd$$

$$\Delta V = -Ed$$

Units

- ΔU: change in electrical potential energy (J)
- q: charge moved (C)
- ΔV: potential difference (V)

$$\left[\frac{\text{Joules}}{\text{Coulomb}}\right] = \left[\frac{\text{J}}{\text{C}}\right] = \text{Volt} = \text{V}$$

- Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt
- One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt

  1 eV = 1.60×10^{-19} J

In short

$$\Delta V = \frac{\Delta U}{q} = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

If E and ds are at the same direction: ΔV is –

If E and ds are at an opposite direction: ΔV is +

ΔV = -Ed for 2 oppositly charged plates

Quick Quiz 25.1 In Figure 25.1, two points A and B are located within a region in which there is an electric field. The potential difference $\Delta V = V_B - V_A$ is (a) positive (b) negative (c) zero.

Quick Quiz 25.2 In Figure 25.1, a negative charge is placed at A and then moved to B. The change in potential energy of the charge–field system for this process is (a) positive (b) negative (c) zero.

Q.25.1: ∵ $\Delta V = -\bar{E}\cdot\bar{d}$

∵ $\Delta V = V_A - V_B$

∴ d is in directio →

∵ E " " " →

∴ E and d at same direction → (·)=1

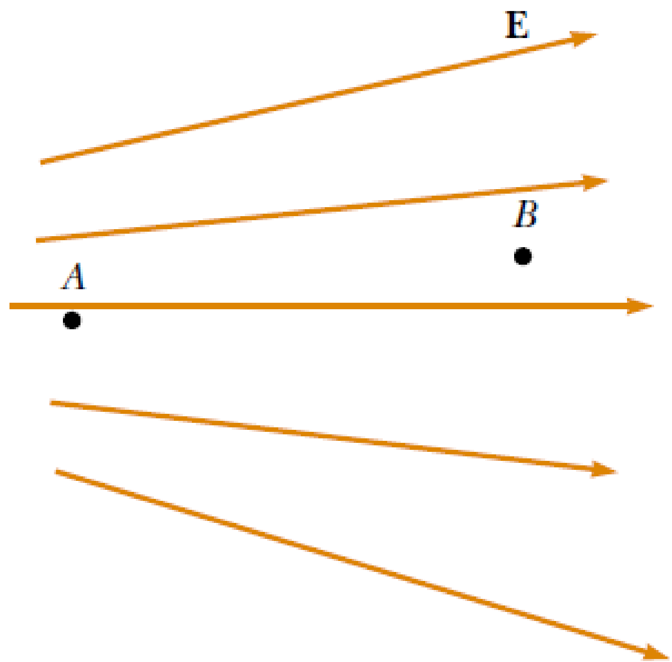
∴ $\Delta V = -Ed$ (−tive)

Q 25.2: ∵ $\Delta U = q\,\Delta V = -qEd$

∵ q is −tive

→ $\Delta U = --qEd = +$tive

∴ change in potential energy ΔU is positive #



Example

If a 9 V battery has a charge of 46 C how much chemical energy does the battery have?

$$\because \Delta V = \frac{\Delta U}{q}$$

$$V = \frac{U}{q}$$

U = V x q = 9 V x 46C = 414 Joules

Example

A pair of oppositely charged, parallel plates are separated by 5.33 mm. A potential difference of 600 V exists between the plates. (a) What is the magnitude of the electric field strength between the plates? (b) What is the magnitude of the force on an electron between the plates?

