جامعة الملك سعود

كلية العلوم

قسم الفيزياء والفلك

مذكرة المقرر 104 فيز

تنسيق: أ.د. ناصر بن صالح الزايد

الباب 26

محاضرة رقم 6 (صيفي)

أجزاء كبيرة من هذه المذكرة معتمدة على عروض الأستاذة نورة علي المنيف - قسم الفيزياء.

2019

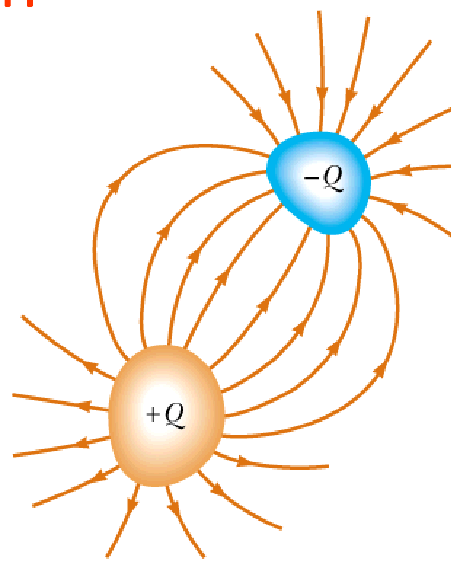
Physics 104
Chapter 26

capacitance and dielectric

Lecture No. 08

26-1 Definition of Capacitance

A capacitor consists of two conductors (known as plates) carrying charges of equal magnitude but opposite signs.



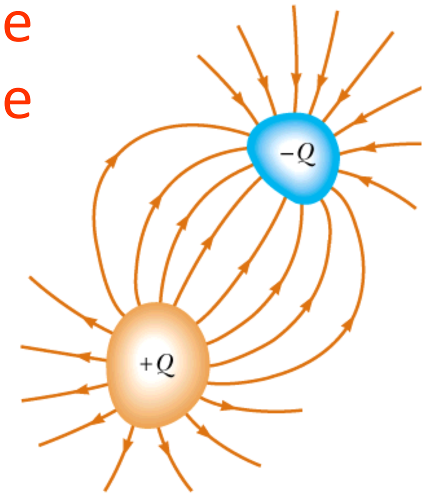
A potential difference V exists between the conductors due to the presence of the charges.

What is the capacity of the device for storing charge at particular value of V?

Experiments show the quantity of electric charge Q on a capacitor is linearly proportional to the potential difference between the conductors, that is Q ~ V. Or we write Q = CV

Definition of Capacitance

The capacitance C of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:



$$C \equiv \frac{Q}{V}$$

SI Unit: farad (F), 1F = 1 C/V

The farad is an extremely large unit, typically you will see

microfarads ($\mu F=10^{-6}F$),

nanofarads ($nF=10^{-9}F$), and

picofarads ($pF=10^{-12}F$)

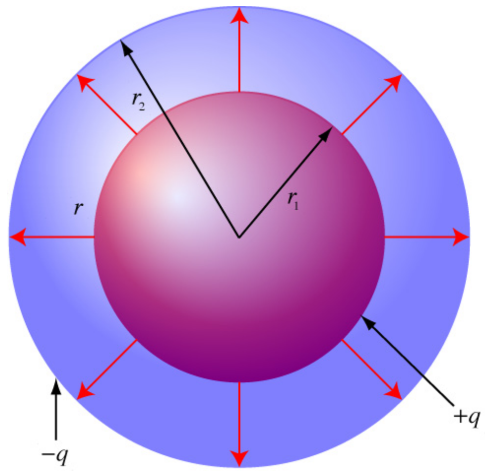
- Capacitance will always be a positive quantity
- The capacitance of a given capacitor is constant
- The capacitance is a measure of the capacitor's ability to store charge

