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قسم الفيزياء والفلك  
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الباب 29  
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المنيف - قسم الفيزياء.

2019

# *Physics 104*

## *Chapter 29*

### **MAGNETIC FIELD**

**29.1 MAGNETIC FIELDS AND FORCES**

**29.2 MAGNETIC FORCE ACTING ON A CURRENT- CARRYING  
CONDUCTOR**

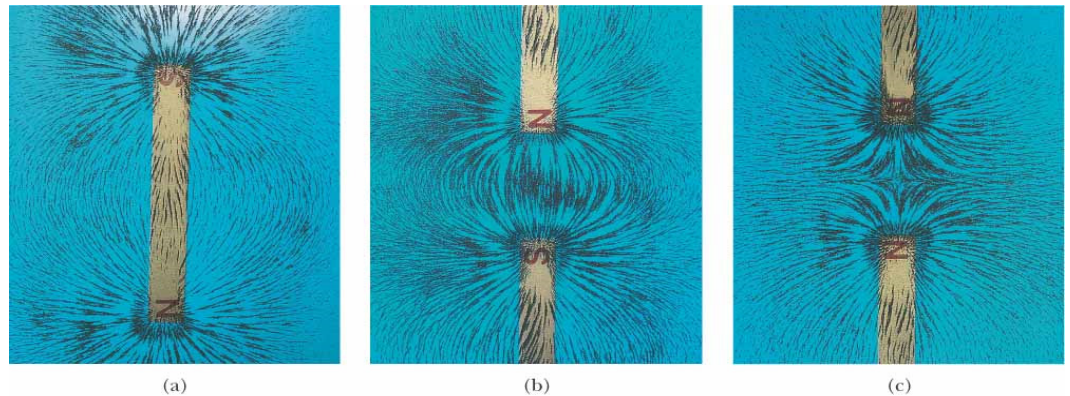
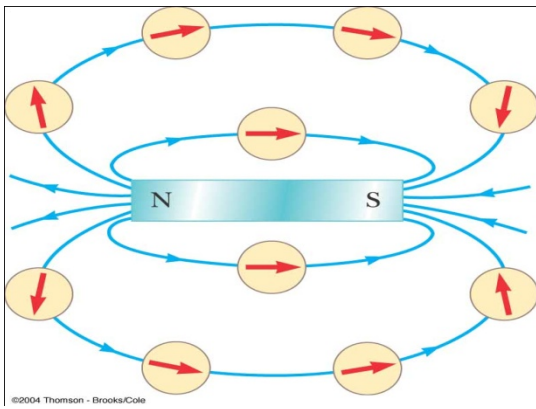
**29.4 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC  
FIELD**

**29.5 APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN  
A MAGNETIC FIELD**

## *Lecture No. 09*

## 29.1 MAGNETIC FIELDS AND FORCES

- The direction of the magnetic field  $\mathbf{B}$  at any location is the direction in which a compass needle points at that location.
- the magnetic field lines outside the magnet point away from north poles and toward south poles.

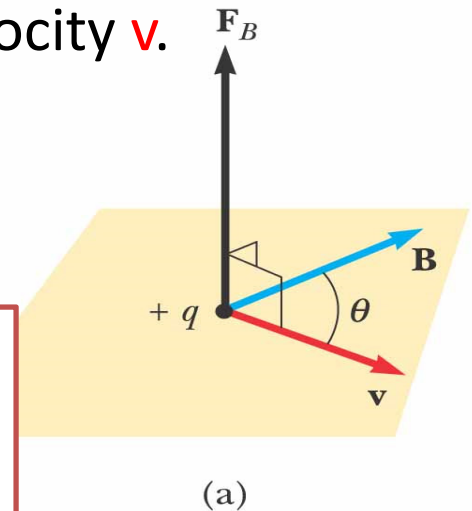


**Active Figure 29.1** Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet

**Active Figure 29.1** Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet

- We can define a magnetic field  $\mathbf{B}$  at some point in space in terms of the magnetic force  $\mathbf{F}_B$  that the field exerts on a test object, for which we use a charged particle moving with a velocity  $\mathbf{v}$ .
- ❖ assuming that no electric ( $\mathbf{E}$ ) or gravitational ( $\mathbf{g}$ ) fields are present at the location of the test object.

Figure 29.3 The direction of the magnetic force  $\mathbf{F}_B$  acting on a charged particle moving with a velocity  $\mathbf{v}$  in the presence of a magnetic field  $\mathbf{B}$ . (a) The magnetic force is perpendicular to both  $\mathbf{v}$  and  $\mathbf{B}$ . *Henry Leap and Jim Lehman*