$$d = 0.00533m$$

$$\Delta V = 600V$$

$$E = ?$$

$$\Delta V = Ed$$

$$600 = E(0.0053)$$

$$E = 113{,}207.55 N / C$$

$$q_{e^-} = 1.6x10^{-19} C$$

$$E = \frac{F_e}{q} = \frac{F_e}{1.6x10^{-19} C}$$

$$F_e = 1.81 \times 10^{-14}\ \text{N}$$

Example

Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V

$$q_{p^+} = 1.6x10^{-19}C$$

$$m_{p^+} = 1.67x10^{-27}kg$$

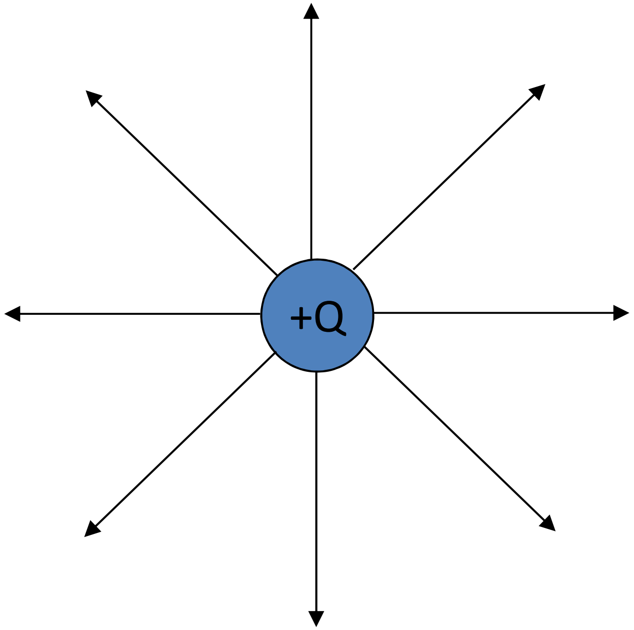
$$V = 120V$$

$$v = ?$$

$$\Delta V = \frac{U}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$$

$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.6x10^{-19})(120)}{1.67x10^{-27}}} = 1.52\times10^5 \ \textbf{m/s}$$

25-3 Electric Potential and Potential energy due to point charges



$$V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

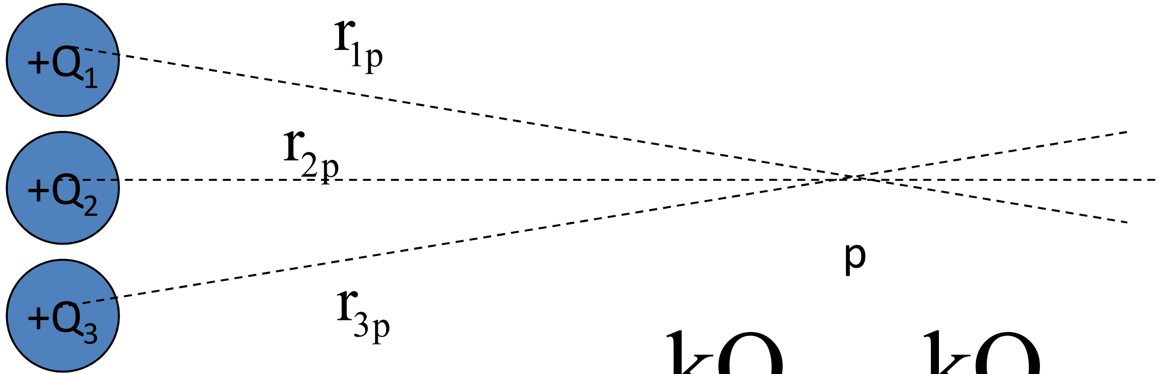
$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

$$V_{ba} = V_b - V_a = -\frac{kq}{r^2}\int_a^b \hat{r} \cdot d\vec{s}$$

$$V(r) = \frac{kq}{r}$$

Superposition of potentials

$$V_p = V_1 + V_2 + V_3 + \ldots$$

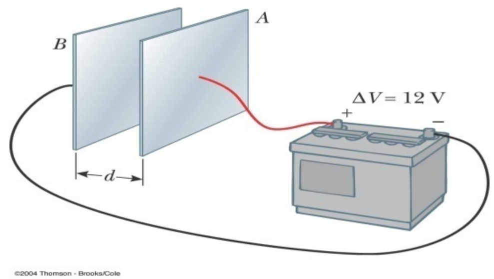


$$V_p = \frac{kQ_1}{r_{1p}} + \frac{kQ_2}{r_{2p}} + \frac{kQ_3}{r_{3p}} + \ldots$$

$$V_p = \sum_{i=1}^{N} \frac{kQ_i}{r_{ip}}$$

Example (25.1)

A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference *V* divided by the plate separation d =0.3 cm



$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = 4.0 \times 10^3\text{ V/m}$$

Example (25.2)

