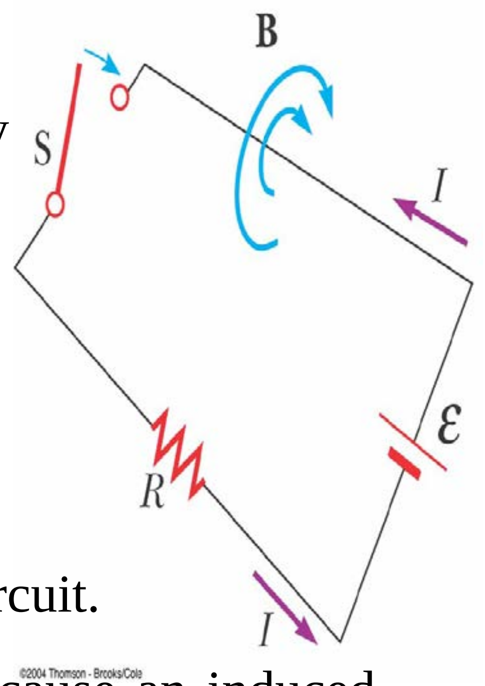
CHAPTER 32
INDUCTANCE

32-1 Self-Inductance
32-3 Energy of a Magnetic Field

32-1 SELF-INDUCTANCE

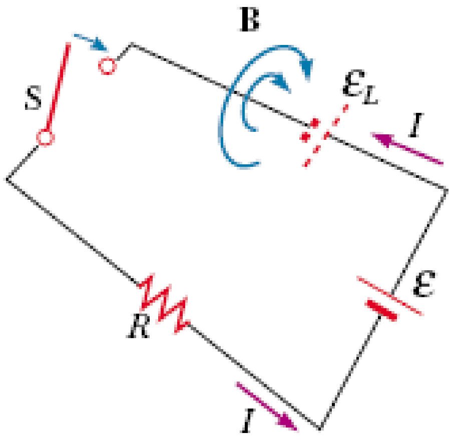


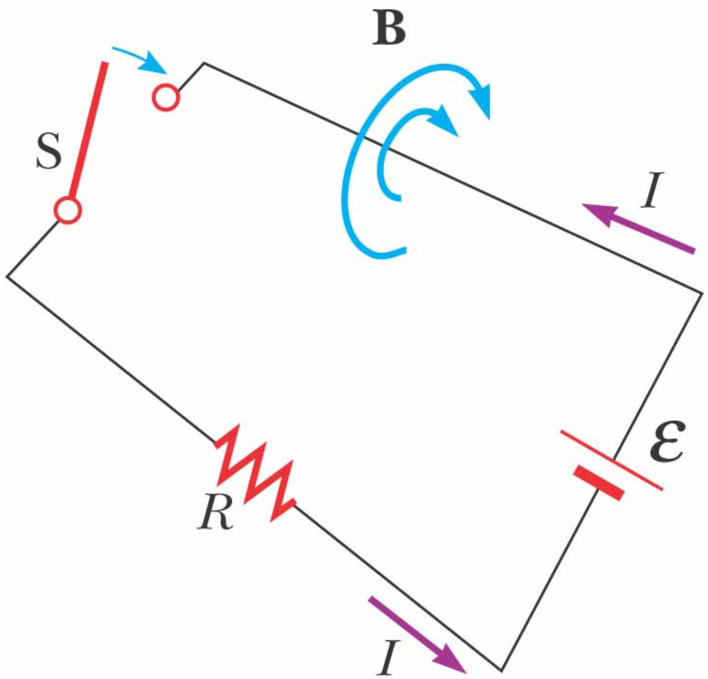
- When the switch is closed, the current does not immediately reach its maximum value
- Faraday’s law can be used to describe the effect

- As the source current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit.
- The direction of the induced emf is such that it would cause an induced current in the loop (if a current were not already flowing in the loop), which would establish a magnetic field that would oppose the change in the source magnetic field.

