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الباب 27

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Physics 104
Chapter 27

CHAPTER 27 CURRENT AND RESISTANCE

27.1 ELECTRIC CURRENT

27.2 RESISTANCE AND OHM'S LAW

27.3 RESISTANCE AND TEMPERATURE

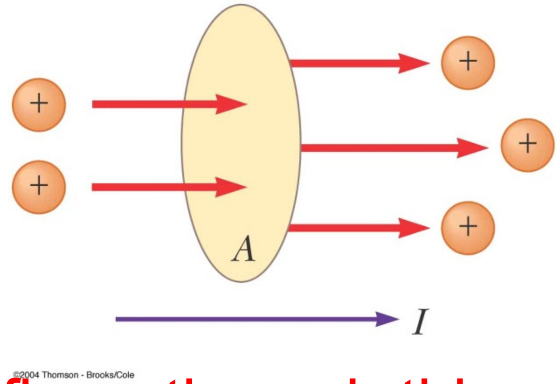
27.6 ELECTRICAL ENERGY AND POWER

Lecture No. 10

27-1 Electric Current

Now consider a system of electric charges in motion. Whenever there is a net flow of charge through some region, a current is said to exist.

the charges are moving perpendicular to a surface of area *A*,



The current is the rate at which charge flows through this surface.

average current

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

instantaneous current

$$I \equiv \frac{dq}{dt}$$

The SI unit of current is the ampere (A): $1\,A = \frac{1\,C}{1\,s}$

$$\Delta Q = Ne$$

$$I_{av} = \frac{Ne}{\Delta t}$$

That is, **1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.**

- It is conventional to assign to the current the same direction as the flow of ***positive charge***

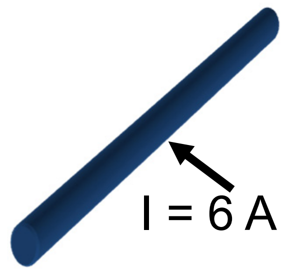
• the direction of the current is opposite the direction of flow of electrons.

• It is common to refer to a moving charge (positive or negative) as a mobile charge carrier. For example, the mobile charge carriers in a metal are electrons.

Example:

The electric current in a wire is 6 A. How many electrons flow past a given point in a time of 3 s?

$$I_{ave} = \frac{\Delta Q}{\Delta t} = \frac{Ne}{\Delta t}$$



$$N = \frac{I\ \Delta t}{e} = \frac{6 \times 3}{1.6 \times 10^{-19}} = 1.125 \times 10^{20}\, electrons$$

Example :

The quantity of charge q (in coulombs) that has passed through a surface of area 2.00 cm^2 varies with time according to the equation $q = 4t^3 + 5t + 6$, where t is in seconds.

(a) What is the instantaneous current through the surface at t = 1.00 s?

(b) What is the value of the current density?

$$a) \qquad I = \frac{dq}{dt} = 12t^2 + 5$$

$$at \qquad \mathrm{t} = 1\sec$$

$$\mathrm{I} = 12 \times (1)^2 + 5 = 17\ A$$

$$J = \frac{I}{A} = \frac{17.0\ \mathrm{A}}{2.00 \times 10^{-4}\ \mathrm{m}^2} = \boxed{85.0\ \mathrm{kA/m}^2}$$