26-2 Calculating Capacitance

Capacitance of an Isolated Sphere

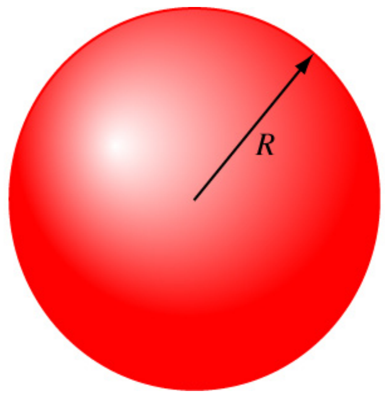
•Let's assume that the inner sphere has charge $+q$ and the outer sphere has charge $-q$

We obtain the capacitance of a single conducting sphere by taking our result for a spherical capacitor and moving the outer spherical conductor infinitely far away



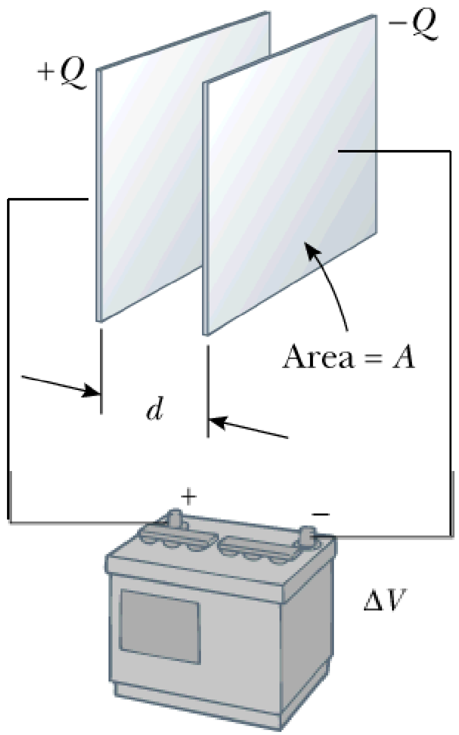
- Assume a spherical charged conductor
- Assume $V = 0$ at infinity

$$C = \frac{q}{V} = \frac{q}{\dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{R}} = 4\pi\varepsilon_0 R$$



Note, this is independent of the charge and the potential difference

Parallel - Plate Capacitors



A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d. When the capacitor is charged, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.

The plates are charged by connection to a battery.
Describe the process by which the plates get charged up.

- For example, a `parallel plate' capacitor, has capacitance

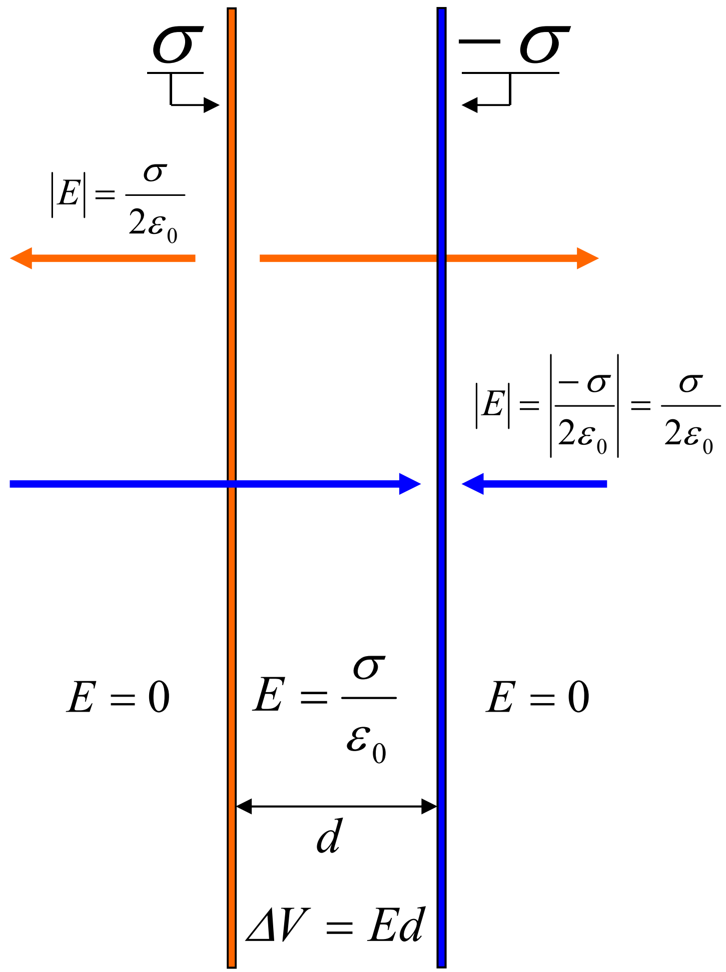
$$C = \frac{Q}{V}$$

$$\because V = Ed = \frac{\sigma}{\varepsilon_o} d = \frac{Qd}{\varepsilon_o A}$$