The formula:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

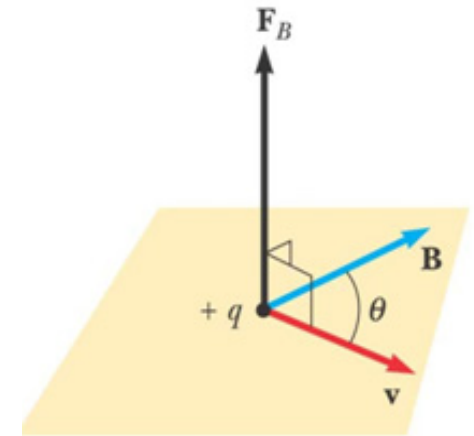
$q$  is the charge

$\mathbf{v}$  is velocity of the charge

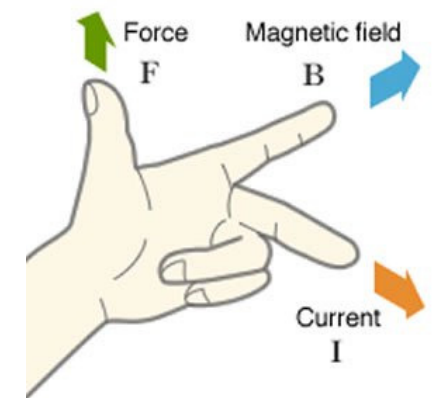
$\mathbf{F}_B$  is the magnetic force

$\mathbf{B}$  is the magnetic field

- The magnitude  $F_B$  of the magnetic force exerted on the particle is proportional to the charge  $q$  and to the velocity  $v$  of the particle
- The magnitude and direction of  $F_B$  depend on the velocity of the particle  $v$  and on the magnitude and direction of the magnetic field  $B$ .
- When a charged particle moves parallel to the magnetic field vector (i.e.,  $\theta = 0$ ), the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle with the magnetic field, the magnetic force acts in a direction perpendicular to both  $v$  and  $B$ ;  $F_B$  is perpendicular to the plane formed by  $v$  and  $B$



(a)



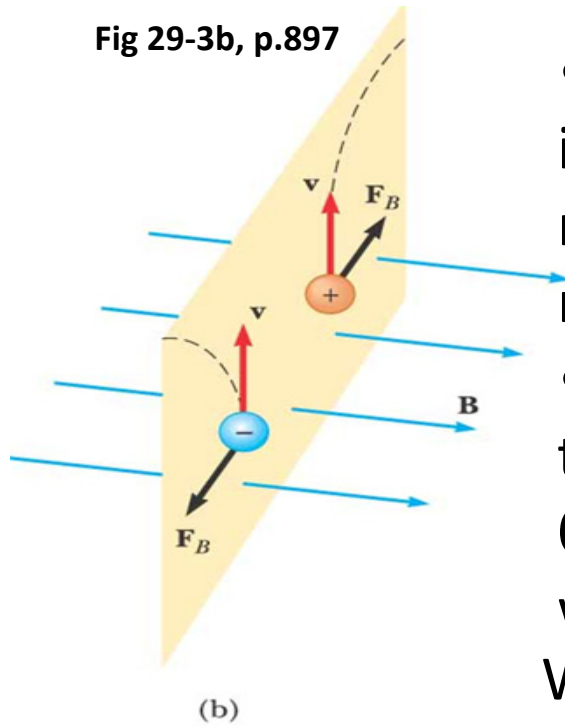
The magnitude:

For a positive charge

$$F_B = qvB \sin \theta$$

or  $F_B = qvB$  When the velocity and the field are perpendicular to each other.

Fig 29-3b, p.897



- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.
- The magnitude of the magnetic force exerted on the moving particle is proportional to  $\sin\theta$ , where  $\theta$  is the angle the particle's velocity vector makes with the direction of  $\mathbf{B}$ .

We can summarize these observations by writing the magnetic force in the form

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

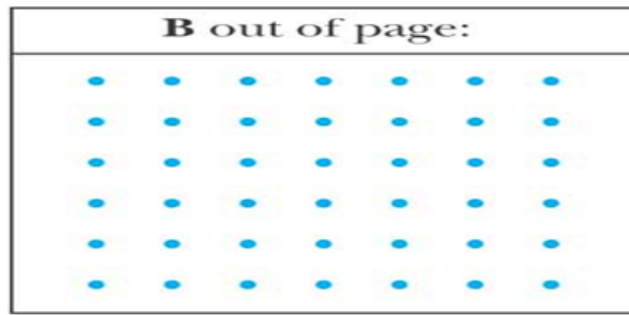
- The **SI unit** of magnetic field is the **tesla (T)**

$$T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

– **Wb (Weber) is the unit for magnetic field flux.**

- A non-SI commonly used unit is a gauss (G)