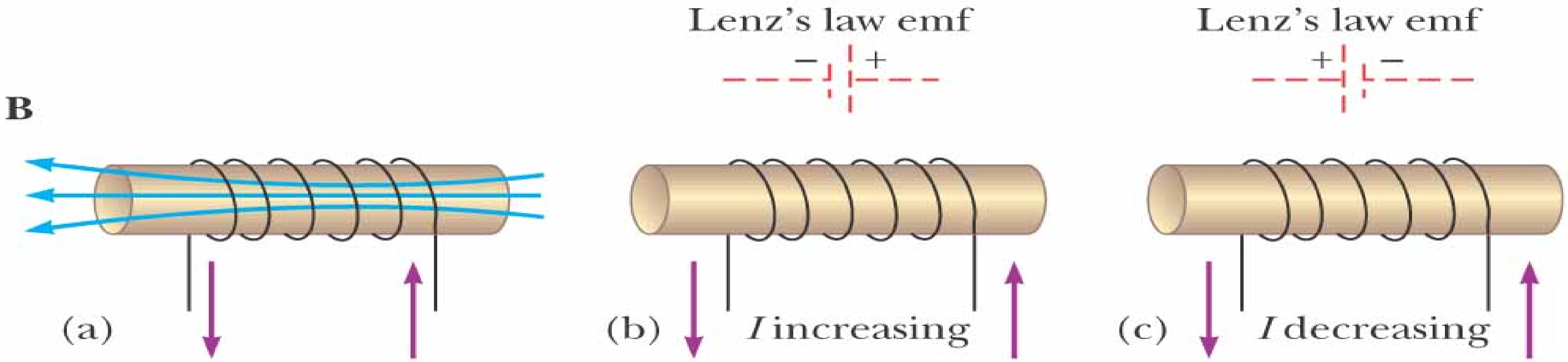
After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop.

As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop (back emf).



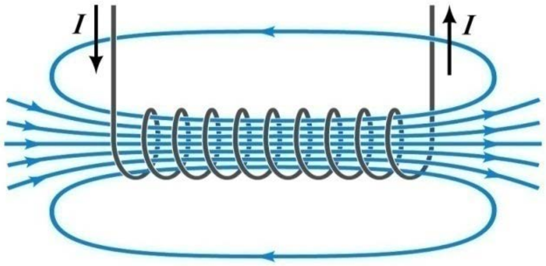


The emf set up in this case is called a self-induced emf.



- A current in the coil produces a magnetic field directed toward the left (a)
- If the current increases, the increasing flux creates an induced emf of the polarity shown (b)
- The polarity of the induced emf reverses if the current decreases (c)

$$B = \mu_o n I$$



$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d\left(\mu_o n I A\right)}{dt} = -N\mu_o n A\frac{dI}{dt} = -\frac{NBA}{I}\frac{dI}{dt}$$

$$\varepsilon = -\frac{N\Phi_B}{I}\frac{dI}{dt} = -L\frac{dI}{dt}$$

Define: Self Inductance $$L = \frac{N\Phi_B}{I}$$

Inductance Units

$$L = \left[\frac{V}{A/s}\right] = [\Omega - s] = [\text{Henry}] = [H]$$

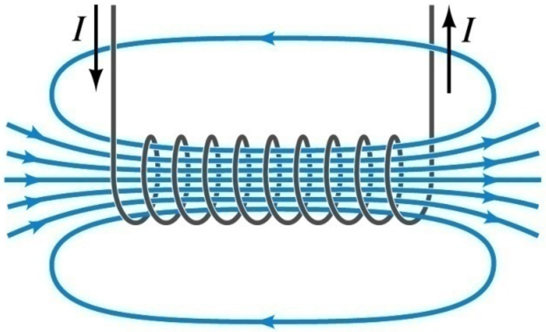
Inductance of a Solenoid

- The magnetic flux through each turn is

$$\Phi_B = BA = \left(\mu_o \frac{N}{l} I \right) A$$

- Therefore, the inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_o N^2 A}{l}$$



L = inductance of the solenoid
N = # of turns in solenoid
l = length of solenoid
A = cross sectional area of solenoid
n = # of turns per unit length

- This shows that L depends on the <u>geometry</u> of the object
- The inductance is a measure of the **opposition to a change in current**

Example 32-2 calculating Inductance and emf

(a) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

$N = 300$
$\ell = 0.25$
$A = 04 \times 10^{-4}$

Solution Using Equation 32.4, we obtain

$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$= (4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}) \frac{(300)^2(4.00 \times 10^{-4}\ \mathrm{m}^2)}{25.0 \times 10^{-2}\ \mathrm{m}}$$

$$= 1.81 \times 10^{-4}\ \mathrm{T \cdot m^2/A} = \boxed{0.181\ \mathrm{mH}}$$

(b) Calculate the self-induced emf in the solenoid if the current through it is decreasing at the rate of 50.0 A/s.