Microscopic Model of Current

n = number of electrons/volume

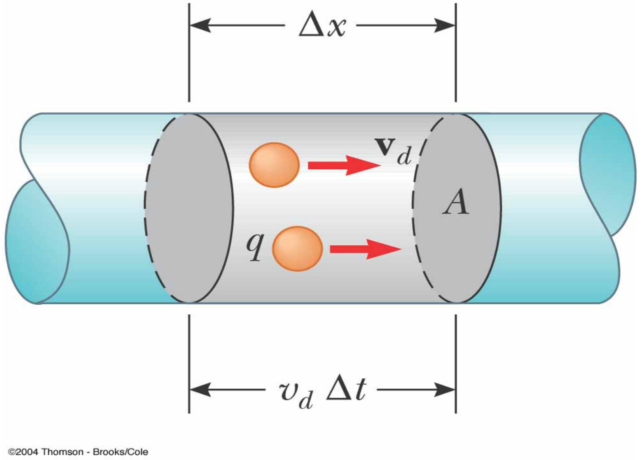
N = n.Volume = n.A.ΔX

electrons travel distance ΔX = v Δt

➔N = n.A.v.Δt

ΔQ = number of carriers in section x charge per carrier

ΔQ = Ne



➔ *ΔQ = (nAv Δt)e*

$$\therefore I_{av} = \frac{\Delta Q}{\Delta t} = vAne$$

The speed of the charge carriers v is an average speed called the *drift speed*.

27-2 resistance and ohm's law

Consider a conductor of cross-sectional area *A carrying a current I*.

The current density **J** in the conductor is defined as the current per unit area. the current density is:

$$J = \frac{I}{A} = ven \qquad (\mathrm{A/m^2})$$

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor.

Georg Simon Ohm

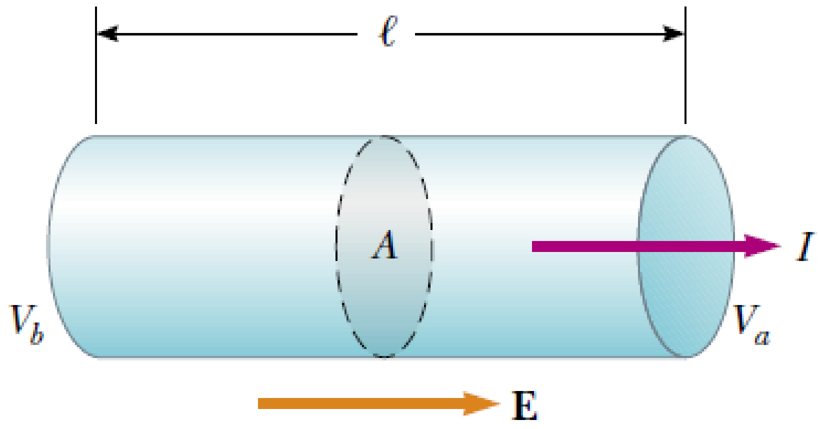
German physicist (1789–1854)

Ohm, a high school teacher and later a professor at the University of Munich, formulated the concept of resistance and discovered the proportionalities expressed in Equations 27.7 and 27.8. *(© Bettmann/Corbis)*

If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field: $J = \sigma E$

σ is called the conductivity

Deriving ohm's law



$$\because V = E\ell$$

$$\therefore J = \sigma E = \sigma \frac{V}{\ell}$$

$$\rightarrow V = \frac{\ell}{\sigma} J = \frac{\ell}{\sigma}\frac{I}{A} = \left(\frac{\ell}{\sigma A}\right) I$$