–  **$1 T = 10^4 G$**



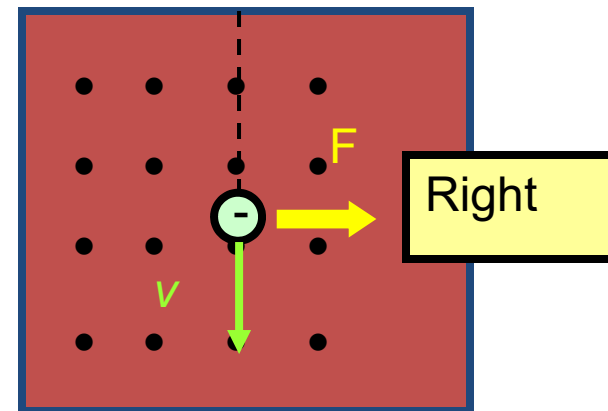
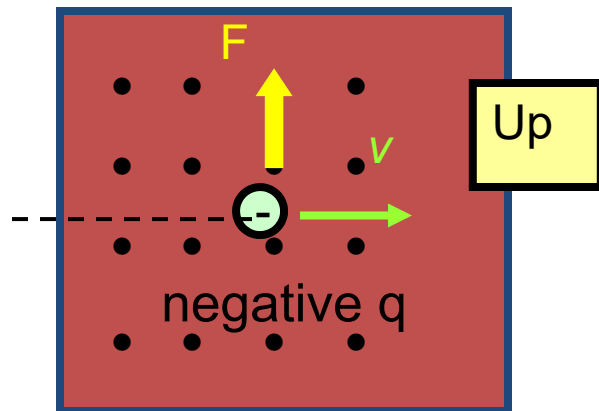
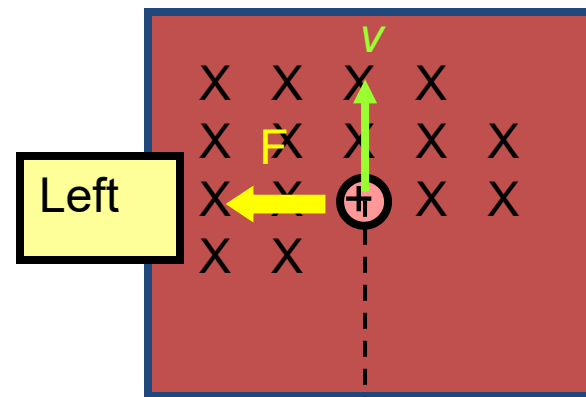
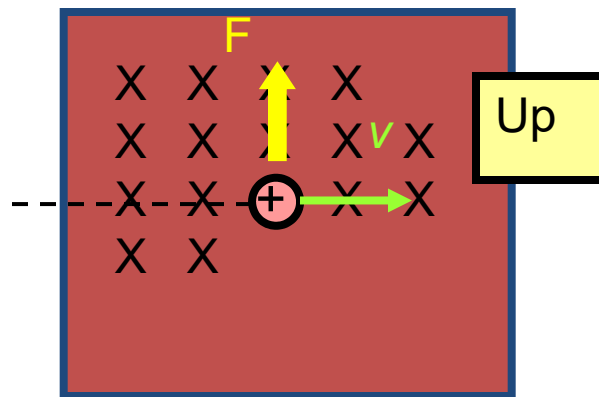
(a)



(b)

**Figure 29.6** (a) Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward. (b) Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

What is the direction of the force  $F$  on the charge in each of the examples described below?





# Differences Between Electric and Magnetic Fields

- **Direction of force**

- The electric force acts *parallel* or *antiparallel* to the electric field ( $F_E = qE$ )
- The magnetic force acts *perpendicular* to the magnetic field

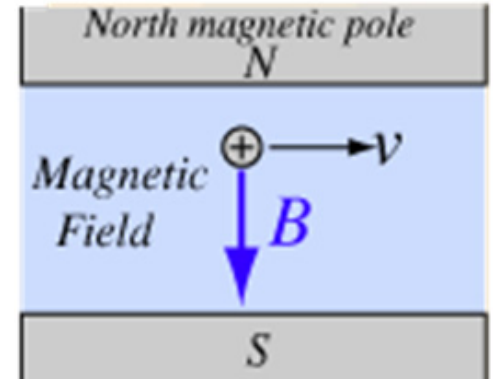
- $F_B = |q| v B \sin \theta$

- $\theta$  is the angle between the velocity and the field

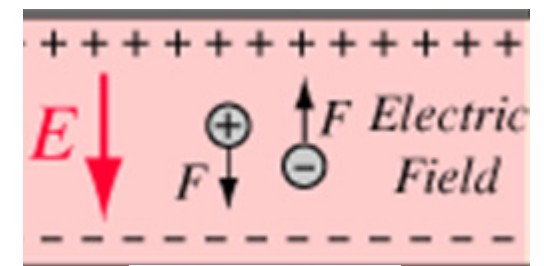
- The force is zero when the velocity and the field are parallel or antiparallel

$\theta = 0$                       or                       $\theta = 180^\circ$

- The force is a maximum when the velocity and the field are perpendicular  
 $\theta = 90^\circ$



*Magnetic force of magnitude  $qvB\sin\theta$  perpendicular to both  $v$  and  $B$ , away from viewer.*



*Electric force  $qE$*

## Motion

The electric force acts on a charged particle regardless of its velocity

The magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity

- Work

- The electric force does work in displacing a charged particle

- The magnetic force associated with a steady magnetic field does no work when a particle is displaced

- This is because the force is perpendicular to the displacement

- The kinetic energy of a charged particle moving through a constant magnetic field cannot be altered by the magnetic field alone
- When a charged particle moves with a velocity through a magnetic field, the field can alter the *direction* of the velocity,  $\hat{\mathbf{v}}$  but not the speed or the kinetic energy

# Magnetic Field and Electric Field

<b>Deflection of a charged particle in a magnetic field</b>	<b>Deflection of a charged particle in an electric field</b>
The magnetic field can exert <b>a magnetic force only on a moving charged particle moving perpendicular to the field</b> . Stationary charged particles and charges moving parallel to the field experience no force.	The electric field <b>always exerts an electric force on a charged particle</b> , whether it is stationary or moving.
The magnetic force is <b>perpendicular to the magnetic field and the direction of motion</b> of the charged particle. Direction of force is deduced by <b>Fleming's Left Hand Rule</b> .	The electric force <b>acts in the direction of the electric field</b> . Deduced from the law of electrostatics (i.e. Like charges repel; unlike charges attract.)
Magnetic force is <b>dependent on the speed and direction of motion</b> of the charged particle: $F_B = Bqv \sin \theta$	Electric force is <b>independent of the speed and direction of motion</b> of the charged particle. ( $F_E = qE$ )
<b>Uniform circular motion</b> is obtained when a charged particle enters a magnetic field perpendicularly.	<b>Parabolic motion</b> is obtained when a charged particle enters an electric field perpendicularly.