$$\therefore V = (\frac{d}{\varepsilon_o A})Q \rightarrow \quad Q = (\frac{\varepsilon_o A}{d})V$$

$$Q = CV$$

$$C = \frac{\varepsilon_o A}{d}$$



example

What is the AREA of a 1F capacitor that has a plate separation of 1 mm?

$$C = \varepsilon_o \frac{A}{d}$$

$$1 = 8.85 \times 10^{-12} \frac{A}{0.001}$$

$$A = 1.13 \times 10^{8} m^{2}$$

$$Sides = 10629m$$

Example

(a) If a drop of liquid has capacitance 1.00 pF, what is its radius ? (b) If another drop has radius 2.00 mm, what is its capacitance ? (c) What is the charge on the smaller drop if its potential is 100V ?

$C = 4\pi\varepsilon_o R$

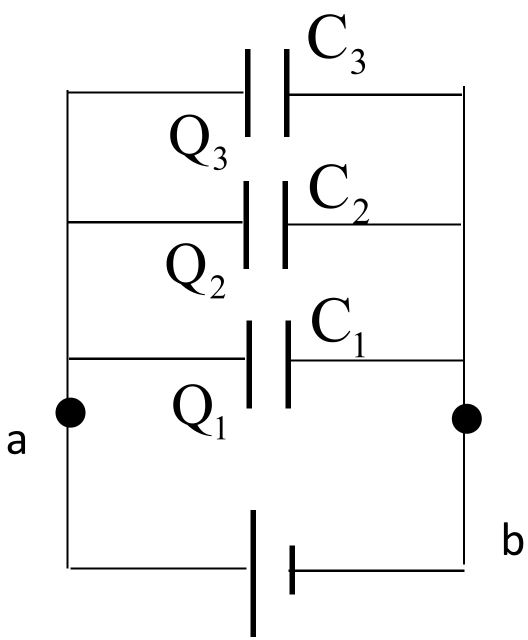
$R = (8.99 \times 10^{9}\ N \cdot m^2/C^2)(1.00 \times 10^{-12}\ F) = 8.99\ mm$

