CHAPTER 2
Introduction to vectors

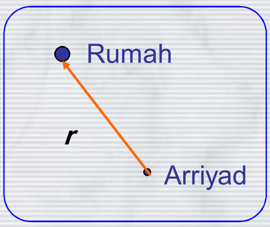
Physical quantities are classified as scalars, vectors, etc.

Scalar : described by a real number with units
examples - mass, charge, energy . . .

Vector : described by a scalar (the magnitude) and a direction in space
examples - displacement, velocity, force . . .

- In 1 dimension, we could specify direction with a + or - sign.
- In 2 or 3 dimensions, we need more than a sign to specify the direction of something:
- To illustrate this, consider the *position vector* $\boldsymbol{r}$ in 2 dimensions.

Example: Where is Arriyad?
- Choose origin at Arriyad
- Choose coordinates of distance (km), and direction (N,S,E,W)
- In this case $\boldsymbol{r}$ is a vector that points 100 km Northwest



What is Vector

Observe the following physical quantities:

1. Velocity, displacement, acceleration and force.
2. Mass, time, temperature

What is/are the distinct different between quantities in “1” and in “2” ?

The answer is:

1. Velocity, displacement, acceleration and force.
   All quantities that have direction associated with them, apart from magnitude.
2. Mass, time, temperature
   All quantities that have only magnitude.

Generally speaking,
All physical quantities can be divided into 2 categories:

1. Vector quantity

needs magnitude and direction to describe it.
Example: Position of the car is 11.2 km, east.

2. Scalar quantity

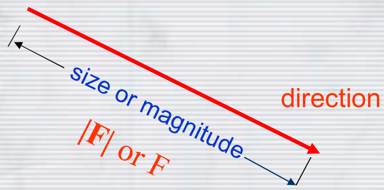
needs only magnitude to describe it.
Example: Temperature of the copper bar is 223 ^{o}C

Physical quantities that have both a size and direction are represented by vectors, which are drawn as arrows.

Notation: bold face **F**

underline <u>F</u>

over arrow $\vec{F}$



The magnitude is a scalar: it has only a size, in some defined units.

Sometimes there are separate names for the magnitude of vectors alone: e.g. "speed" is the magnitude of the velocity vector.

PROPERTIES OF VECTOR

Vector-Addition/Subtraction

Basically, there are 3 methods of adding/subtracting vectors:

1. Graphical method
   - ⇨ Tail-to-tip method
   - ⇨ Parallelogram method
2. Components method (most important !)

Notes:

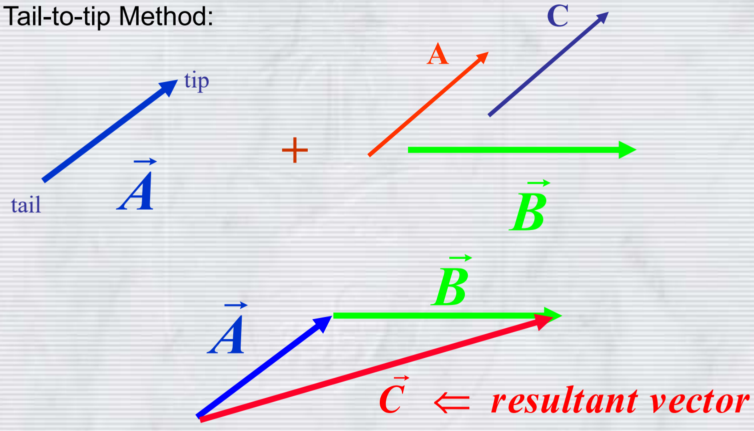
In vector, a negative sign means opposite direction

whereas in scalar quantity, a negative is used to denote "lost" (positive means "gain")

Vector-Addition/Subtraction

Equality: vectors with the same magnitude and direction are equal, regardless of location. So both displacement and position are vectors.

⇨ Tail-to-tip Method:



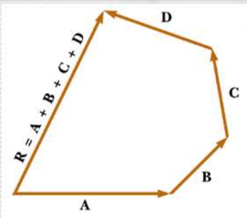
The resultant vector **C = A + B** is drawn from the tail of **A** to the tip of **B**.