## Example.

A 2-nC charge is projected with velocity  $5 \times 10^4$  m/s at an angle of  $30^\circ$  with a 3 mT magnetic field as shown. What are the magnitude and direction of the resulting force?

$$q = 2 \times 10^{-9} \text{ C} \quad v = 5 \times 10^4 \text{ m/s} \quad B = 3 \times 10^{-3} \text{ T} \quad \theta = 30^\circ$$

$$F = qvB \sin \theta = (2 \times 10^{-9} \text{ C})(5 \times 10^4 \text{ m/s})(3 \times 10^{-3} \text{ T}) \sin 30^\circ$$

Resultant Magnetic Force:  $F = 1.50 \times 10^{-7}$  N, upward

## Example

A proton moves with a speed of  $1.0 \times 10^5$  m/s through the Earth's magnetic field, which has a value of  $55 \mu\text{T}$  at a particular location. When the proton moves **eastward**, the magnetic force is a maximum, and when it moves **northward**, no magnetic force acts upon it. What is the magnitude and direction of the magnetic force acting on the proton?

$$\theta = 90^\circ \quad \sin 90^\circ = 1$$

$$F_B = qvB$$

$$F_B = (1.6 \times 10^{-19})(1.0 \times 10^5)(55 \times 10^{-6})$$

$$F_B = 8.8 \times 10^{-19} \text{ N}$$

The direction cannot be determined precisely by the given information. Since no force acts on the proton when it moves northward (meaning the angle is equal to ZERO), we can infer that the magnetic field must either go northward or southward.

## Example

A charge  $q_1 = 25.0 \mu\text{C}$  moves with a speed of  $4.5 \times 10^3 \text{ m/s}$  perpendicularly to a uniform magnetic field. The charge experiences a magnetic force of  $7.31 \times 10^{-3} \text{ N}$ . A second charge  $q_2 = 5.00 \mu\text{C}$  travels at an angle of  $40.0^\circ$  with respect to the same magnetic field and experiences a  $1.90 \times 10^{-3} \text{ N}$  force. Determine

- (i) The magnitude of the **magnetic field** and
- (ii) The **speed of  $q_2$** .

$$q_1 = 25.0 \mu\text{C}, v_1 = 4.5 \times 10^3 \text{ m/s}, \Theta_1 = 90.0^\circ$$
$$F_1 = 7.31 \times 10^{-3} \text{ N}, q_2 = 5.00 \mu\text{C}, \Theta_2 = 40.0^\circ,$$
$$F_2 = 1.90 \times 10^{-3} \text{ N force.}$$

$$(i) \quad B = B_1 = \frac{F_1}{q_1 v_1} = 6.50 \times 10^{-2} \text{ T} = B_2$$

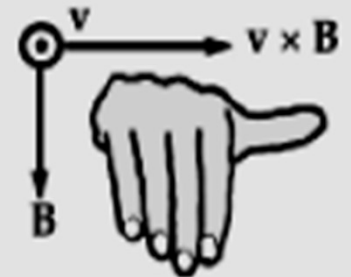
$$(ii) \quad v_2 = 9.10 \times 10^3 \text{ m/s}$$

## Example

5- A proton moves in a direction perpendicular to a uniform magnetic field  $B$  at  $1.0 \times 10^7$  m/s and experiences an acceleration of  $2.0 \times 10^{13}$  m/s<sup>2</sup> in the **+ x direction** when its velocity is in the **+z direction**. Determine the magnitude and direction of the field.

$$F = ma = (1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^{13} \text{ m/s}^2) = 3.34 \times 10^{-14} \text{ N} = qvB \sin 90^\circ$$

$$B = \frac{F}{qv} = \frac{3.34 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^7 \text{ m/s})} = \boxed{2.09 \times 10^{-2} \text{ T}}$$



The right-hand rule shows that  $B$  must be in the  $-y$  direction to yield a force in the  $+x$  direction when  $v$  is in the  $z$  direction.



## Example

$$F = qvB \sin \theta$$

1. Calculate the **magnitude of the force** on a proton travelling  $3.1 \times 10^7 \text{ m s}^{-1}$  in the uniform magnetic flux density of  $1.6 \text{ Wb m}^{-2}$ , if :

(i) the velocity of the proton is **perpendicular** to the magnetic field.

$$F = 7.9 \times 10^{-12} \text{ N}$$

(ii) the velocity of the proton makes an angle  **$60^\circ$**  with the magnetic field.