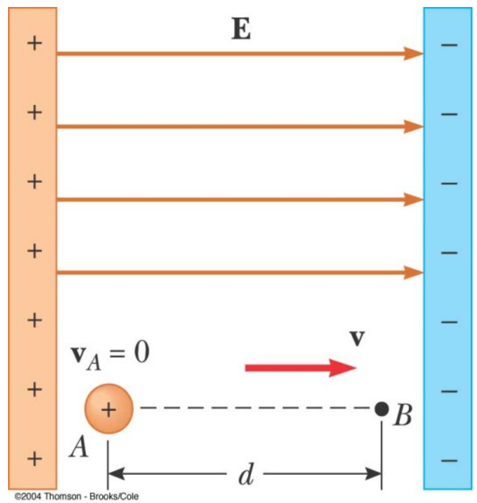
A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of **E**.

(A) Find the change in electric potential between points *A* and *B*.

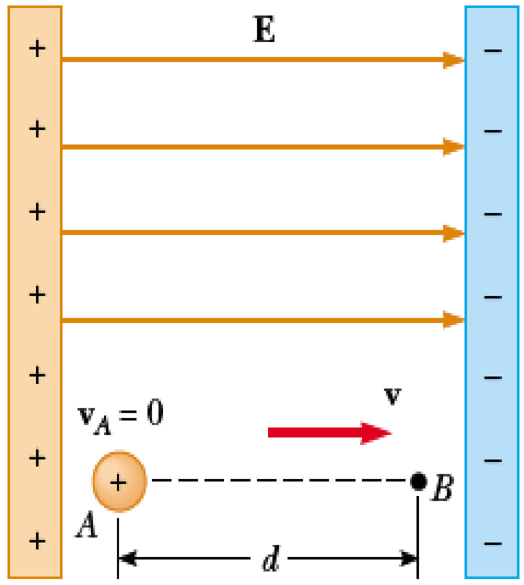
$$\Delta V = -Ed = -(8.0 \times 10^4\text{ V/m})(0.50\text{ m}) = -4.0 \times 10^4\text{ V}$$

(B) Find the change in potential energy of the proton–field system for this displacement.

$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})$$
$$= -6.4 \times 10^{-15}\text{ J}$$



The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.



(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

$$\Delta K + \Delta U = 0$$

$$(\tfrac{1}{2}mv^2 - 0) + e\,\Delta V = 0$$

$$v = \sqrt{\frac{-(2e\Delta V)}{m}}$$

$$= \sqrt{\frac{-2(1.6 \times 10^{-19}\ \text{C})(-4.0 \times 10^{4}\ \text{V})}{1.67 \times 10^{-27}\ \text{kg}}}$$

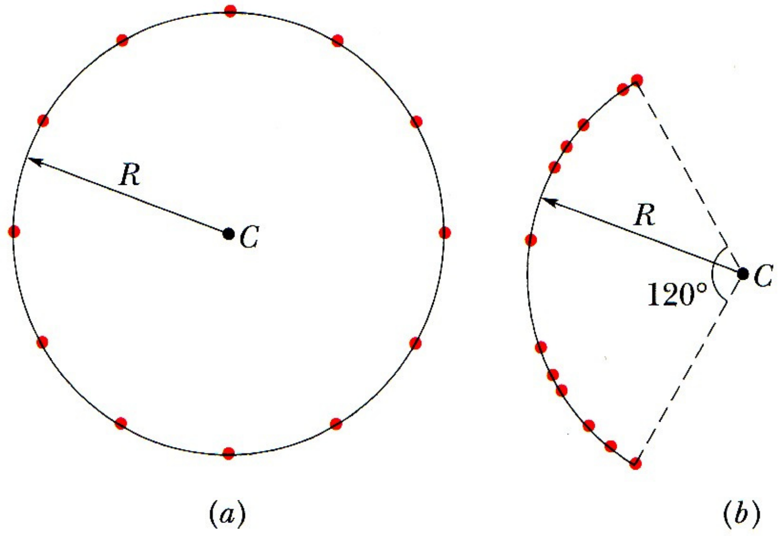
$$= 2.8 \times 10^{6}\ \text{m/s}$$

Example: (a) In figure a, 12 electrons are equally spaced and fixed around a circle of radius R. Relative to V=0 at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons? (b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° are (figure b), what then is the potential at C?