Adding several vectors together.

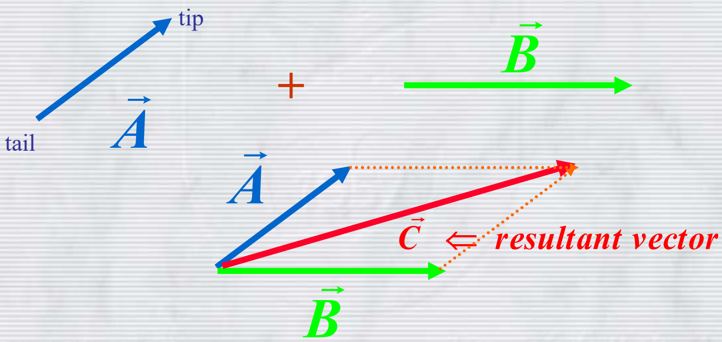
Resultant vector

R=A+B+C+D

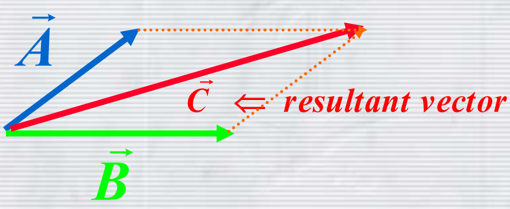
is drawn from the tail of the first vector to the tip of the last vector.



⇨ Parallelogram Method:



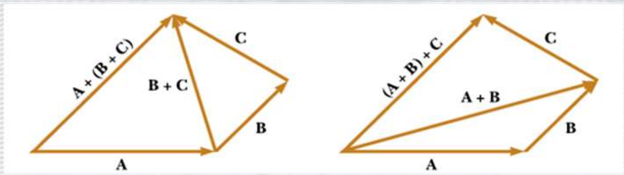
Is A + B = B + A



A + B = B + A YES

Commutative Law of vector addition

Associative Law of vector addition

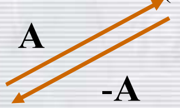


A+(B+C) = (A+B)+C

Negative of a vector.

The vectors **A** and –**A** have the same magnitude but opposite directions.

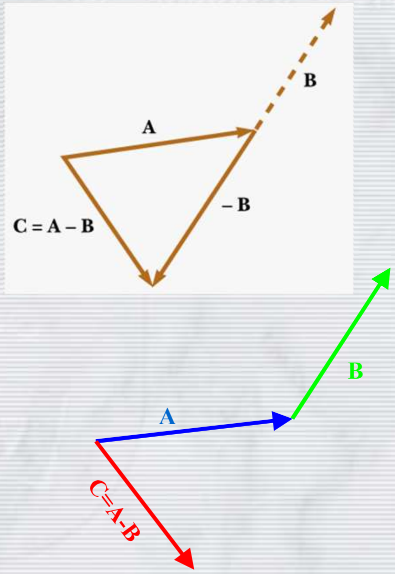
A + (-**A**) = 0

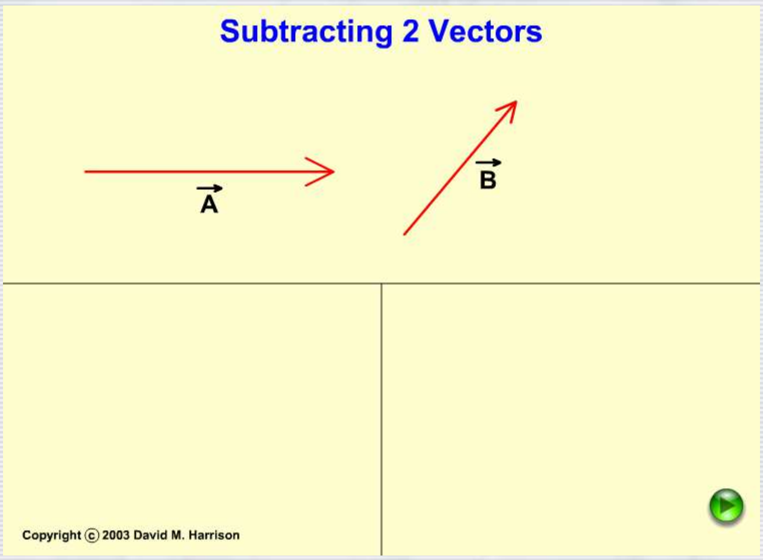


Subtracting vectors:

A - B = A + (-B)

Subtraction: Head to head





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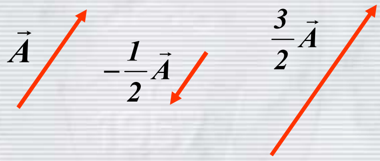
Important Properties

$$\vec{a}+\vec{b}=\vec{b}+\vec{a} \qquad (commutative\ law\ ).$$

$$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c}) \qquad (associative\ law\ ).$$

$$\vec{a}=\vec{a}-\vec{b}=\vec{a}+(-\vec{b}) \qquad (vector\ substraction).$$

Multiplication by a scalar:

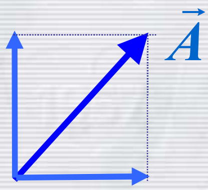


Other complicated combination rules, the **dot product** and the **vector product**, will come later

COMPONENTS METHOD:

- graphical method is less accurate and not useful for vectors in 3D.
- Need a precise and powerful method ==> components method.

Any vector that lies in a particular plane (2D) can be resolved into 2 perpendicular components.



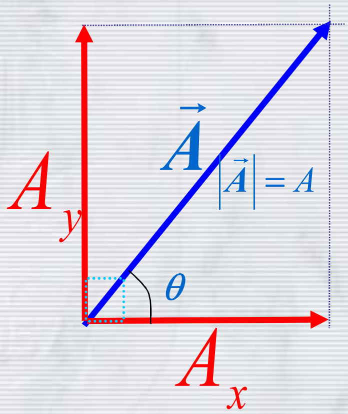
2.5 Components of a vector (Resolving a Vector)

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

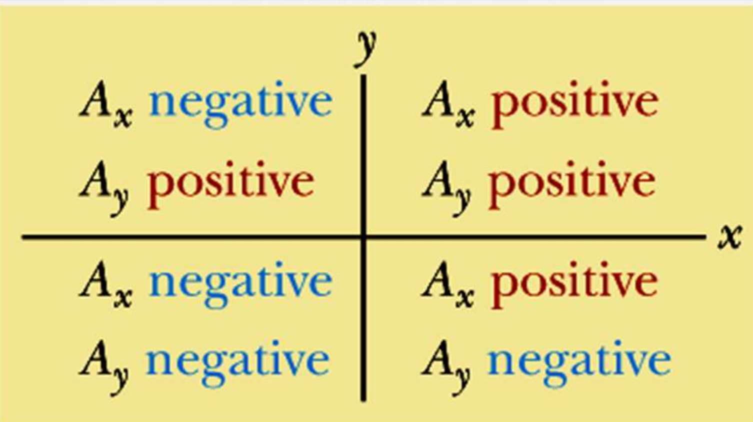
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$



The signs of the components A_x and A_y depend on the angle θ and they can be positive or negative.

(Examples)



2.6 Victor Addition by Components

1. The vector in component form:

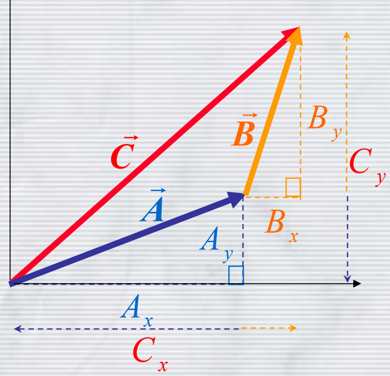
$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$

2. The vector in magnitude-angle form:

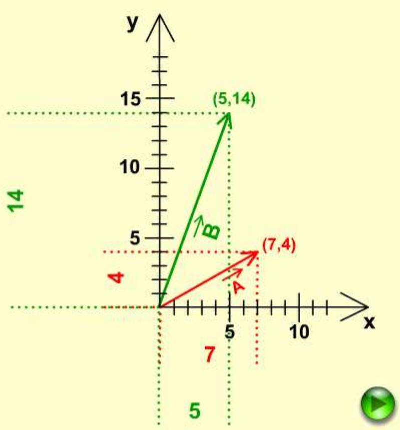
$$|\vec{C}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$
$$= \sqrt{C_x^2 + C_y^2}$$

$$\tan\theta = \frac{A_y + B_y}{A_x + B_x} = \frac{C_y}{C_x}$$

$$\vec{C} = \vec{A} + \vec{B}$$



To Add 2 Vectors Numerically ...