(charge of the proton =  $+1.60 \times 10^{-19} \text{ C}$ )

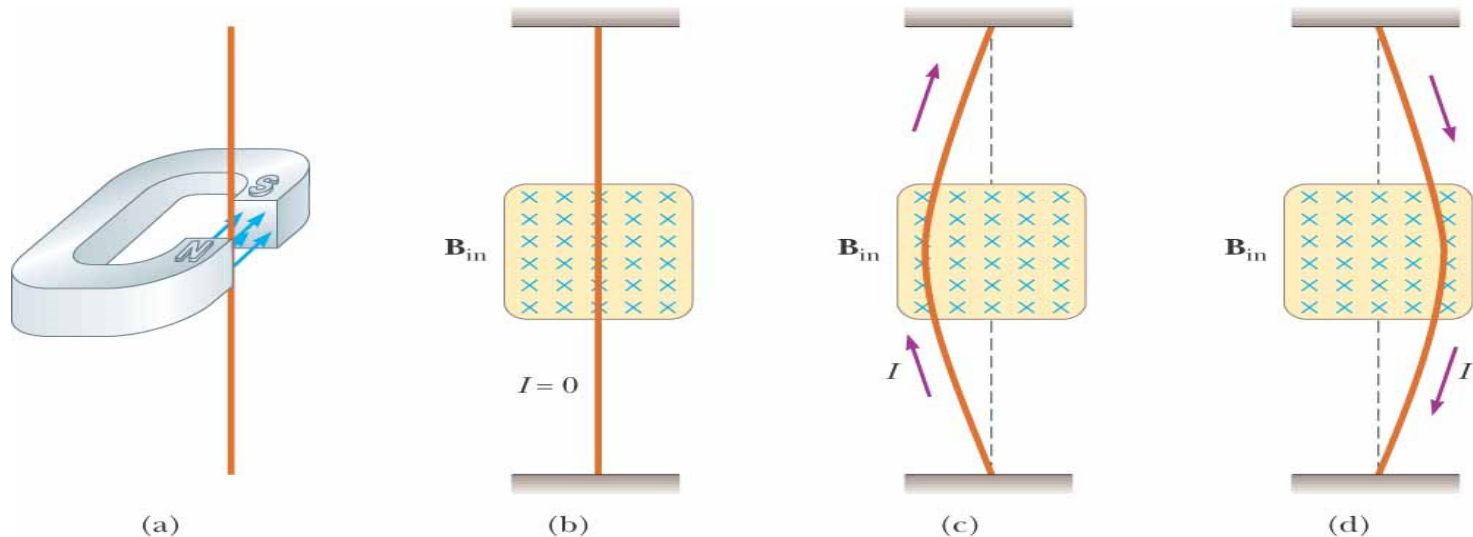
$$F = 6.9 \times 10^{-12} \text{ N}$$

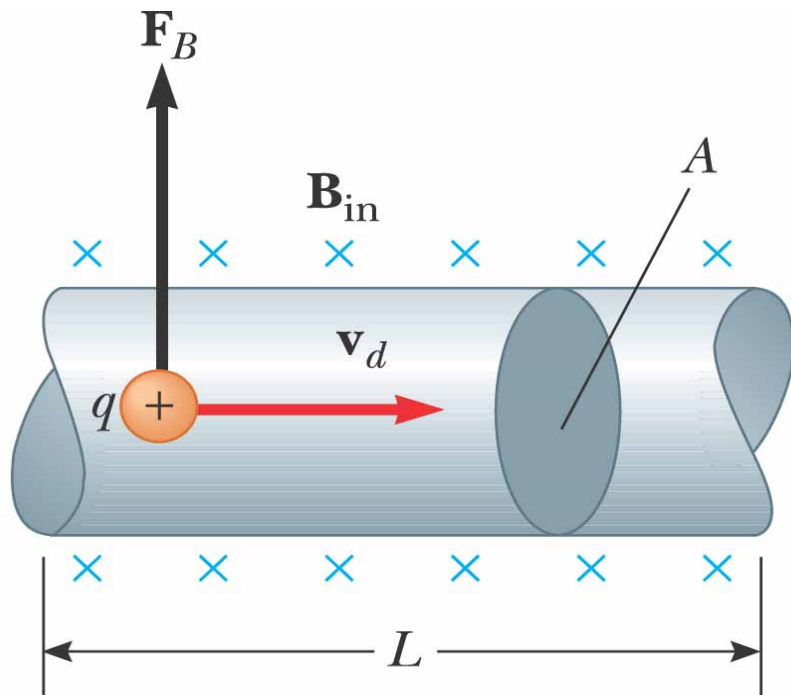
## 29.2 MAGNETIC FORCE ACTING ON A CURRENT-CARRYING CONDUCTOR

If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field, it should not surprise you that a current-carrying wire also experiences a force when placed in a magnetic field.

The current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current.

The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.