Solution:

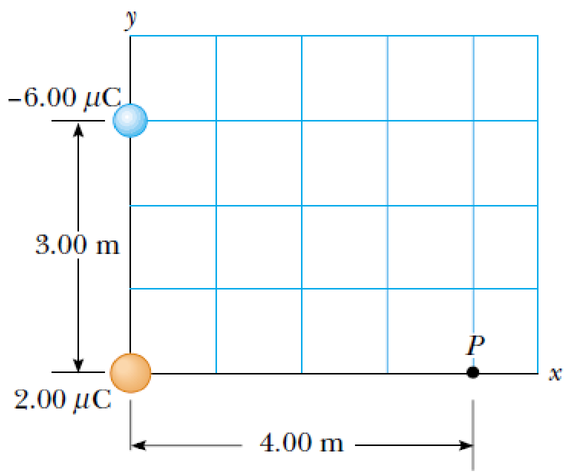
$$(a): \quad V = -K\frac{12e}{R} \qquad \mathbf{E} = 0$$

$$(b): \quad V = -K\frac{12e}{R}$$



Potential due to a group of point charges

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i}^{n}\frac{q_i}{r_i}$$



(a)

Example (25.3)

(a) The electric potential at *P* due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge q_3 = 3.00 C is brought from infinity to a position near the other charges.

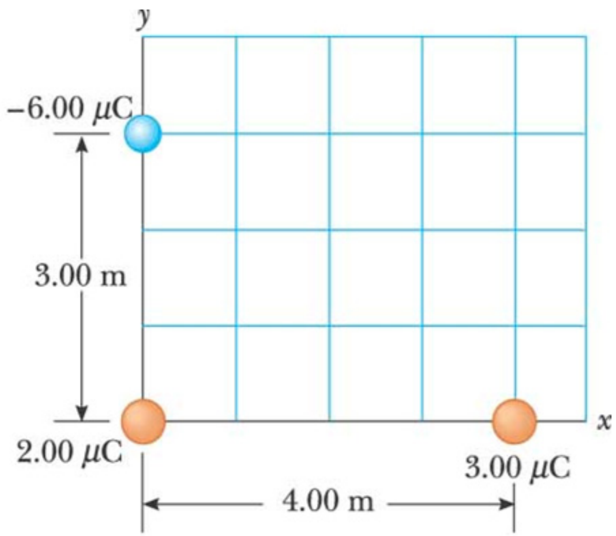
Solution For two charges, the sum

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})$$

$$\times\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} - \frac{6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \mathrm{V}}$$



(b)

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu\text{C}$ as the latter charge moves from infinity to point P

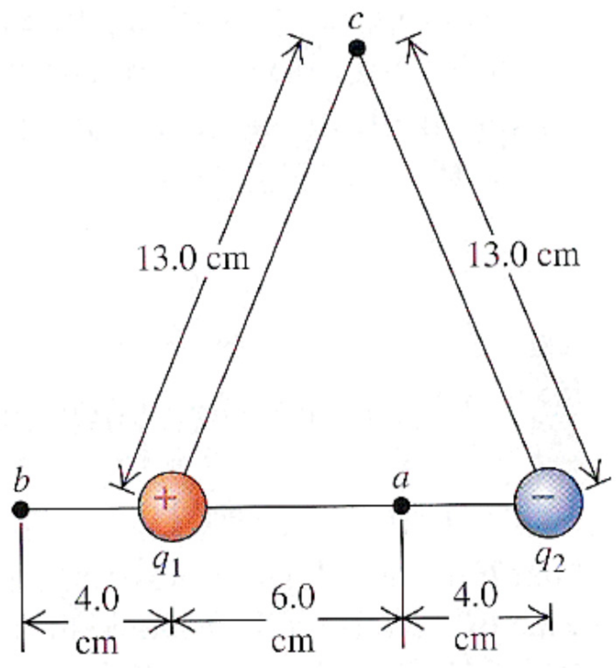
Solution When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^{3}\ \text{V})$$

$$= -1.89 \times 10^{-2}\ \text{J}$$

Example

An electric dipole consists of two charges q_1 = +12nC and q_2 = -12nC, placed 10 cm apart as shown in the figure. Compute the potential at points a,b, and c.