VectorAddComponents

Unit Vectors:

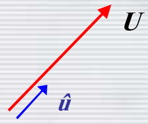
- A **Unit Vector** is a vector having length 1 and no units.
- It is used to specify a direction.
- Unit vector u points in the direction of U. Often denoted with a "hat": $u = \hat{u}$

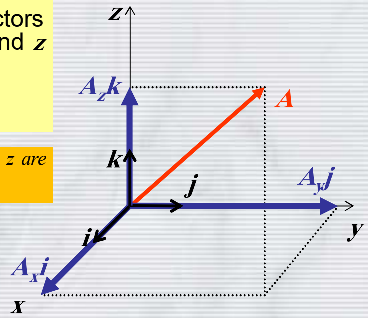
$$|\hat{u}| = 1$$

- Useful examples are the Cartesian unit vectors [i, j, k] point in the direction of the x, y and z axes.

In your textbook the notation for the unit vector along x, y, z are $\hat{x}$, $\hat{y}$, $\hat{z}$

$$A = A_x i + A_y j + A_z k$$





A_x, A_y, and A_z are scalars multiplying i, j, k They are called "components".

The components of a vector can be found from trigonometry.

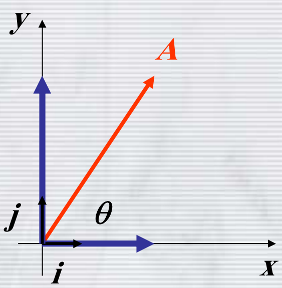
$A_x = |\mathbf{A}| \cos\theta$

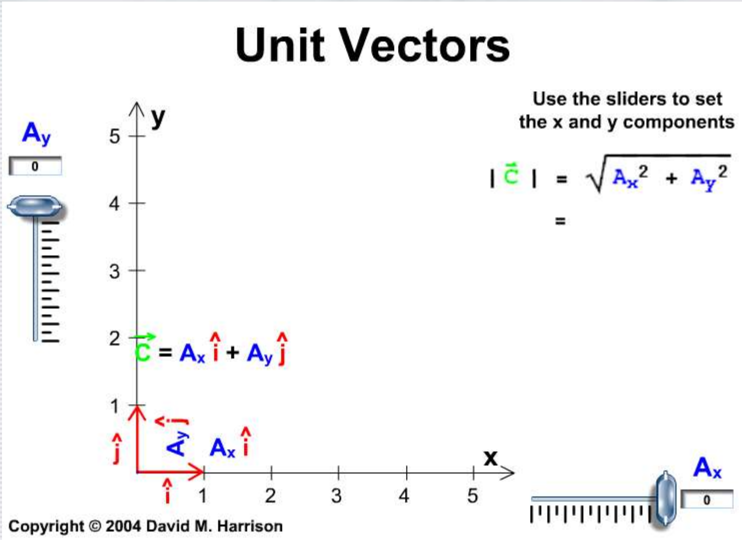
$A_y = |\mathbf{A}| \sin\theta$

Going backwards,

$\tan\theta = A_y / A_x$

$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2}$



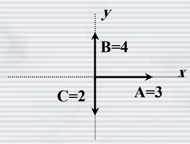


Unit Vectors

Problems from past EXAMS

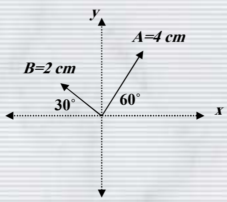
The magnitude of of the vectors shown in the figure is;