**Figure 29.7** A segment of a current-carrying wire located in a magnetic field  $\mathbf{B}$ . The magnetic force exerted on each charge making up the current is  $q\mathbf{v}_d \times \mathbf{B}$ , and the net force on the segment of length  $L$  is  $I\mathbf{L} \times \mathbf{B}$ .

$$\mathbf{F}_B = (q\mathbf{v}_d \times \mathbf{B}) nAL$$

the current in the wire is  $I = nqv_dA$ . Therefore,

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B}$$

Direction: Left-hand rule

$F=0$  when  $\theta = 0^\circ$

$F_{max}=IB$  when  $\theta = 90^\circ$

- $F$  is maximum when  $\Theta=90^\circ$

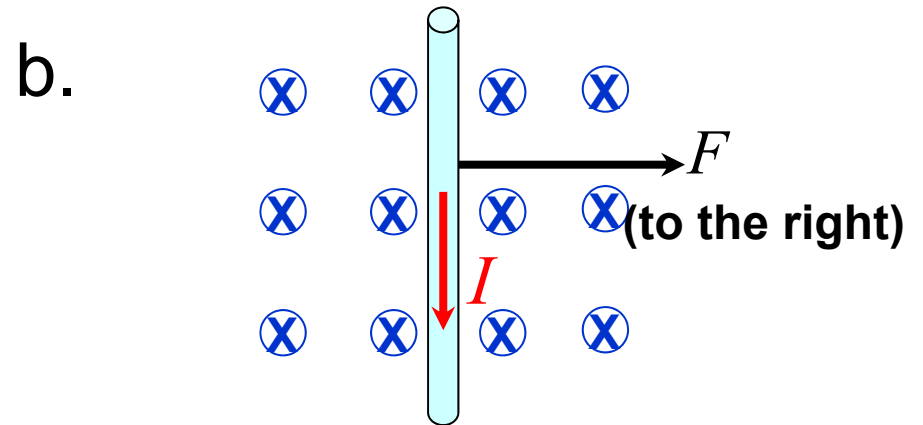
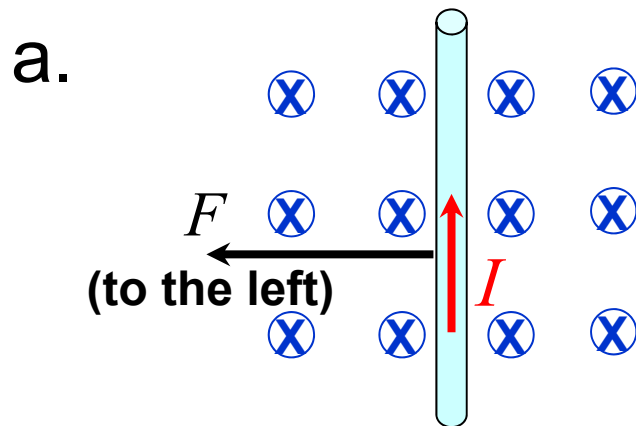
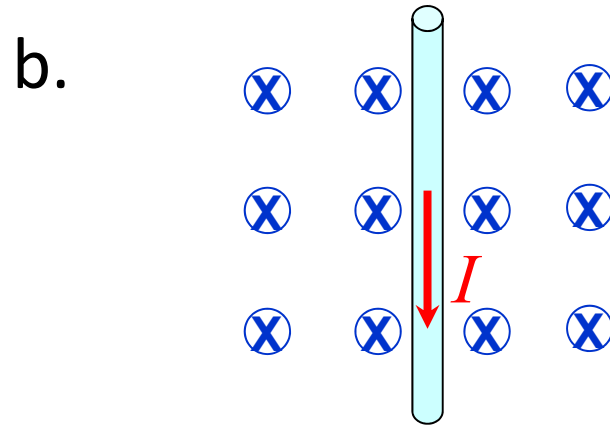
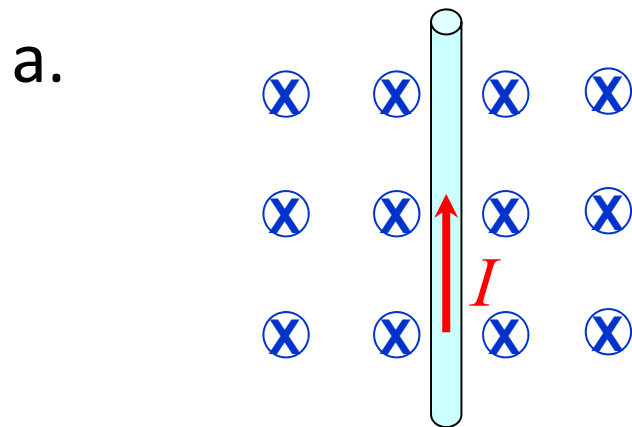
## Example:

A wire carries a current of 22 A from east to west. Assume that at this location the magnetic field of the earth is horizontal and directed from south to north, and has a magnitude of  $0.50 \times 10^{-4}$  T. Find the magnetic force on a 36-m length of wire. What happens if the direction of the current is reversed?

$$\begin{aligned} F_{\max} &= BIl \\ &= (0.50 \times 10^{-4} T)(22 A)(36 m) \\ &= 4.0 \times 10^{-2} N \end{aligned}$$

## Example

Determine the **direction of the magnetic force**, exerted on a conductor carrying current,  $I$  in each problem below.



