جامعة الملك سعود
كلية العلوم
قسم الفيزياء والفلك
مذكرة المقرر 104 فيز

تنسيق: أ.د. ناصر بن صالح الزايد

الباب 28
محاضرة رقم 7 (صيفي)

أجزاء كبيرة من هذه المذكرة معتمدة على عروض الأستاذة نورة علي المنيف – قسم الفيزياء.

2019

Physics 104
Chapter **28**

28.1 Electromotive Force
28.2 Resistors in Series and Parallel
28.3 Kirchhoff's Rules

28.1 Electromotive Force

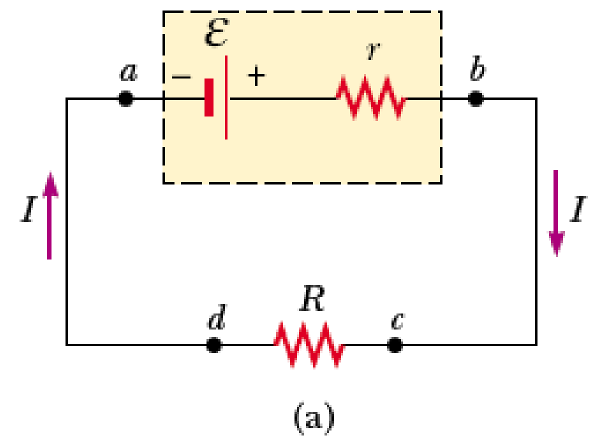


Figure 28.2 (a) Circuit diagram of a source of emf (in this case, a battery), of internal resistance r, connected to an external resistor of resistance R.

- As we pass from the negative terminal to the positive terminal, the potential *increases by an amount ε.*

- *As we move through the resistance r, the potential decreases by an amount Ir, where I is the current in the circuit (between a and b).*

$$V = \varepsilon - Ir$$

- *ε :* is equivalent to the open-circuit voltage—that is, the *terminal voltage when the current is zero. The emf is the voltage labeled on a* battery,….

- the terminal voltage *V must equal the potential difference across the external* resistance *R, often called the load resistance.*

the changes in electric potential as the circuit is traversed in the clockwise direction

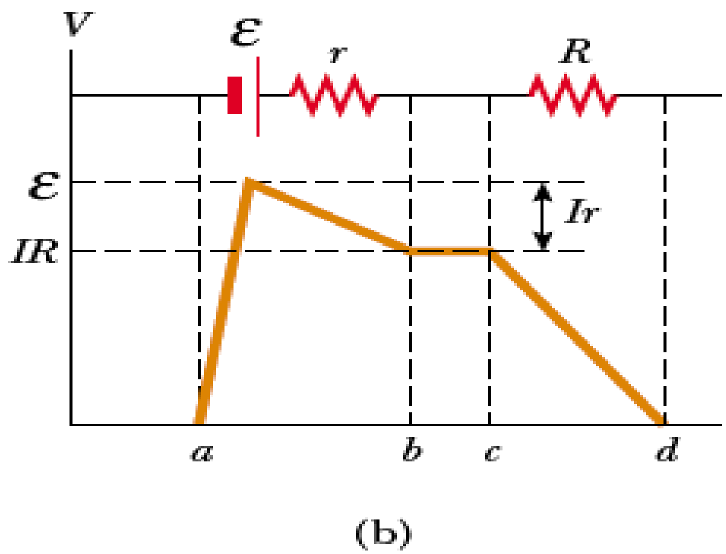


Figure 28.2 (b) Graphical representation showing how the electric potential changes as the circuit in part (a) is traversed clockwise.

- The resistor represents a *load on the battery because the battery must supply energy to* operate the device. The potential difference across the load resistance is

$$V = IR$$

$$\varepsilon = IR + Ir \qquad (28\text{-}2)$$

$$I = \frac{\varepsilon}{R + r} \qquad (28\text{-}3)$$

$$I\varepsilon = I^2R + I^2r \qquad (28\text{-}4)$$

- the total power output $I\varepsilon$ *of the battery is delivered to the external load resistance in the* amount I^2R *and to the internal resistance in the amount* I^2r.

Example 28.1 **Terminal Voltage of a Battery**

A battery has an emf of 12V and an internal resistance of 0.05Ω. Its terminals are connected to a load resistance of 3Ω. Find:

a) The current in the circuit and the terminal voltage

b) The power dissipated in the load, the internal resistance, and the total power delivered by the battery

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = \boxed{3.93\ \text{A}}$$

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = \boxed{11.8\ \text{V}}$$

To check this result, we can calculate the voltage across the load resistance R:

$$\Delta V = IR = (3.93\ \text{A})(3.00\ \Omega) = 11.8\ \text{V}$$

$$\mathcal{P}_R = I^2R = (3.93\ \text{A})^2\ (3.00\ \Omega) = \boxed{46.3\ \text{W}}$$

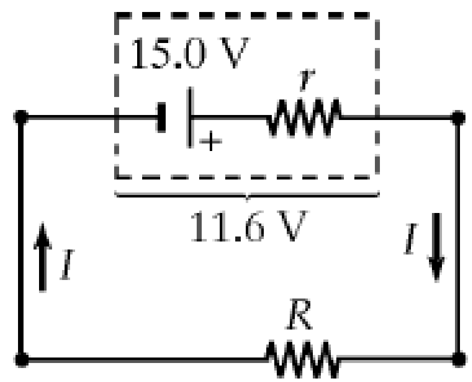
The power delivered to the internal resistance is

$$\mathcal{P}_r = I^2r = (3.93\ \text{A})^2\ (0.05\ \Omega) = \boxed{0.772\ \text{W}}$$

Hence, the power delivered by the battery is the sum of these quantities, or 47.1 W. You should check this result, using the expression $\mathcal{P} = I\mathcal{E}$.