Solution Using Equation 32.1 and given that $dI/dt = -50.0$ A/s, we obtain

$$\mathcal{E}_L = -L\frac{dI}{dt} = -(1.81 \times 10^{-4}\ \mathrm{H})(-50.0\ \mathrm{A/s})$$

$$= \boxed{9.05\ \mathrm{mV}}$$

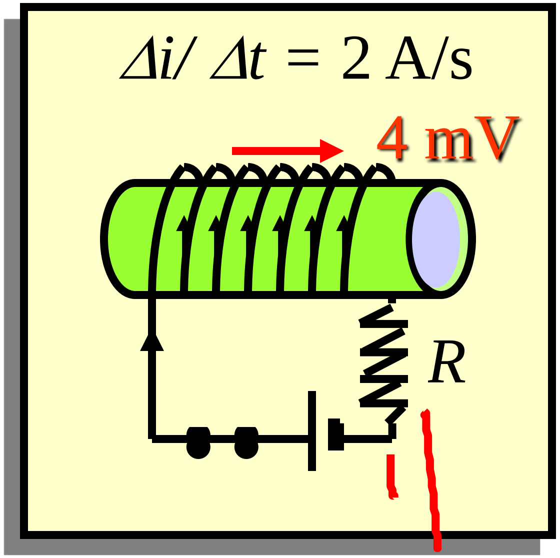
Example :

A solenoid with L=1x10^{-4} H is in a circuit and experiences a change in current from 0 to 10 A in 1.0 ms. Find the self-induced emf in the solenoid

$$\varepsilon = -L\frac{\Delta I}{\Delta t} = -10^{-4}\,\frac{10}{10^{-3}} = -1.0V$$

The emf opposes the current direction

Example : A coil having 20 turns has an induced emf of 4 mV when the current is changing at the rate of 2 A/s. What is the inductance?



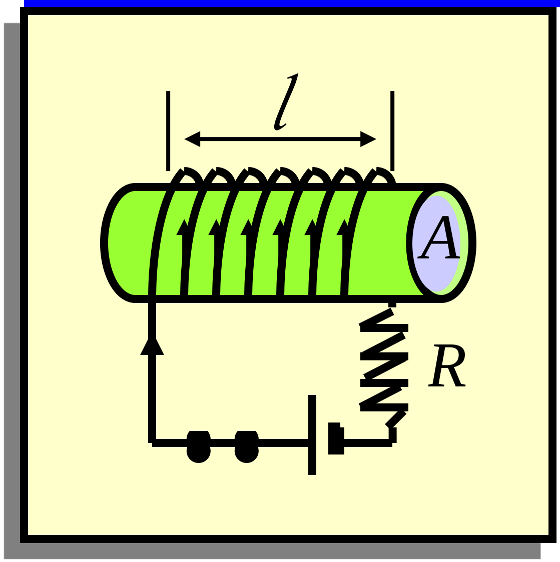
$$L = \frac{-(-0.004\ \text{V})}{2\ \text{A/s}}$$

L = 2.00 mH

Example A solenoid of area 0.002 m^2 and length 30 cm, has 100 turns. If the current increases from 0 to 2 A in 0.1 s, what is the inductance of the solenoid?