$C = 4\pi\ (8.85 \times 10^{-12}) \times 2.0\times10^{-3} = 0.222\ pF$

$Q = CV = 0.222\ pF \times 100\ V = 2.22 \times 10^{-11}\ C$

26.3 Combinations of Capacitors

Parallel Combination



$$V = V_{ab}$$

$$Q_{total} = Q_1 + Q_2 + Q_3 \qquad V = V_1 = V_2 = V_3$$

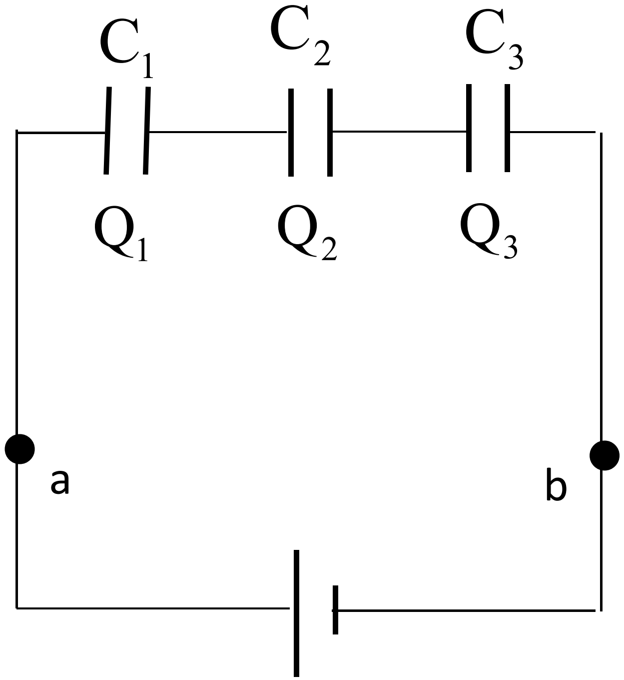
$$Q_{total} = C_{eq} V$$

$$Q_1 = C_1 V \qquad Q_2 = C_2 V \qquad Q_3 = C_3 V$$

$$C_{eq} V = C_1 V + C_2 V + C_3 V$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

Capacitors in Series



$$V = V_{ab}$$

$$Q = C_{eq}V \rightarrow V = \frac{Q}{C_{eq}}$$

$$V = V_1 + V_2 + V_3 \qquad Q_{total} = Q_1 = Q_2 = Q_3$$