Example

A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor *R.*
(a) What is the value of *R? (b) What is the internal resistance of the* battery?



(a) $\mathcal{P} = \dfrac{(\Delta V)^2}{R}$

becomes $20.0\text{ W} = \dfrac{(11.6\text{ V})^2}{R}$

so $R = \boxed{6.73\ \Omega}$.

(b) $\Delta V = IR$

so $11.6\text{ V} = I(6.73\ \Omega)$

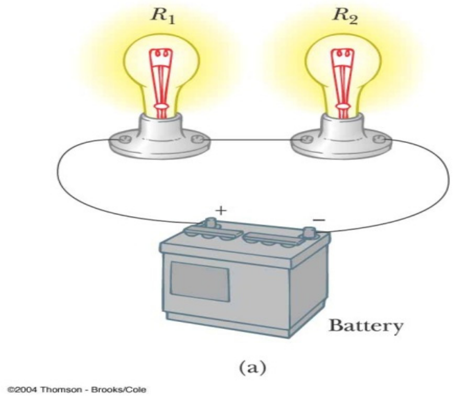
and $I = 1.72\text{ A}$

$\varepsilon = IR + Ir$

so $15.0\text{ V} = 11.6\text{ V} + (1.72\text{ A})r$

$r = \boxed{1.97\ \Omega}$.

28.2 Resistors in Series and Parallel

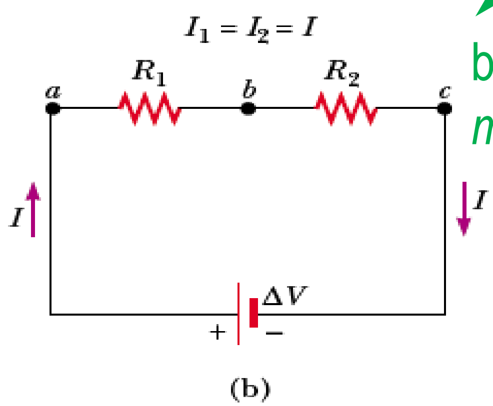


series connection

When two or more resistors are connected together as are the light- bulbs, they are said to be in *series.*

In a series connection, all the charges moving through one resistor must also pass through the second resistor.

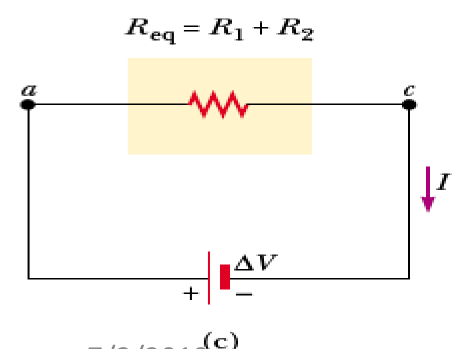
- for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 *must also pass through* R_2



$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

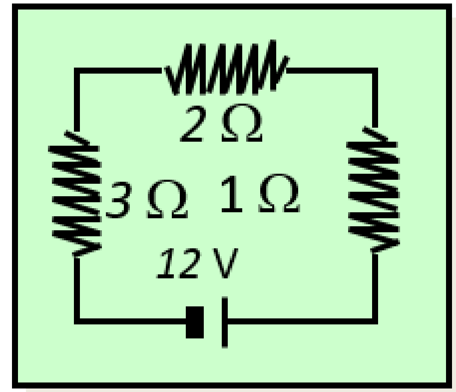


This relationship indicates that the equivalent resistance of a series connection of resistors is always greater than any individual resistance.

Example :

Find the equivalent resistance R_e. What is the current I in the circuit?

$$R_{eq} = R_1 + R_2 + R_3$$



Equivalent $R_{eq} = 3\ \Omega + 2\ \Omega + 1\ \Omega = 6\ \Omega$

The current is found from Ohm's law: $V = IR_e$

$$I = \frac{V}{R_e} = \frac{12\ \text{V}}{6\ \Omega}$$

I = 2 A

Current I = 2 A same in each R.

$V_1 = IR_1$; $V_2 = IR_2$; $V_3 = IR_3$

$V_1 = (2\ \text{A})(1\ \Omega) = 2\ \text{V}$

$V_1 = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$

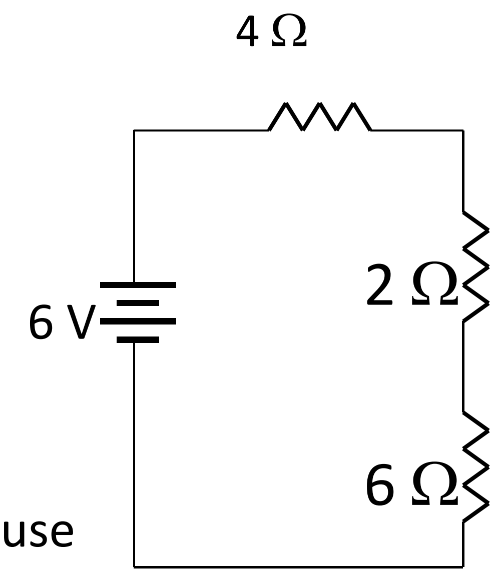
$V_1 = (2\ \text{A})(3\ \Omega) = 6\ \text{V}$