$Or:$

$$V = \left(\frac{\ell}{\sigma A}\right) I$$

$$with: R = \frac{\ell}{\sigma A}$$

$$\therefore V = IR$$

$$\rho \text{ (resistivity)} = \frac{1}{\sigma}$$

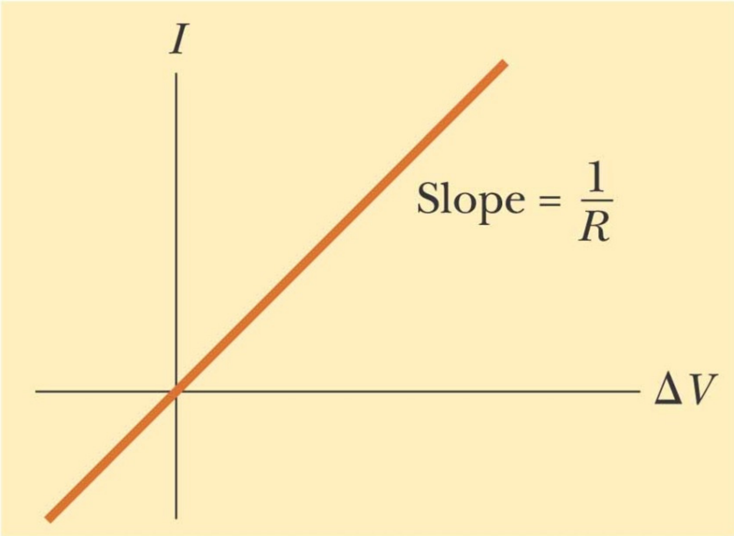
$$R \equiv \frac{\rho \ell}{A}$$

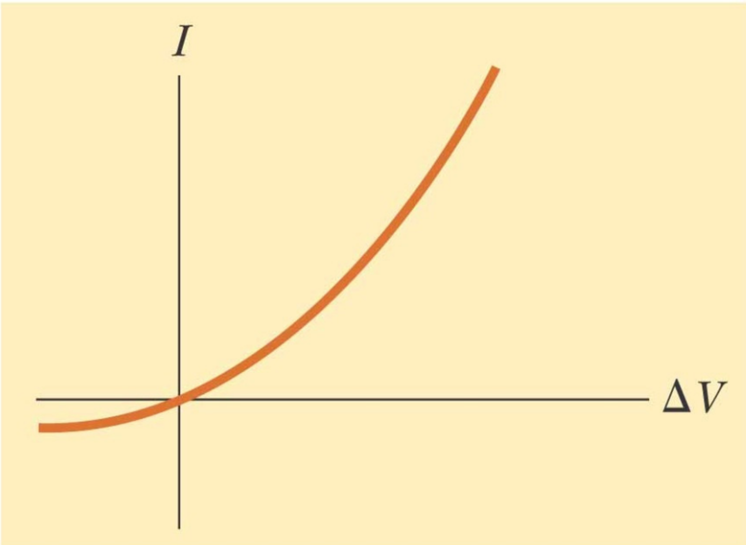
From this result we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be 1 ohm (Ω):

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

$$I = \frac{V}{R}; \qquad V = IR; \qquad R = \frac{V}{I}$$

Quick Quiz 27.4 A cylindrical wire has a radius r and length ℓ. If both r and ℓ are doubled, the resistance of the wire (a) increases (b) decreases (c) remains the same.





(b)

(a) The current–potential difference curve for an **ohmic material**. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor.

(b) A nonlinear current–potential difference curve for a semiconducting diode. This device does not obey Ohm’s law.

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Example:

When a 3V battery is connected to a light, a current of 6 mA is observed. What is the resistance of the light filament?

$$R = \frac{V}{I} = \frac{3}{6 \times 10^{-3}} = 500\ \Omega$$

Example :

What length ℓ of copper wire is required to produce a 4 mΩ resistor? Assume the diameter of the wire is 1 mm and that the resistivity ρ of copper is 1.72 x 10^{-8} Ω·m .

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = 3.14 \times \left(\frac{1\times 10^{-3}}{2}\right)^2 = 7.85 \times 10^{-7} m^2$$

$$R = \rho \frac{l}{A}$$

$$l = \frac{RA}{\rho} = \frac{0.004 \times 7.85 \times 10^{-7}}{1.72 \times 10^{-8}} = 0.185\ m$$

Example :

Calculate the resistance of a rectangular strip of copper length 0.08 m. thickness15 mm and width 0.8 mm . The resistivity of copper = 1.7 x 10^{-8} Ω.m

$$R = \rho \frac{l}{A}$$

$$R = 1.7 \times 10^{-8} \times \frac{0.08}{0.8 \times 10^{-3} \times 15 \times 10^{-3}} = 11 \mu \Omega$$

Table 27.1

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] $(\Omega \cdot m)$	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements.

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Table 27-1, p.837

EXAMPLE 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of 2.0x 10^{-4} m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of 3×10^{10} Ω

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

EXAMPLE 27.3 The Resistance of Nichrome Wire

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega\cdot\text{m}$ (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 27.8 gives

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

27-3 RESISTANCE AND TEMPERATURE

Over a limited temperature range, the resistivity of a metal varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad \textbf{(27.19)}$$

Variation of ρ with temperature

where ρ is the resistivity at some temperature *T (in degrees Celsius)*, ρ_0 *is the resistivity* at some reference temperature T_0 *(usually taken to be 20°C), and* α is *the* temperature coefficient of resistivity.