$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2} \qquad V_3 = \frac{Q}{C_3}$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Example

A 1-megabit computer memory chip contains many 60.0-fF capacitors. Each capacitor has a plate area of 21.0 x 10^{-12} m^2. Determine the plate separation of such a capacitor (assume a parallel-plate configuration). The characteristic atomic diameter is 10^{-10} m =0.100nm. Express the plate separation in nanometers.

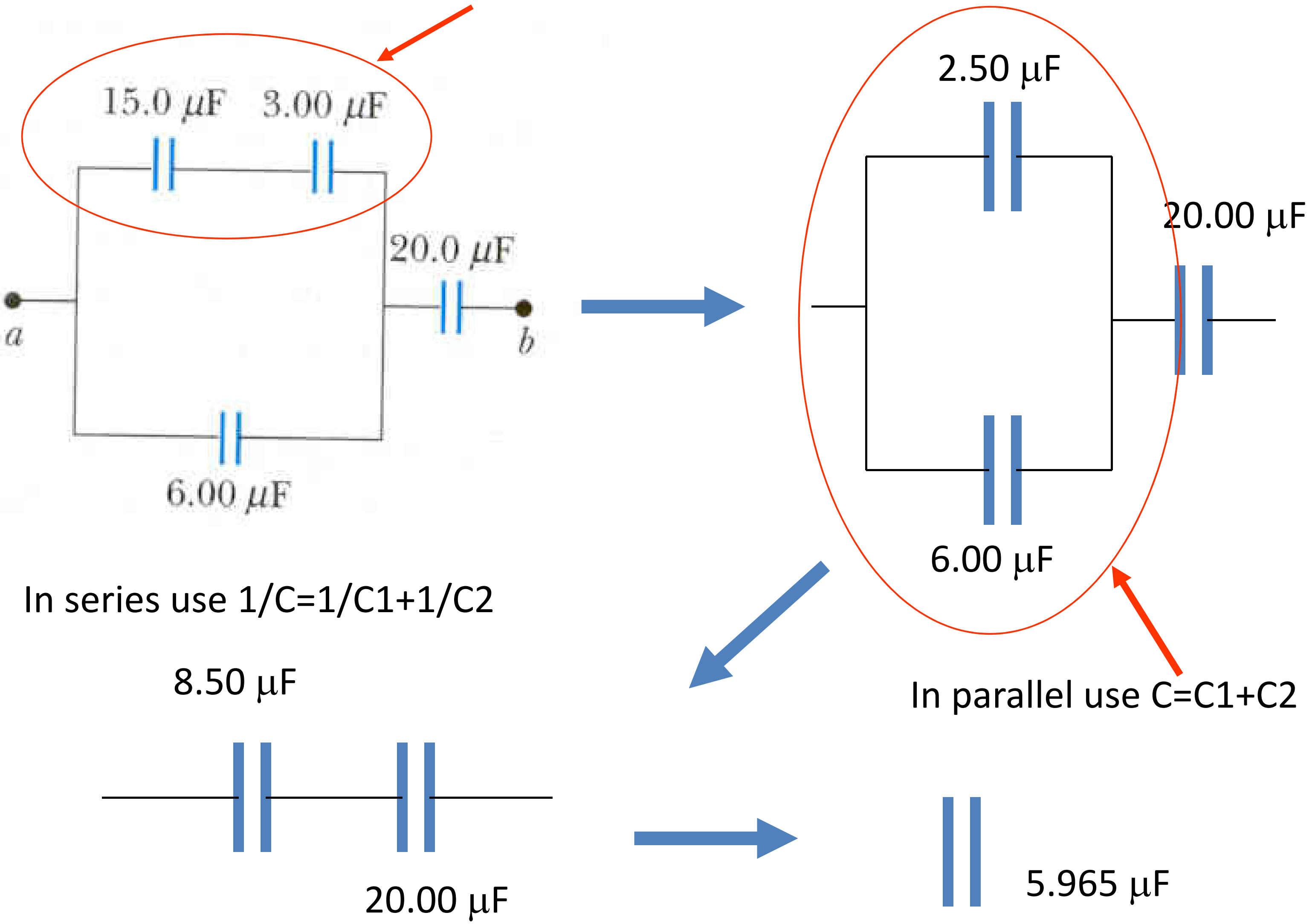
$$C = \frac{\varepsilon_0 A}{d} = 60.0\times 10^{-15}\ \mathbf{F}$$