$$V_a = k\sum\left(\frac{q_1}{r_a} + \frac{q_2}{r_a}\right)$$

$$V_a = 8.99x10^9\left(\frac{12x10^{-9}}{0.06} + \frac{-12x10^{-9}}{0.04}\right)$$

$$V_a = -899\ \mathrm{V}$$

$$V_b = k\sum\left(\frac{q_1}{r_b} + \frac{q_2}{r_b}\right)$$

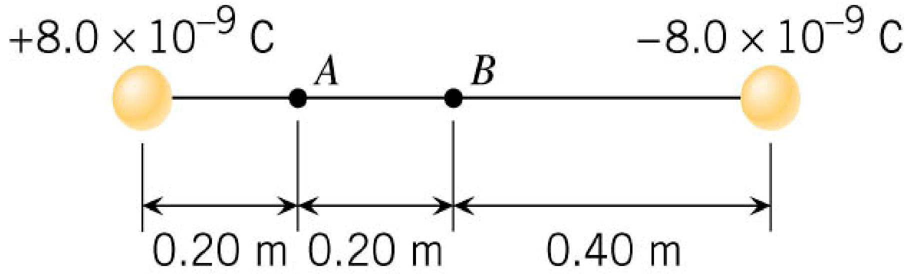
$$V_b = 8.99x10^9\left(\frac{12x10^{-9}}{0.04} + \frac{-12x10^{-9}}{0.14}\right)$$

$$V_b = 1926.4\ V$$

$$V_c = 0\ \mathrm{V}$$

Example The Total Electric Potential

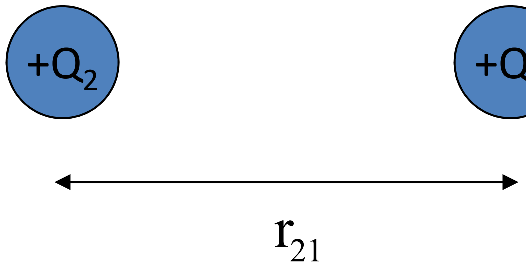
At locations A and B, find the total electric potential.



$$V_A = \frac{\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(+8.0\times10^{-8}\,\mathrm{C}\right)}{0.20\mathrm{m}} + \frac{\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(-8.0\times10^{-8}\,\mathrm{C}\right)}{0.60\mathrm{m}} = +240\mathrm{V}$$

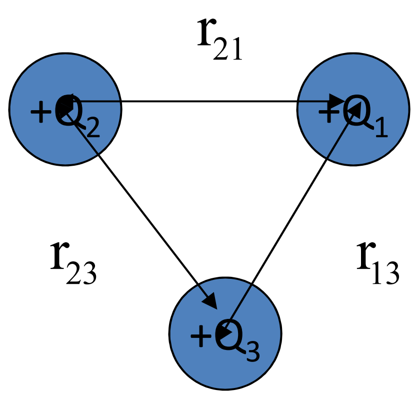
$$V_B = \frac{\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(+8.0\times10^{-8}\,\mathrm{C}\right)}{0.40\mathrm{m}} + \frac{\left(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\left(-8.0\times10^{-8}\,\mathrm{C}\right)}{0.40\mathrm{m}} = 0\,\mathrm{V}$$

Potential energy due to multiple point charges



$$V(r) = \frac{kq}{r} \qquad V = \frac{kq_1}{r_{12}}$$

$$U = q_2 V = \frac{kq_1 q_2}{r_{12}}$$



$$V = \frac{kq_1}{r_{13}} + \frac{kq_2}{r_{23}}$$

$$U = \frac{kq_1 q_2}{r_{12}} + \frac{kq_1 q_3}{r_{13}} + \frac{kq_2 q_3}{r_{23}}$$

Summary

- Electric potential energy: $PE_b - PE_a = -qEd$

- Electric potential difference: work done to move charge from one point to another

- Relationship between potential difference and field:

$$E = -\frac{V_{ba}}{d}$$

- Equipotential: line or surface along which potential is the same

- Electric potential of a point charge:

$$V = k\frac{Q}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$