First we find the inductance of the solenoid:

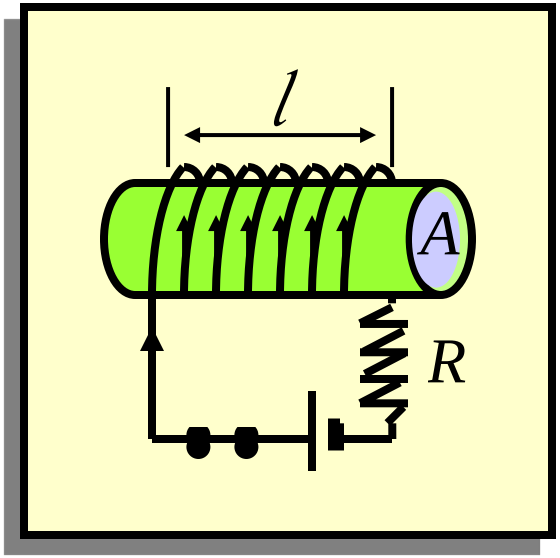
$$L = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi \text{ x } 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}})(100)^2(0.002 \text{ m}^2)}{0.300 \text{ m}}$$



$L = 8.38 \text{ x } 10^{-5}$ H

Note: L does NOT depend on current,.

Example (Cont.): If the current in the 83.8-μH solenoid increased from 0 to 2 A in 0.1 s, what is the induced emf?



$L = 8.38 \times 10^{-5}$ H

$$\mathcal{E} = -L\frac{\Delta i}{\Delta t}$$

$$\mathcal{E} = \frac{-(8.38 \times 10^{-5} \text{H})(2 \text{ A} - 0)}{0.100 \text{ s}}$$

$$\mathcal{E} = -1.68 \text{ mV}$$

$$U_B = \int_0^{U_B} dU_B = \int_0^{I} LI\,dI$$

Remember for a capacitor: $U = \frac{1}{2}CV^2$

$$U_B = \frac{1}{2} L I^2$$

mech $U = \frac{1}{2} m v^2$

cap $U = \frac{1}{2} C V^2$

ind $U = \frac{1}{2} L I^2$

Q $L = \mu_0 n^2 A\ell$ Q $B = \mu_0 n I$

$$\therefore U_B = \frac{1}{2} L I^2 = \frac{1}{2}(\mu_0 n^2 A\ell)\left(\frac{B}{\mu_0 n}\right)^2$$

$A\ell$ is the volume of the solenoid

$$U_B = \frac{B^2}{2\mu_0}(A\ell)$$

Energy of a Magnetic Field

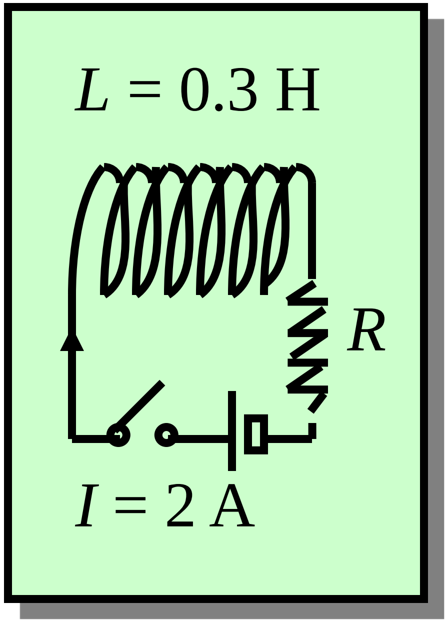
$$\text{energy density} = \frac{\text{energy}}{\text{volume}}$$

Energy Density in a coil

$$u_B = \frac{U_B}{A\ell} = \frac{B^2}{2\mu_0}$$

$$u = \frac{1}{2}\varepsilon_o E^2$$

Example : What is the potential energy stored in a 0.3 H inductor if the current rises from 0 to a final value of 2 A?