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times 10^{-12}\right)\left(21.0\times 10^{-12}\right)}{60.0\times 10^{-15}}$$

$$d = 3.10\times 10^{-9}\ \mathbf{m} = 3.10nm$$

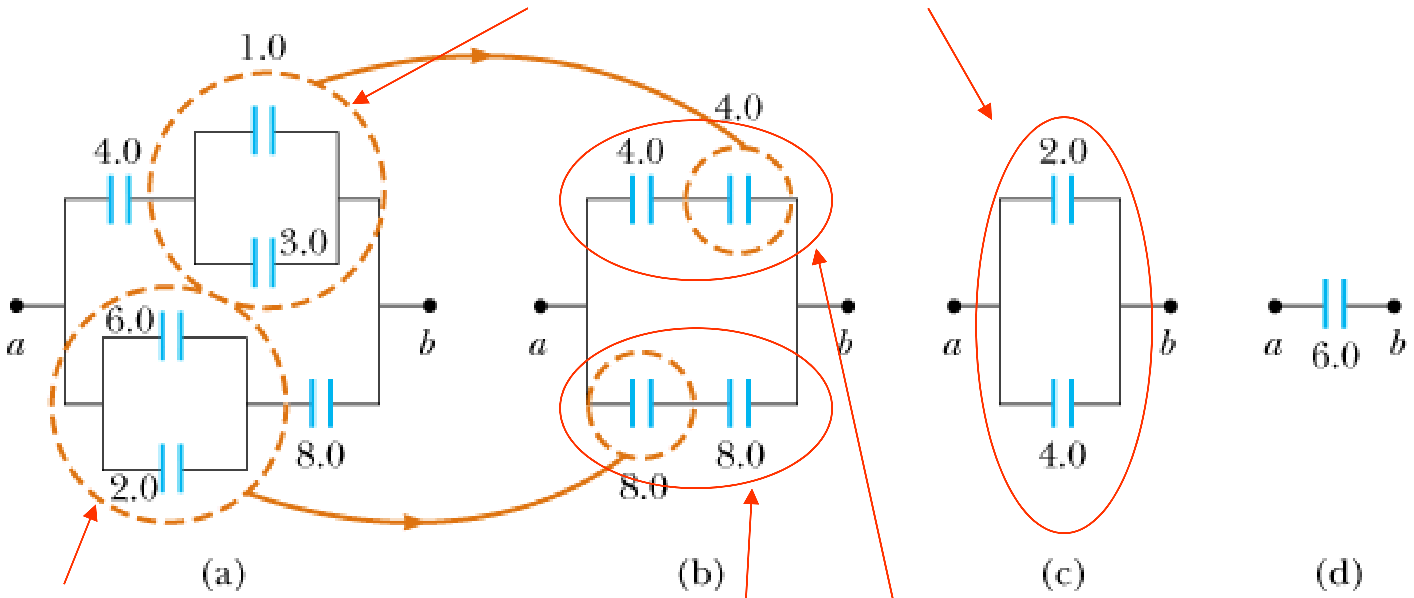
Example: Equivalent Capacitance

In series use 1/C=1/C1+1/C2



Example: Equivalent Capacitance

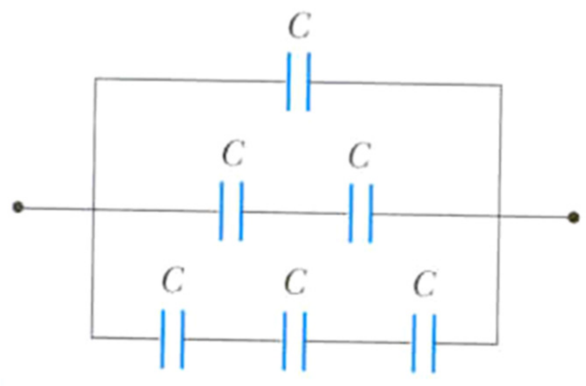
In parallel use C=C1+C2



In parallel use C=C1+C2

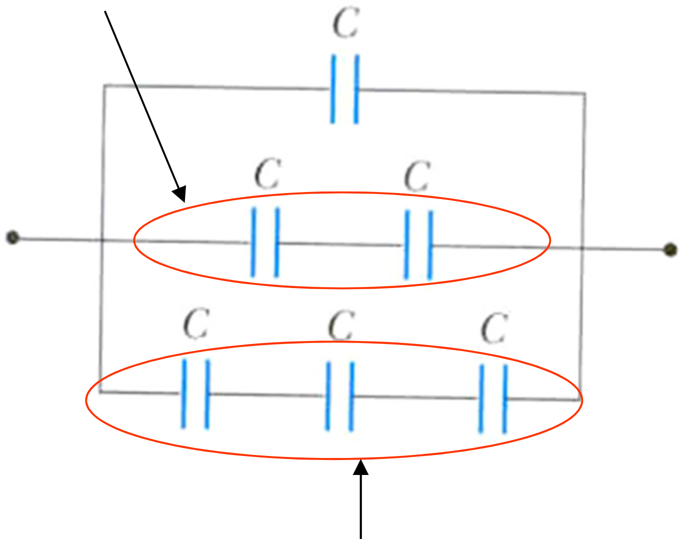
In series use 1/C=1/C1+1/C2

Example: Equivalent Capacitance 26.22

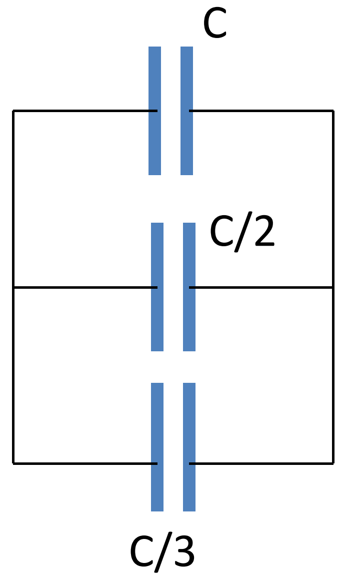


In parallel use $C_{eq}=C+C/2+C/3$

In series use $1/C_A=1/C+1/C$



In series use $1/C_B=1/C+1/C+1/C$



26-4 Energy Stored in a Charged Capacitor

- When a capacitor has charge stored in it, it also stores electric potential energy that is

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \rightarrow U = \frac{1}{2}\frac{Q}{V}V^2 = \frac{1}{2}QV$$

- This applies to a capacitor of any geometry
- The energy stored increases as the charge increases and as the potential difference (voltage) increases
- In practice, there is a maximum voltage before discharge occurs between the plates