$V_1 + V_2 + V_3 = V_{Total}$

$2\ \text{V} + 4\ \text{V} + 6\ \text{V} = 12\ \text{V}$

Exmple

1. Find R_{eq} 2. Find I_{total} 3. Find the V drops across each resistor.



1. Since the resistors are in series, simply add the three resistances to find R_{eq}:

   $R_{eq} = 4\ \Omega + 2\ \Omega + 6\ \Omega = 12\ \Omega$

2. To find I_{total} (the current through the battery), use $V = IR$:

   $6 = 12\ I$. So, $I = 6/12 = 0.5$ A

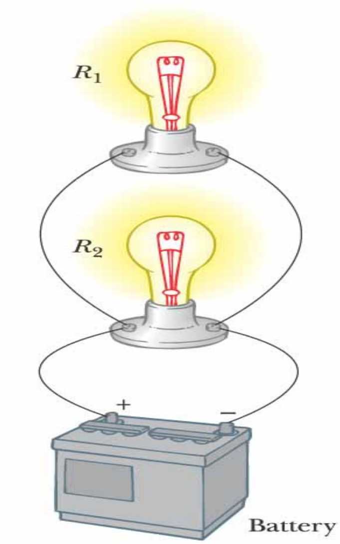
3. Since the current throughout a series circuit is constant, use $V = IR$ with each resistor individually to find the V drop across each. Listed clockwise from top:

   $V_1 = (0.5)(4) = 2$ V

   $V_2 = (0.5)(2) = 1$ V

   $V_3 = (0.5)(6) = 3$ V

Note the voltage drops sum to 6 V.



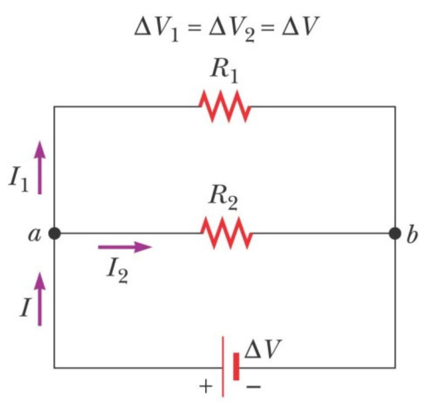
(a)

Now consider two resistors connected in *parallel,*

When the current *I reaches point a in Figure 28.5b, called a junction, it splits into* two parts, with I_1 *going through* R_1 *and* I_2 *going through* R_2 *. A* ***junction is any*** point in a circuit where a current can split

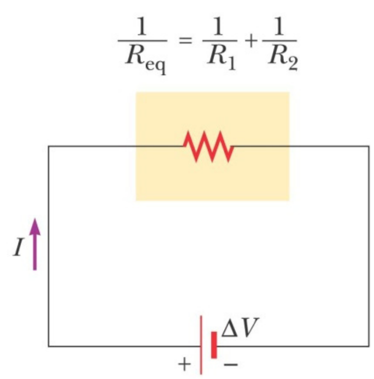
$$I = I_1 + I_2$$

when resistors are connected in parallel, the potential differences across the resistors is the same.



(b)

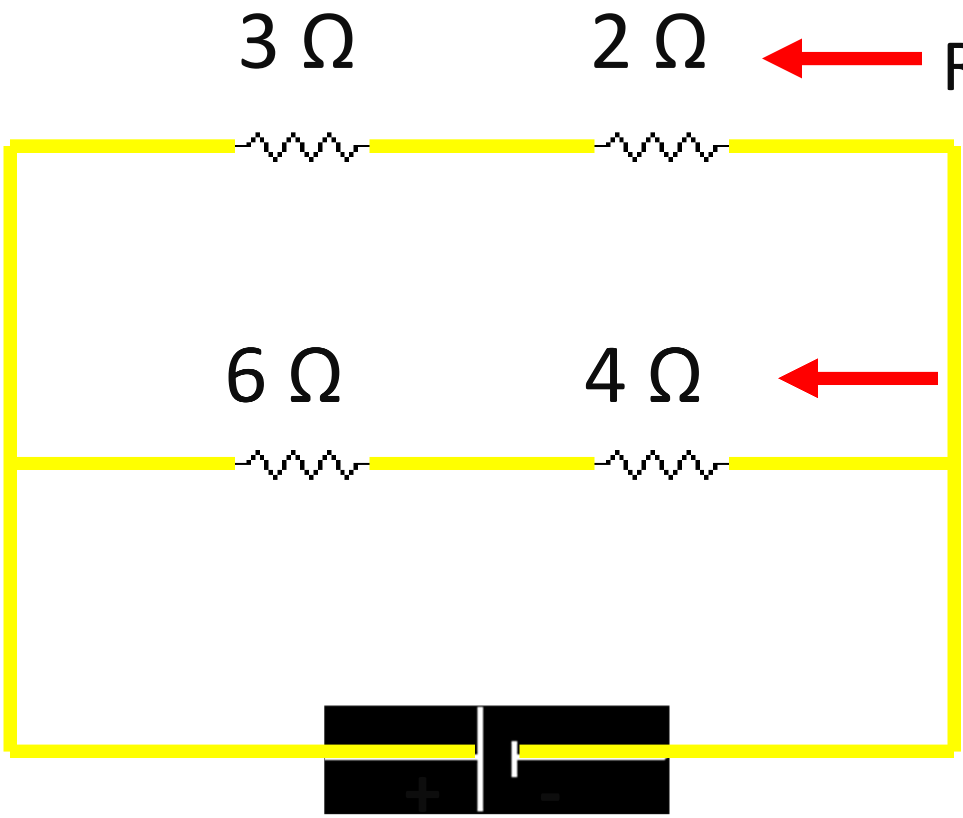
$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$