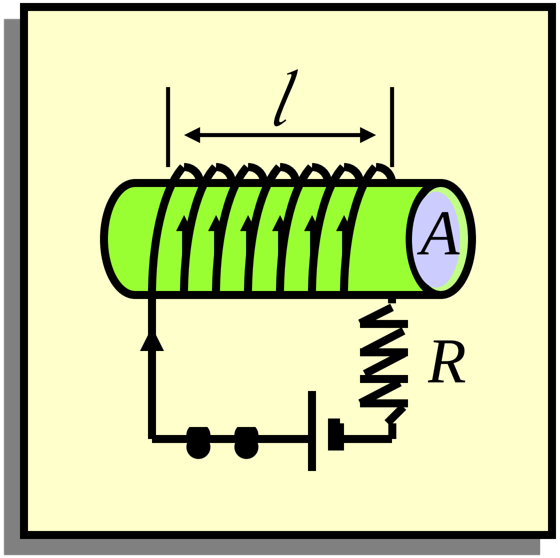
$$U = \tfrac{1}{2} L i^2$$

$$U = \tfrac{1}{2}(0.3\ \text{H})(2\ \text{A})^2 = 0.600\ \text{J}$$

U = 0.600 J

This energy is equal to the work done in reaching the final current I; it is when the current decreases to zero. returned

Example 4: The final steady current in a solenoid of 40 turns and length 20 cm is 5 A. What is the energy density?



$$B = \frac{\mu_0 NI}{l} = \frac{(4\pi \text{ x } 10^{-7})(40)(5\text{ A})}{0.200\text{ m}}$$

B = 1.26 mT

$$u = \frac{B^2}{2\mu_0} = \frac{(1.26 \text{ x } 10^{-3}\text{T})^2}{2(4\pi \text{ x } 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}})}$$

u = 0.268 J/m^3

Energy density is important for the study of electro-magnetic waves.

Example

a) Find The Inductance of a long solenoid length ~~L~~ ℓ=2m and radius=2cm with 2000 turns?

b) if current decreased from 4A to 0 in 2 microseconds what is magnitude and direction of the self induced emf ?

c) what is the energy stored in the solenoid at the beginning of the 2 microsecond interval?

a) Inductance value

$$L = \frac{\mu_o N^2 A}{l}$$

$$A = \pi r^2$$

$A = 1.257\text{x}10^{-3}\text{m}^2$

$L = (1.2566\text{x}10^{-6})(2000)\ ^2(1.257\text{x}10^{-3})/2.0 =$

$= 3.159\text{x}10^{-3}$ H

b) $$\varepsilon = -L\frac{dI}{dt} = -L\frac{I_2 - I_1}{\Delta t}$$

emf = $(3.159 \times 10^{-3})(4-0)/(2.0 \times 10^{-6})$ = 6318V

in direction of current trying to stop field collapse by trying to maintain current

c) Energy? $$U_1 = \frac{1}{2}LI_1^2$$

$U_1 = (1/2)\,(3.158 \times 10^{-3})(4)^2 = 2.52 \times 10^{-2}$ Joules

Quick Quiz 32.5 You are performing an experiment that requires the highest possible energy density in the interior of a very long solenoid. Which of the following increases the energy density? (More than one choice may be correct.) (a) increasing the number of turns per unit length on the solenoid (b) increasing the cross-sectional area of the solenoid (c) increasing only the length of the solenoid while keeping the number of turns per unit length fixed (d) increasing the current in the solenoid.

SUMMARY

When the current in a coil changes with time, an emf is induced in the coil according to Faraday's law. The **self-induced emf** is

$$\mathcal{E}_L = -L\frac{dI}{dt} \qquad (32.1)$$

where L is the **inductance** of the coil. Inductance is a measure of how much opposition an electrical device offers to a change in current passing through the device. Inductance has the SI unit of **henry** (H), where $1\ \text{H} = 1\ \text{V}\cdot\text{s/A}$.

The inductance of any coil is

$$L = \frac{N\Phi_B}{I} \qquad (32.2)$$

where Φ_B is the magnetic flux through the coil and N is the total number of turns. The inductance of a device depends on its geometry. For example, the inductance of an air-core solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} \quad \textbf{(32.4)}$$

where A is the cross-sectional area, and ℓ is the length of the solenoid.

The energy stored in the magnetic field of an inductor carrying a current I is

$$U = \tfrac{1}{2} L I^2 \quad \textbf{(32.12)}$$

Summary

- Inductance (units, henry H) is given by $$L = \frac{N\Phi_B}{i}$$
- Inductance of a solenoid is: $$L = \frac{\mu_0 N^2 A}{l}$$ (depends only on geometry)
- EMF, in terms of inductance, is: $$\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$$
- Energy in inductor: $$U_B = \frac{1}{2}Li^2$$ Energy in magnetic field