## Example

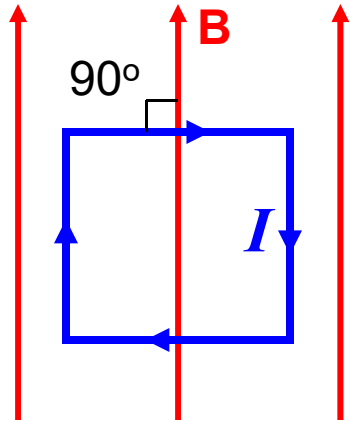
A wire of length 0.655 m carries a current of 21.0 A. In the presence of a 0.470 T magnetic field, the wire experiences a force of 5.46 N . What is the **angle** (less than 90°) between the wire and the magnetic field?

$$F = BIL \sin \theta$$

$$\theta = \sin^{-1} \frac{F}{BIL} = \sin^{-1} \left( \frac{5.46}{0.47 \times 21 \times 0.655} \right) = 57.63^\circ$$

## Example

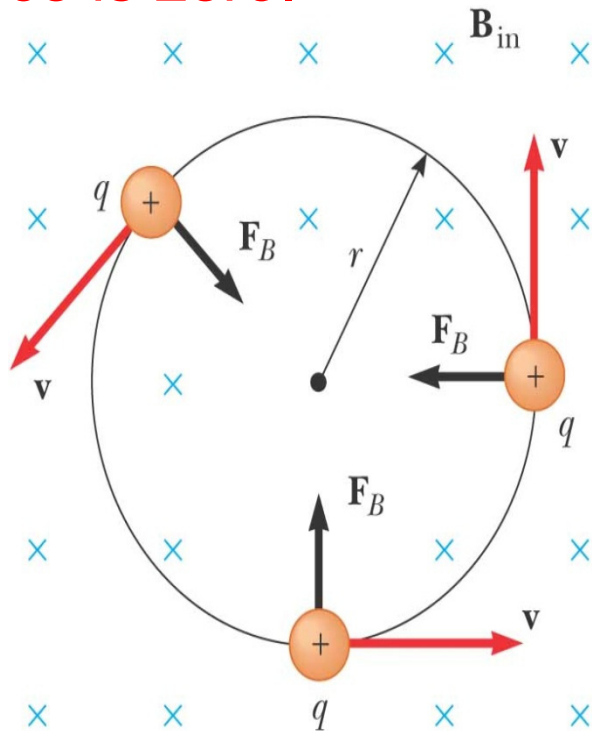
1. A square coil of wire containing a single turn is placed in a uniform 0.25 T magnetic field. Each side has a length of 0.32 m, and the current in the coil is 12 A. Determine the magnitude of the **magnetic force** on each of the four sides.



**0.96 N (top and bottom sides)**  
**0 N (left and right sides)**

## 29.4 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

The magnetic force acting on a charged particle moving in a magnetic field is perpendicular to the velocity of the particle and that consequently **the work done on the particle by the magnetic force is zero.** ( $W = F \cdot S = F s \cos \theta$ )



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Because  $F_B$  always points toward the center of the circle, **it changes only the direction of  $v$  and not its magnitude.**

the rotation is counterclockwise for a **positive charge**. If  $q$  were **negative**, the rotation would be clockwise.



$$F_B = F_c$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

$r$  is proportional mass of the particle and inversely proportional to the magnetic field and the charge

The angular speed of the particle

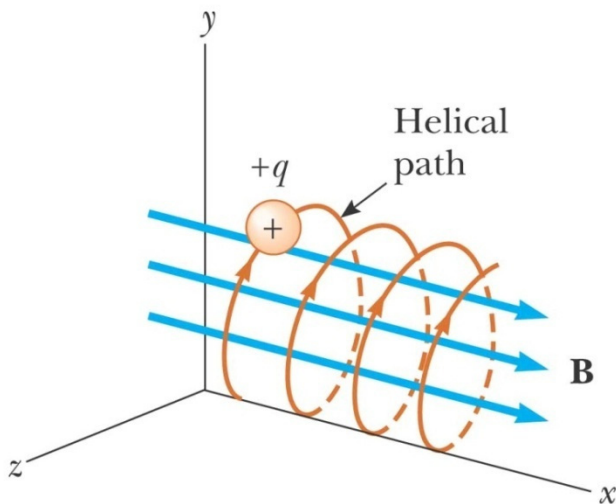
$$\omega = \frac{v}{r} = \frac{qB}{m}$$

The period of the motion  $T$  (the time that the particle takes to complete one revolution) is equal to the circumference of the circle divided by the linear speed of the particle:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

$$T = \frac{1}{f}$$

$$f = \frac{Bq}{2\pi m}$$



**Figure 29.19** A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.

**EXAMPLE 29.6** A Proton Moving Perpendicular to a Uniform Magnetic Field

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton.

$$\begin{aligned}v &= \frac{qBr}{m_p} = \frac{(1.60 \times 10^{-19} \text{ C})(0.350 \text{ T})(14.0 \times 10^{-2} \text{ m})}{1.67 \times 10^{-27} \text{ kg}} \\ &= 4.69 \times 10^6 \text{ m/s}\end{aligned}$$

Find the period of the circular motion of the proton.

$$\begin{aligned}T &= \frac{2\pi m_p}{qB} = \frac{2\pi (1.67 \times 10^{-27} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.350 \text{ T})} \\ &= 1.87 \times 10^{-7} \text{ s}\end{aligned}$$

**Exercise** If an electron moves in a direction perpendicular to the same magnetic field with this same linear speed, what is the radius of its circular orbit?