(c)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Calculate the total resistance in the circuit below



$R_{eq} = 3\ \Omega + 2\ \Omega = 5\ \Omega$

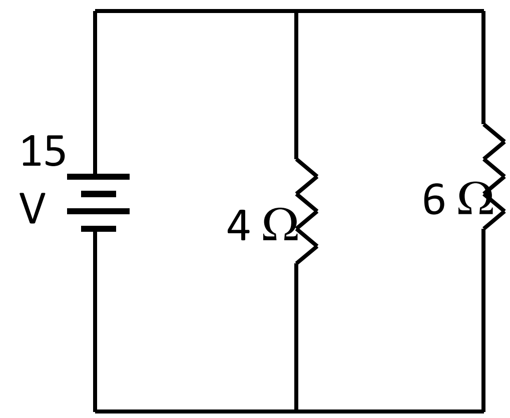
$R_{eq} = 6\ \Omega + 4\ \Omega = 10\ \Omega$

$1/Req = 2/10\ \Omega + 1/10\ \Omega = 3/10\ \Omega$ $R_{eq} = 3.33\ \Omega$

Parallel Example

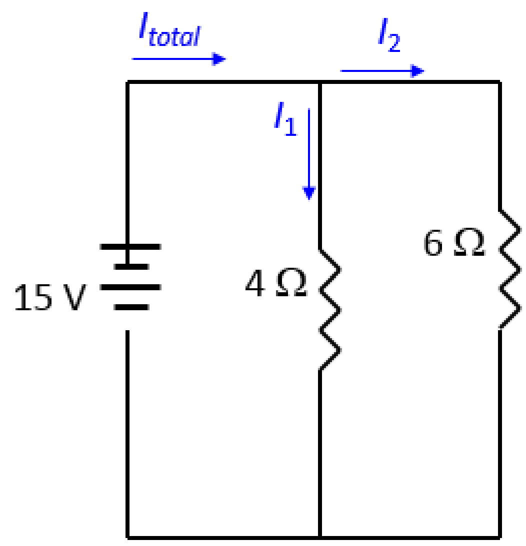
1. Find R_{eq} 2. Find I_{total}

3. Find the current through, and voltage drop across, each resistor.



1. $1/R_{eq} = 1/R_1 + 1/R_2 = 1/4 + 1/6$
   $= 6/24 + 4/24 = 5/12$
   $R_{eq} = 12/5 =$ **2.4 Ω**

2. $I_{total} = V / R_{eq}$
   $= 15 / (12/5)$
   $= 75/12 =$ **6.25 A**



3. The voltage drop across each resistor is the same in parallel.

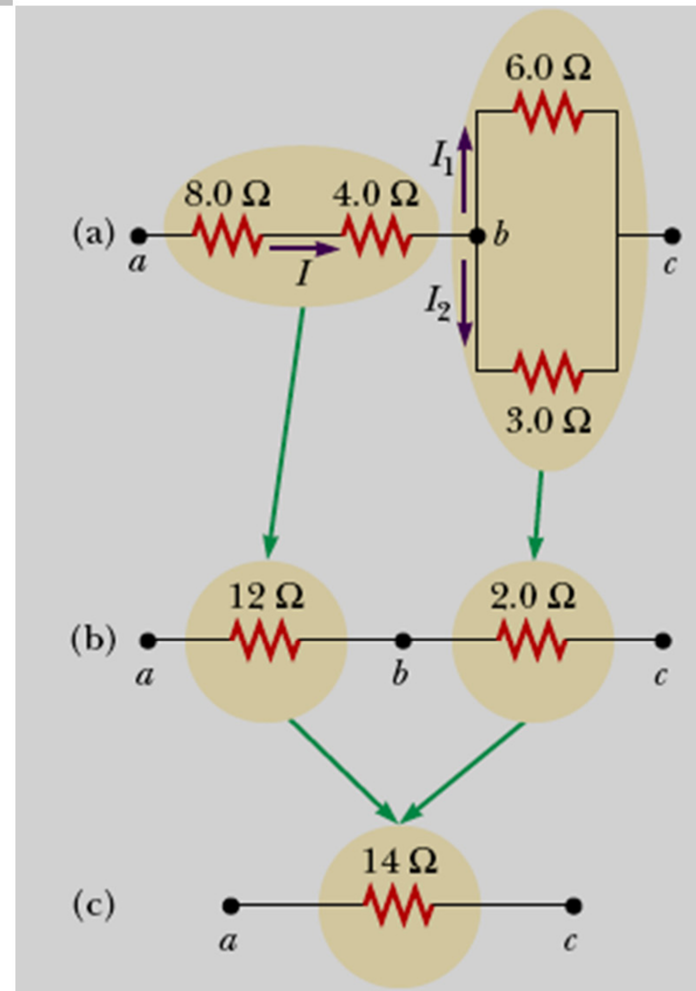
Each drop is 15 V. The current through the 4 Ω resistor is (15 V)/(4 Ω) = 3.75 A. The current through the 6 Ω resistor is (15 V)/(6 Ω) = 2.5 A. Check:

EXAMPLE 28.3 Find the Equivalent Resistance

(a) Find the equivalent resistance between points *a and c.*

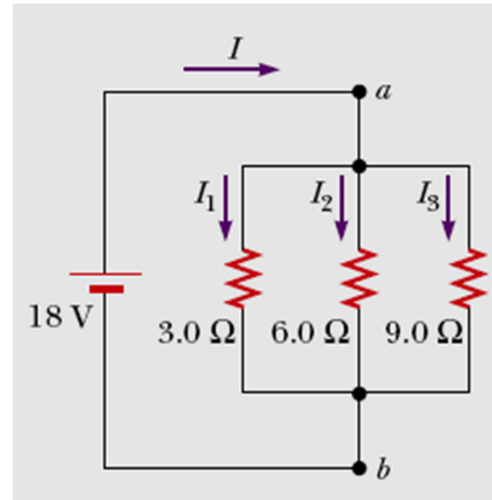
(b) What is the current in each resistor if a potential difference of 42V is maintained between *a and c ?*

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\ \text{V}}{14\ \Omega} = 3.0\ \text{A}$$



EXAMPLE 28.4 Three Resistors in Parallel

Three resistors are connected in parallel as shown in Figure 28.7. A potential difference of 18 V is maintained between points *a and b.*



(a) Find the current in each resistor.

$$I_1 = \frac{\Delta V}{R_1} = \frac{18\text{ V}}{3.0\ \Omega} = 6.0\text{ A}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\text{ V}}{6.0\ \Omega} = 3.0\text{ A}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\text{ V}}{9.0\ \Omega} = 2.0\text{ A}$$

(b) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

$$\mathcal{P}_1 = \frac{\Delta V^2}{R_1} = \frac{(18\text{ V})^2}{3.0\ \Omega} = 110\text{ W}$$

$$\mathcal{P}_2 = \frac{\Delta V^2}{R_2} = \frac{(18\text{ V})^2}{6.0\ \Omega} = 54\text{ W}$$

$$\mathcal{P}_3 = \frac{\Delta V^2}{R_3} = \frac{(18\text{ V})^2}{9.0\ \Omega} = 36\text{ W}$$

(c) Calculate the equivalent resistance of the circuit.

$$\frac{1}{R_{eq}} = \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} + \frac{1}{9.0\ \Omega}$$

$$= \frac{6}{18\ \Omega} + \frac{3}{18\ \Omega} + \frac{2}{18\ \Omega} = \frac{11}{18\ \Omega}$$

$$R_{eq} = \frac{18\ \Omega}{11} = \boxed{1.6\ \Omega}$$

***Exercise :Use R_{eq} to calculate the total power delivered by* the battery**.

Answer :200 W.

28.3 KIRCHHOFF'S RULES

- we can analyze simple circuits using the expression *V IR and the rules for series and parallel combinations of resistors.* Very often, however, it is not possible to reduce a circuit to a single loop.

- The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff's rules:

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out} \quad (28.9)$$

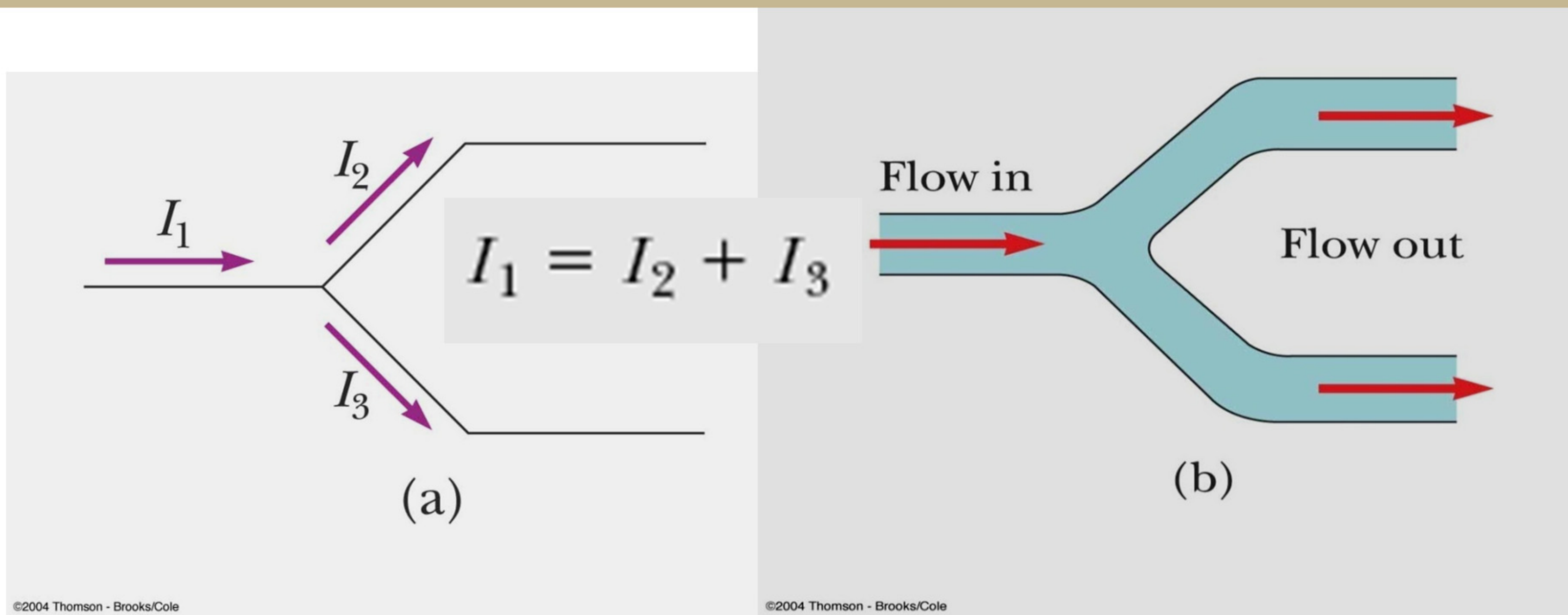
2. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \quad (28.10)$$

Kirchhoff's first rule is a statement of conservation of electric charge.