Energy Density

the energy density (energy per unit volume) $u = \dfrac{U}{\text{Volume}}$

Consider a Parallel Plate Capacitor: $U = \dfrac{1}{2}CV^2$

$$\because C = \frac{A\varepsilon_0}{d} \quad \text{and} \quad V = Ed$$

$$U = \frac{1}{2}\frac{A\varepsilon_0}{d}E^2d^2 = \frac{1}{2}(Ad)\varepsilon_0E^2$$

$$u = \frac{U}{\text{Volume}} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0E^2$$

Example

A parallel plate capacitor consists of two metal disks, 5.00 cm in radius. The disks are separated by air and are a distance of 4.00 mm apart. A potential of 50.0 V is applied across the plates by a battery. Find
(a) the capacitance *C of the capacitor, and (b) the charge q on the plate.*

a. The area of the plate is

$$A = \pi r^2 = \pi(0.0500 \text{ m})^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$$C = \frac{\varepsilon_o A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)(7.85 \times 10^{-3} \text{ m}^2)}{(4.00 \times 10^{-3} \text{ m})}$$

$$C = 17.4 \times 10^{-12} \text{ F}$$
$$C = 17.4 \text{ pF}$$

b. The charge on the plate is

$$q = CV$$
$$q = (17.4 \times 10^{-12} \text{ F})(50.0 \text{ V})$$
$$q = 8.70 \times 10^{-10} \text{ C}$$

(c) Find the energy stored in the capacitor

$$W = 1/2\ CV^2 = 1/2\ (17.4 \times 10^{-12}\ \text{F})(50.0\ \text{V})^2$$

$$W = 2.18 \times 10^{-8} \frac{\text{C V}^2}{\text{V}} \left(\frac{\text{J/C}}{\text{V}}\right)$$

$$W = 2.18 \times 10^{-8}\ \text{J}$$

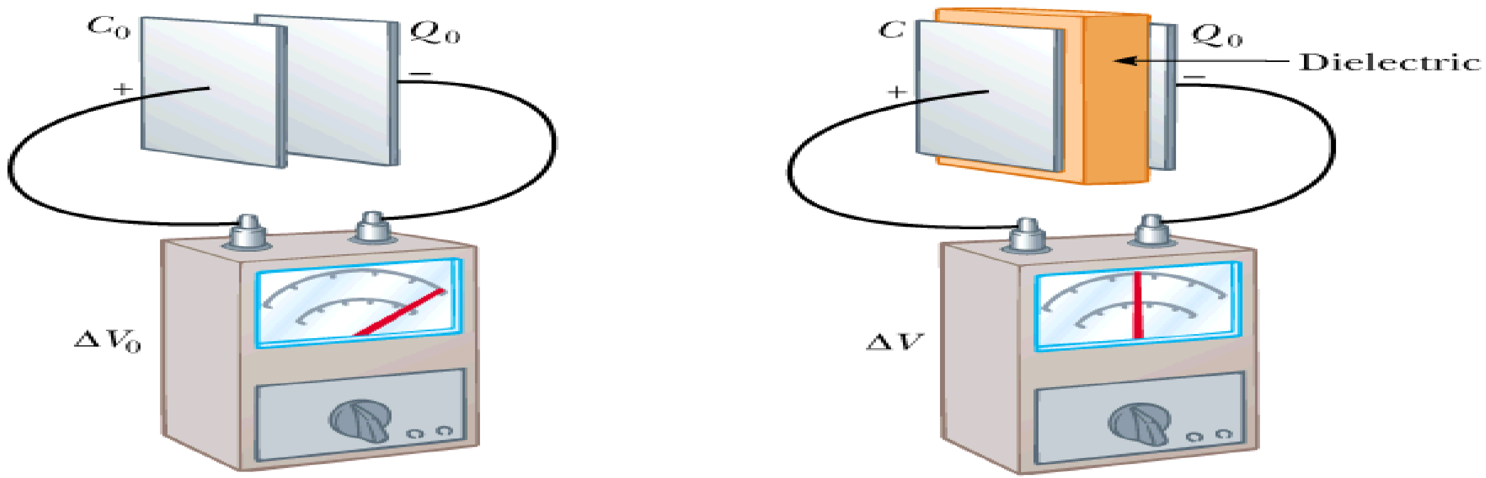
(d) Find the energy density in the electric field between the plates of the above parallel plate capacitor.