Temperature coefficient of resistivity

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \qquad \text{Celsius}^{-1}\ [(°\text{C})^{-1}] \qquad \textbf{(27.20)}$$

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

Because resistance is proportional to resistivity $R = \rho \frac{L}{A}$ (Eq. 27.11),

For most materials, the resistance *R* changes in proportion to the initial resistance R_o and to the change in temperature Δt.

$$R = R_o\left[1 + \alpha\left(T - T_0\right)\right]$$ (Eq. 27.21),

Change in resistance:

$$\Delta R = \alpha R_0 \Delta t$$

The temperature coefficient of resistance α, is the change in resistance per unit resistance per unit degree change of temperature.

$$\alpha = \frac{\Delta R}{R_0 \Delta t}; \quad \text{Units: } \frac{1}{C^0}$$

Example:

The resistance of a copper wire is 4.00 mΩ at 20^0C. What will be its resistance if heated to 80^0C? Assume that α = 0.004 /C°.

R_o = 4.00 mΩ; Δt = 80°C – 20°C = 60 C°

$$\Delta R = \alpha R_0 \Delta t; \quad \Delta R = (0.004/C^0)(4\ m\Omega)(60\ C^0)$$

ΔR = 0.96 mΩ

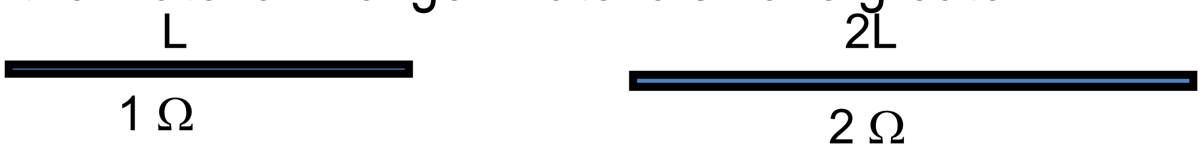
$R = R_o + \Delta R$

R = 4.00 mΩ + 0.96 mΩ

R = 4.96 mΩ

Factors Affecting Resistance

1. The length L of the material. Longer materials have greater resistance.



2. The cross-sectional area A of the material. Larger areas offer LESS resistance.

3. The temperature T of the material. The higher temperatures usually result in higher resistances.

4. The kind of material. Iron has more electrical resistance than a geometrically similar copper conductor.

EXAMPLE 27.6 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 . Calculate the melting point of the indium.

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}\ (°\mathrm{C})^{-1}](50.0\ \Omega)} = 137°\mathrm{C}$$

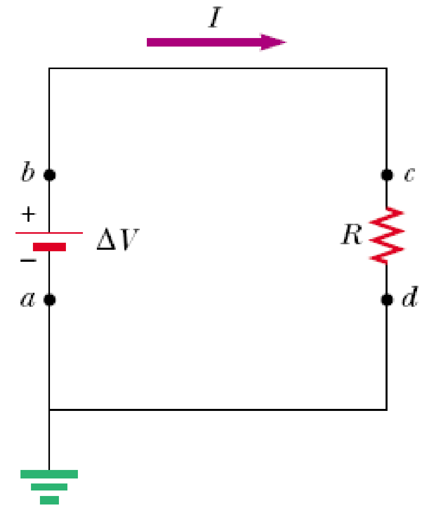
Because $T_0 = 20.0°\mathrm{C}$, we find that T, the temperature of the melting indium sample, is 157°C.

27.6 ELECTRICAL ENERGY AND POWER

Because the rate at which the charge loses energy equals the power delivered to the resistor (which appears as internal energy), we have

$$\mathcal{P} = I\Delta V \qquad \textbf{(27.22)}$$

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R} \qquad \textbf{(27.23)}$$



When the internal resistance of the battery is neglected, the potential difference between points *a* and *b* in Figure 27.14 is equal to the emf $\mathcal{E}$ of the battery—that is, $\Delta V = V_b - V_a = \mathcal{E}$. This being true, we can state that the current in the circuit is $I = \Delta V/R = \mathcal{E}/R$. Because $\Delta V = \mathcal{E}$, the power supplied by the emf source can be expressed as $\mathcal{P} = I\mathcal{E}$, which equals the power delivered to the resistor, I^2R.