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out} \qquad \textbf{(28.9)}$$



the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy decreases whenever the charge moves through a potential drop *IR* across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.

Fig 28-14, p.870

Kirchhoff's second rule follows from the law of conservation of energy.

2. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \qquad \textbf{(28.10)}$$

Let us imagine moving a charge around the loop. When the charge returns to the starting point, the charge–circuit system must have the same energy as when the charge started from it.

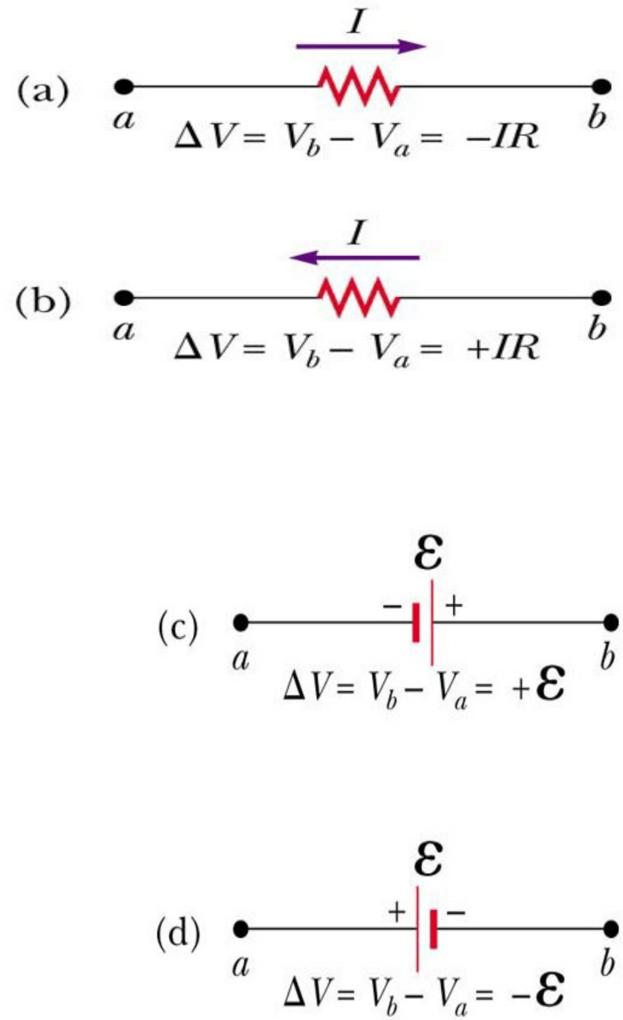
The sum of the increases in energy in some circuit elements must equal the sum of the decreases in energy in other elements.

The potential energy decreases whenever the charge moves through a potential drop *IR* across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases when-ever the charge passes through a battery from the negative terminal to the positive terminal.

Kirchhoff's *rules* for the second rule

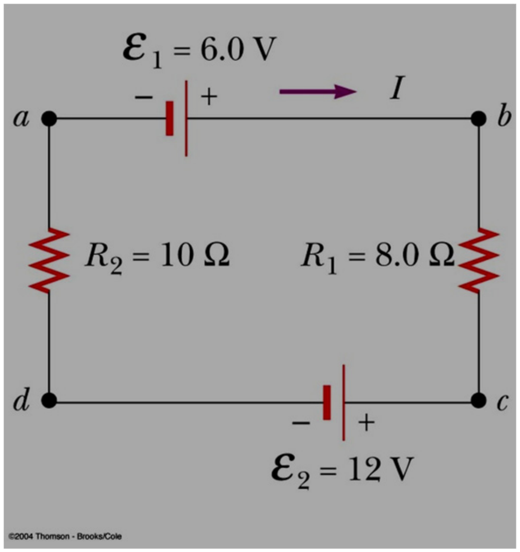
- Traveling around the loop from a to b In a, the resistor is transversed in the direction of the current, the potential across the resistor is –IR
- In b, the resistor is transversed in the direction opposite of the current, the potential across the resistor is +IR

- In c, the source of emf is transversed in the direction of the emf (from – to +), the change in the electric potential is +ε
- In d, the source of emf is transversed in the direction opposite of the emf (from + to -), the change in the electric potential is -ε



EXAMPLE 28.7 A Single-Loop Circuit

(a) Find the current in the circuit. (Neglect the internal resistances of the batteries.)



Traversing the circuit in the clockwise direction, starting at *a, we* see that *a →b represents a potential change of* $+\varepsilon_1$*, b → c* represents a potential change of $-IR_1$*, c → d represents a potential* change of $-\varepsilon_2$, and *d → a represents a potential* change of $-IR_2$*. Applying Kirchhoff's loop rule gives*

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.13, we obtain

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\ \text{V} - 12\ \text{V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\ \text{A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction.