$$U_E = \frac{W}{\text{volume}} = \frac{W}{Ad} = \frac{2.18 \times 10^{-8}\ \text{J}}{(7.85 \times 10^{-3}\ \text{m}^2)(4.00 \times 10^{-3}\ \text{m})}$$

$$U_E = 6.94 \times 10^{-4}\ \text{J/m}^3$$

26-5 Capacitors with Dielectrics

- A *dielectric* is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance
  - Dielectrics include rubber, glass, and waxed paper
- With a dielectric, the capacitance becomes $C = \kappa C_o$
  - The capacitance increases by the factor κ when the dielectric completely fills the region between the plates
  - κ is the **dielectric constant** of the material Dielectric constant is a property of a material and varies from one material to another.



(a) (b)

Effect of a dielectric on capacitance

$$E_{Dielectric} = \frac{E_o}{\kappa}$$

$$V_{Dielectric} = \frac{V_o}{\kappa}$$

Potential difference with a dielectric is less than the potential difference across free space

$$C = \frac{Q}{V} = \kappa \frac{Q}{V_o} = \kappa C_o$$

Results in a higher capacitance.

$$C = \kappa \frac{\varepsilon_o A}{d}$$

Allows more charge to be stored before breakdown voltage.

If the dielectric is introduced while the potential difference is being maintained constant by a battery, the charge increases to a value $Q = \kappa Q_o$. The additional charge is supplied by the battery and the capacitance again increases by the factor κ.

Example values of dielectric constant

Table 26.1

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

"Dielectric strength" is the maximum field in the dielectric before breakdown.
(a spark or flow of charge)

$$E_{max} = V_{max} / d$$

Example

26.33: A parallel-plate capacitor is charged and then disconnected from a battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled ?

$U = Q^2/2C$

and $C = \varepsilon_o A/d$

and $d_2 = 2\ d_1$ then

$C_2 = C_1/2.$

Hence, U is doubled

Quick Quiz 26.1 A capacitor stores charge Q at a potential difference ΔV. If the voltage applied by a battery to the capacitor is doubled to $2\Delta V$, (a) the capacitance falls to half its initial value and the charge remains the same (b) the capacitance and the charge both fall to half their initial values (c) the capacitance and the charge both double (d) the capacitance remains the same and the charge doubles.

Quick Quiz 26.3 Two capacitors are identical. They can be connected in series or in parallel. If you want the *smallest* equivalent capacitance for the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same capacitance?

Quick Quiz 26.4 Consider the two capacitors in Quick Quiz 26.3 again. Each capacitor is charged to a voltage of 10 V. If you want the largest combined potential difference across the combination, do you connect them in (a) series, in (b) parallel, or (c) do the combinations have the same potential difference?

Quick Quiz 26.5 You have three capacitors and a battery. In which of the following combinations of the three capacitors will the maximum possible energy be stored when the combination is attached to the battery? (a) series (b) parallel (c) Both combinations will store the same amount of energy.

Quick Quiz 26.6 You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires connected to the plates from touching each other. When you pull the plates apart to a larger separation, do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV; (e) energy stored in the capacitor.

Quick Quiz 26.7 Repeat Quick Quiz 26.6, but this time answer the questions for the situation in which the battery remains connected to the capacitor while you pull the plates apart.

Quick Quiz 26.9 A fully charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates. Do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV.

Summary

Capacitance:	$C \equiv \frac{Q}{V}$
Capacitance of a sphere:	$C = 4\pi\varepsilon_0 R$
Parallel Plate Capacitor:	$C = \frac{\varepsilon_o A}{d}$
Capacitors in Parallel:	$C_{eq} = C_1 + C_2 + C_3$
Capacitors in Series:	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
Energy of a Capcitor:	$U = \frac{1}{2}CV^2 = \frac{1}{2}QV$
Density of Energy Parallel Plate Capacitor:	$u = \frac{1}{2}\varepsilon_0 E^2$
Dielectric Effect:	$C = \kappa C_o$