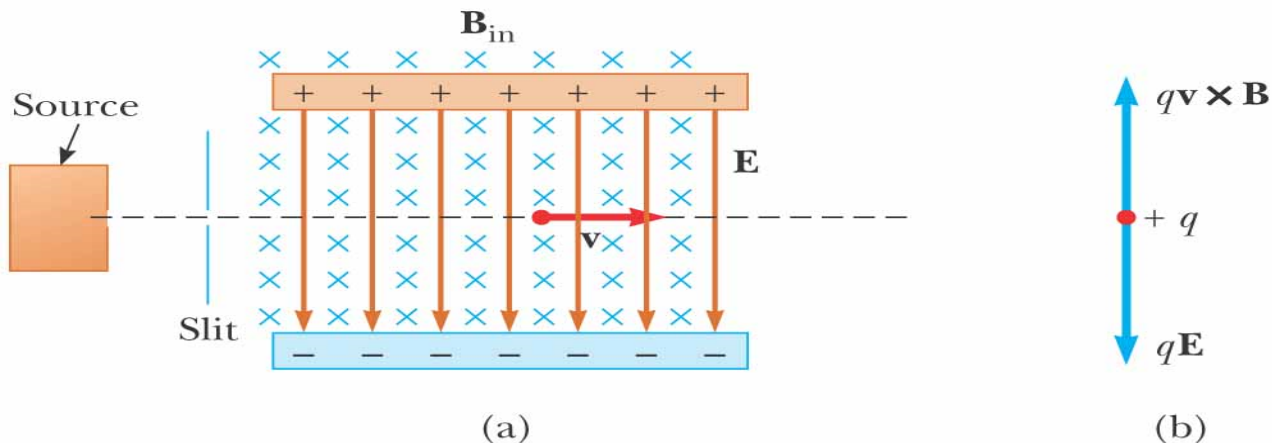
**Answer**  $7.6 \times 10^{-5} \text{ m}$ .

## 29.5 APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

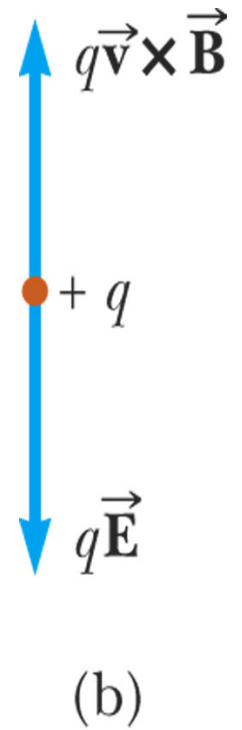
### Velocity Selector:

- In many applications, the charged particle will move in the presence of both magnetic and electric fields
- In that case, the total force is the sum of the forces due to the individual fields
- In general:
  - This force is called the Lorentz force
  - It is the vector sum of the electric force and the magnetic force

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$



- Used when all the particles need to move with the same velocity
- A uniform electric field is perpendicular to a uniform magnetic field
- When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line
- This occurs for velocities of value  $\mathbf{v} = \mathbf{E} / \mathbf{B}$



# Example

Suppose a cyclotron is operated at frequency  $f=12$  MHz and has a dee radius of  $R=53$ cm. What is the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron ( $m=3.34 \cdot 10^{-27}$ kg)?

$$f = \frac{qB}{2\pi m}$$

$$B = \frac{2\pi m f}{q} = \frac{(2\pi)(3.34 \cdot 10^{-27} \text{kg})(12 \cdot 10^6 \text{s}^{-1})}{1.6 \cdot 10^{-19} \text{C}} = 1.57 \text{T}$$

# SUMMARY

The magnetic force that acts on a charge  $q$  moving with a velocity  $\mathbf{v}$  in a magnetic field  $\mathbf{B}$  is

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \quad (29.1)$$

The direction of this magnetic force is perpendicular both to the velocity of the particle and to the magnetic field. The magnitude of this force is

$$F_B = |q|vB \sin \theta \quad (29.2)$$

where  $\theta$  is the smaller angle between  $\mathbf{v}$  and  $\mathbf{B}$ . The SI unit of  $\mathbf{B}$  is the **tesla** (T), where  $1 \text{ T} = 1 \text{ N/A} \cdot \text{m}$ .

When a charged particle moves in a magnetic field, the work done by the magnetic force on the particle is zero because the displacement is always perpendicular to the direction of the force. The magnetic field can alter the direction of the particle's velocity vector, but it cannot change its speed.

If a straight conductor of length  $L$  carries a current  $I$ , the force exerted on that conductor when it is placed in a uniform magnetic field  $\mathbf{B}$  is

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B} \quad (29.3)$$

If a charged particle moves in a uniform magnetic field so that its initial velocity is perpendicular to the field, the particle moves in a circle, the plane of which is perpendicular to the magnetic field. The radius of the circular path is

$$r = \frac{mv}{qB} \quad (29.13)$$

where  $m$  is the mass of the particle and  $q$  is its charge. The angular speed of the charged particle is

$$\omega = \frac{qB}{m} \quad (29.14)$$