(b) What power is delivered to each resistor? What power is delivered by the 12-V battery?

$$\mathcal{P}_1 = I^2 R_1 = (0.33\ \text{A})^2 (8.0\ \Omega) = \boxed{0.87\ \text{W}}$$

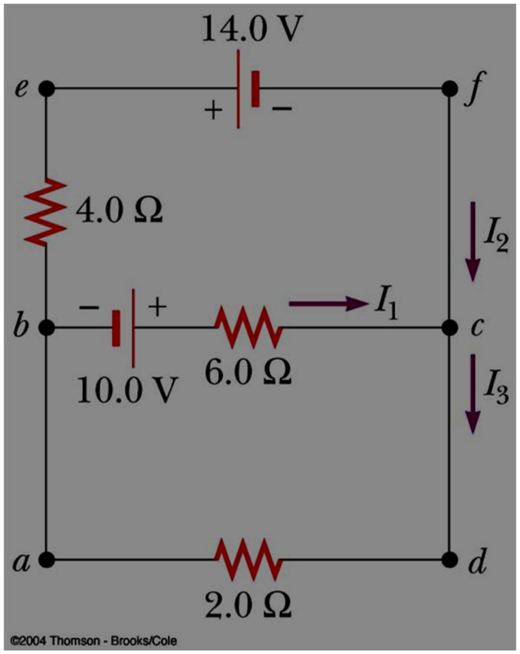
$$\mathcal{P}_2 = I^2 R_2 = (0.33\ \text{A})^2 (10\ \Omega) = \boxed{1.1\ \text{W}}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0\ \text{W}$.

The 12-V battery delivers power $I\varepsilon_2$. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery.

EXAMPLE 28.8 Applying Kirchhoff's Rules

Find the currents I_1, I_2, and I_3 in the circuit



We arbitrarily choose the directions of the currents as labeled in Figure

$$(1) \quad I_1 + I_2 = I_3$$

$$(2) \quad abcda \quad 10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} - (4\ \Omega)I_2 = 0$$

Substituting Equation (1) into Equation (2) gives

$$10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)(I_1 + I_2) = 0$$

$$(4) \quad 10\text{ V} = (8\ \Omega)I_1 + (2\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \quad -12\text{ V} = -(3\ \Omega)I_1 + (2\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22\text{ V} = (11\ \Omega)I_1$$

$$I_1 = 2\text{ A}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2\ \Omega)I_2 = (3\ \Omega)I_1 - 12\text{ V} = (3\ \Omega)(2\text{ A}) - 12\text{ V} = -6\text{ V}$$

$$I_2 = -3\text{ A}$$

$$I_3 = I_1 + I_2 = -1\text{ A}$$

The fact that I_2 *and* I_3 *are both negative indicates only that* the currents are opposite the direction we chose for them. However, the numerical values are correct.

Example 28.8 Solution …

First: $\sum I_{in} = \sum I_{out}$

at ⓒ : $I_1 + I_2 = I_3$ --- (1)

2nd : $\sum V = 0$

Loop (1): $14 + 4I_2 + 10 - 6I_1 = 0$ ---- (2)

Loop (2): $10 - 6I_1 - 2I_3 = 0$ (3)

next: arrange equations:

(1) → $0 + I_1 + I_2 - I_3 = 0$ (4)

(2) → $24 - 6I_1 + 4I_2 + 0 = 0$ (5)

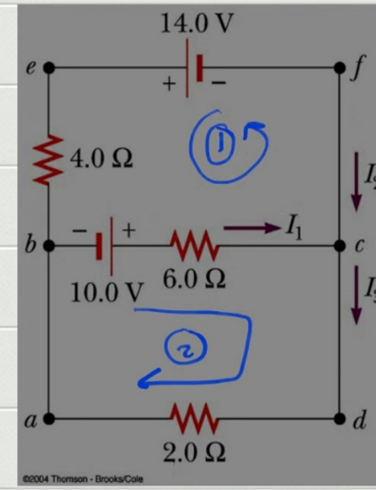
(3) → $10 - 6I_1 + 0 - 2I_3 = 0$ (6)

we have 3 equations and 3 unknowns:

(6) − (4) × 2 : $10 - 8I_1 - 2I_2 + 0 = 0$ (7)

(7) × 2 + (5) : $44 - 22I_1 = 0$ (8)

(8) → $I_1 = \frac{44}{22} = 2A$ ※



Example 28.8 Solution (continued)

put I_1 in (5) $\rightarrow I_2$:

$$24 - 6(2) + 4I_2 = 0$$

$$\rightarrow 24 - 12 + 4I_2 = 0$$

$$\rightarrow 12 = -4I_2$$

$$\therefore I_2 = -\frac{12}{4} = -3\ A\ \#$$

(−) تعني أن الاتجاه المعطى في الرسم معكوس

put I_1 in (6) $\rightarrow I_3$:

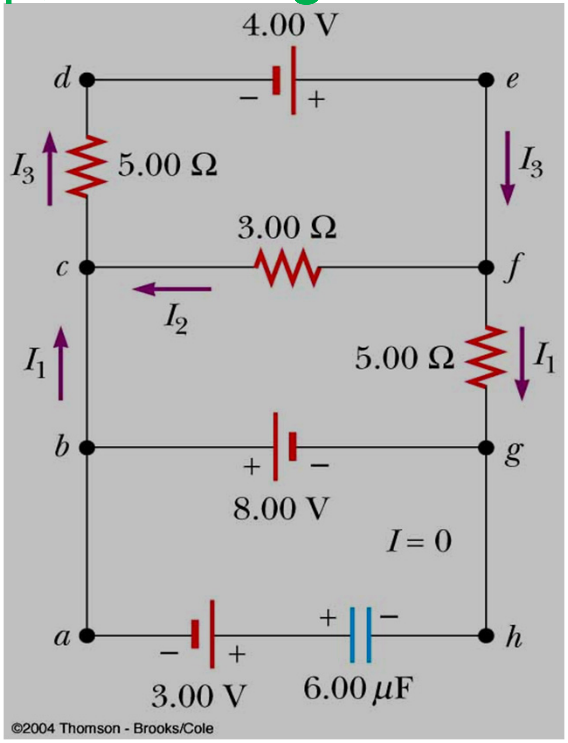
$$10 - 6(2) - 2I_3 = 0$$

$$\rightarrow -2 = 2I_3$$

$$\rightarrow I_3 = -1\ A\ \#$$

(−) تعني أن الاتجاه المبين في الرسم معكوس.