EXAMPLE 27.7 Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V to a Ni-chrome wire that has a total resistance of 8.0 Ω . Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\ \text{A})^2(8.00\ \Omega) = \boxed{1.80\ \text{kW}}$$

If we doubled the applied potential difference, the current would double but the power would quadruple because $\boxed{\mathcal{P} = (\Delta V)^2/R.}$

EXAMPLE 27.8 The Cost of Making Dinner

Estimate the cost of cooking a turkey for 4 h in an oven that operates continuously at 20.0 A and 240 V.

Solution The power used by the oven is

$$\mathcal{P} = I\,\Delta V = (20.0\text{ A})(240\text{ V}) = 4\,800\text{ W} = 4.80\text{ kW}$$

Because the energy consumed equals power × time, the amount of energy for which you must pay is

$$\text{Energy} = \mathcal{P}t = (4.80\text{ kW})(4\text{ h}) = 19.2\text{ kWh}$$

If the energy is purchased at an estimated price of 8.00¢ per kilowatt hour, the cost is

$$\text{Cost} = (19.2\text{ kWh})(\$0.080/\text{kWh}) = \$1.54$$

Example:

An electric heater draws a steady 15.0 A on a 120-V line. How much power does it require and how much does it cost per month (30 days) if it operates 3.0 h per day and the electric company charges 9.2 cents per kWh?

$P = IV = 1800$ W.

➔total = 1800 W x 3.0 h/day x 30 days = 162 kWh.

At 9.2 cents per kWh, this would cost $15.

Summary

The **electric current** I in a conductor is defined as

$$I \equiv \frac{dQ}{dt} \qquad \textbf{(27.2)}$$

where dQ is the charge that passes through a cross-section of the conductor in a time dt. The SI unit of current is the **ampere** (A), where 1 A = 1 C/s.

The average current in a conductor is related to the motion of the charge carriers through the relationship

$$I_{av} = nqv_d A \qquad \textbf{(27.4)}$$

where n is the density of charge carriers, q is the charge on each carrier, v_d is the drift speed, and A is the cross-sectional area of the conductor.

The magnitude of the **current density** J in a conductor is the current per unit area:

$$J \equiv \frac{I}{A} = nqv_d \qquad \textbf{(27.5)}$$

The current density in a conductor is proportional to the electric field according to the expression

$$\mathbf{J} = \sigma \mathbf{E} \tag{27.7}$$

The proportionality constant σ is called the **conductivity** of the material of which the conductor is made. The inverse of σ is known as **resistivity** ρ ($\rho = 1/\sigma$). Equation 27.7 is known as **Ohm's law,** and a material is said to obey this law if the ratio of its current density $\mathbf{J}$ to its applied electric field $\mathbf{E}$ is a constant that is independent of the applied field.

The **resistance** R of a conductor is defined either in terms of the length of the conductor or in terms of the potential difference across it:

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I} \tag{27.8}$$

where ℓ is the length of the conductor, σ is the conductivity of the material of which it is made, A is its cross-sectional area, ΔV is the potential difference across it, and I is the current it carries.

The SI unit of resistance is volts per ampere, which is defined to be 1 **ohm** (Ω); that is, $1\ \Omega = 1\ \text{V/A}$. If the resistance is independent of the applied potential difference, the conductor obeys Ohm's law.

The resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad \textbf{(27.19)}$$

where α is the **temperature coefficient of resistivity** and ρ_0 is the resistivity at some reference temperature T_0.

If a potential difference ΔV is maintained across a resistor, the **power,** or rate at which energy is supplied to the resistor, is

$$\mathcal{P} = I\,\Delta V \qquad \textbf{(27.22)}$$

Because the potential difference across a resistor is given by $\Delta V = IR$, we can express the power delivered to a resistor in the form

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R} \qquad \textbf{(27.23)}$$

The electrical energy supplied to a resistor appears in the form of internal energy in the resistor.