Figure 28.18 (Example 28.10) A multiloop circuit. Kirchhoff's loop rule can be applied to *any* closed loop, including the one containing the capacitor.



(A) Under steady-state conditions, find the unknown currents I_1, I_2, and I_3 in the multiloop circuit shown in Figure 28.18.

(1) $\quad I_1 + I_2 = I_3$

(2) *defcd* $\quad 4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$

(3) *cfgbc* $\quad (3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$

(4) $\quad (8.00\ \Omega)I_2 - (5.00\ \Omega)I_3 + 8.00\text{ V} = 0$

$$I_2 = -\frac{4.00\text{ V}}{11.0\ \Omega} = -0.364\text{ A}$$

Because our value for I_2 is negative, we conclude that the direction of I_2 is from *c* to *f* in the 3.00-Ω resistor. Despite this interpretation of the direction, however, we must continue to use this negative value for I_2 in subsequent calculations because our equations were established with our original choice of direction.

Using $I_2 = -0.364$ A in Equations (3) and (1) gives

$$I_1 = 1.38\text{ A} \qquad I_3 = 1.02\text{ A}$$

Example 28.10 Solution ….

at (2) $I_1 + I_2 = I_3$ --- (1)

Loop (1): $4 - 3I_2 - 5I_3 = 0$ (2)

Loop (2): $8 + 3I_2 - 5I_1 = 0$ (3)

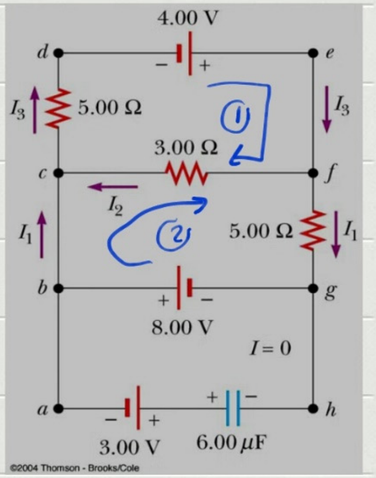
re arrange equations:

(1) → $0 + I_1 + I_2 - I_3 = 0$ (4)

(2) → $4 + 0 - 3I_2 - 5I_3 = 0$ (5)

(3) → $8 - 5I_1 + 3I_2 + 0 = 0$ (6)

} 3 equations in 3 unknowns.



---

(5) − (4)×5: $4 - 5I_1 - 8I_2 = 0$ (7)

(6) − (7): $4 + 11I_2 = 0$

→ $I_2 = -4/11 = -0.364$ A --- (8)

use (8) in (6) → I_1: $8 - 5I_1 - 3\left(\frac{4}{11}\right) = 0$

→ $I_1 = \frac{8 - 3\times\frac{4}{11}}{5} = +1.38$ A ... (9)

(8) in (5) → I_3:

$4 - 3\left(-\frac{4}{11}\right) = 5I_3$

→ $I_3 = \frac{4 + \frac{12}{11}}{5} = +1.02$ A (10)

(B) What is the charge on the capacitor?

Solution We can apply Kirchhoff's loop rule to loop *bghab* (or any other loop that contains the capacitor) to find the potential difference ΔV_{cap} across the capacitor. We use this potential difference in the loop equation without reference to a sign convention because the charge on the capacitor depends only on the magnitude of the potential difference. Moving clockwise around this loop, we obtain

$$-8.00\text{ V} + \Delta V_{cap} - 3.00\text{ V} = 0$$

$$\Delta V_{cap} = 11.0\text{ V}$$

Because $Q = C\,\Delta V_{cap}$ (see Eq. 26.1), the charge on the capacitor is

$$Q = (6.00\ \mu\text{F})(11.0\text{ V}) = \boxed{66.0\ \mu\text{C}}$$

SUMMARY

The **emf** of a battery is equal to the voltage across its terminals when the current is zero. That is, the emf is equivalent to the **open-circuit voltage** of the battery.

The **equivalent resistance** of a set of resistors connected in **series** is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad \textbf{(28.6)}$$

The **equivalent resistance** of a set of resistors connected in **parallel** is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad \textbf{(28.8)}$$

Circuits involving more than one loop are conveniently analyzed with the use of **Kirchhoff's rules:**

1. The sum of the currents entering any junction in an electric circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out} \qquad \textbf{(28.9)}$$

2. The sum of the potential differences across all elements around any circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \qquad \textbf{(28.10)}$$

The first rule is a statement of conservation of charge; the second is equivalent to a statement of conservation of energy.

SUMMARY

The **emf** of a battery is equal to the voltage across its terminals when the current is zero. That is, the emf is equivalent to the **open-circuit voltage** of the battery.

The **equivalent resistance** of a set of resistors connected in **series** is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \quad \textbf{(28.6)}$$

The **equivalent resistance** of a set of resistors connected in **parallel** is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \quad \textbf{(28.8)}$$

Circuits involving more than one loop are conveniently analyzed with the use of **Kirchhoff's rules:**

1. The sum of the currents entering any junction in an electric circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out} \quad \textbf{(28.9)}$$

2. The sum of the potential differences across all elements around any circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \quad \textbf{(28.10)}$$

The first rule is a statement of conservation of charge; the second is equivalent to a statement of conservation of energy.