1. 3.6
2. 5.1
3. 6.7
4. 9.7

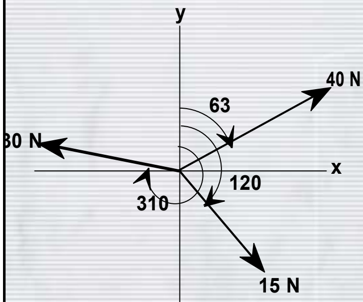


The direction of the vector $\vec{C} = \vec{A} - \vec{B}$ with respect to the *+x-axis* is:

1. 33°
2. 39°
3. 46°
4. 90°

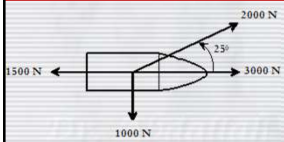


مثال:
احسب مقدار واتجاه محصلة القوي المبينة فى الشكل



ANS: |R|=39.4 N , θ=49.5°

مثال:
إذا أثرت مجموعة من القوى على القارب الموضح في الشكل فان محصلة هذه القوى واتجاهها هما على الترتيب :

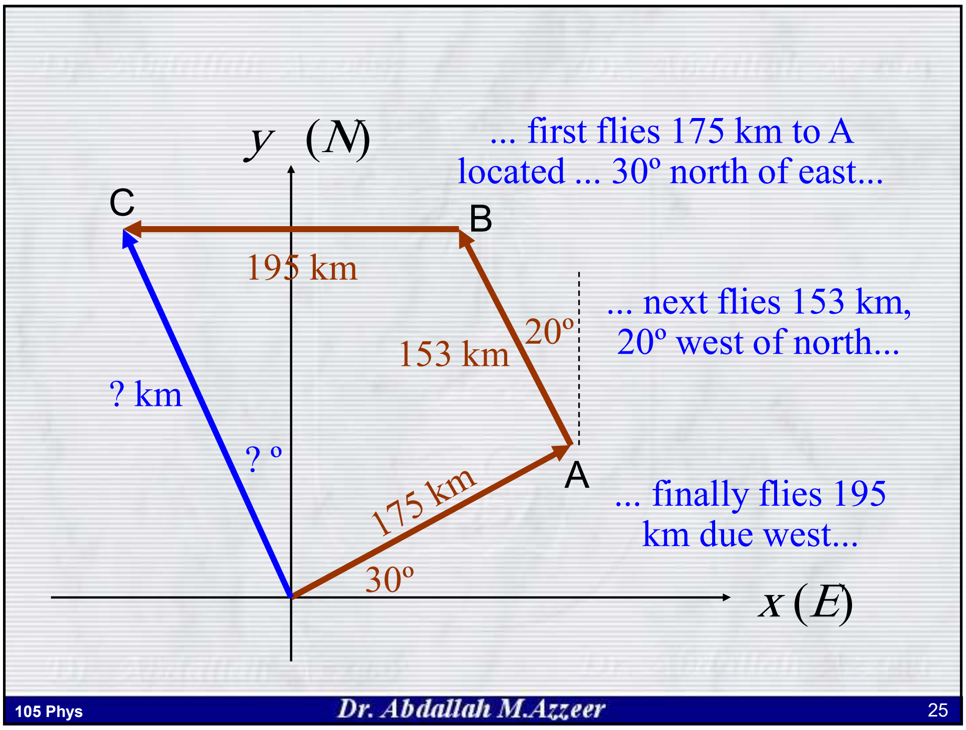


ANS; 3317 N, 357°

Example 3.6

A commuter plane first flies 175 km to minor airport A located in the direction 30° north of east. Next it flies 153 km, 20° west of north to town B. Finally, it flies 195 km due west to city C. What is the location of city C relative to the starting point?

Strategy:
1. It's a vector problem; draw a diagram—a map.
2. Find *x*- (east) and *y*- (north) components of all vectors.
3. Add components ⇒ total displacement vector.
4. Determine magnitude and angle of total displacement.


y (N)
... first flies 175 km to A located ... 30° north of east...
C
B
195 km
153 km
20°
... next flies 153 km, 20° west of north...
? km
? °
A
175 km
... finally flies 195 km due west...
30°
x (E)
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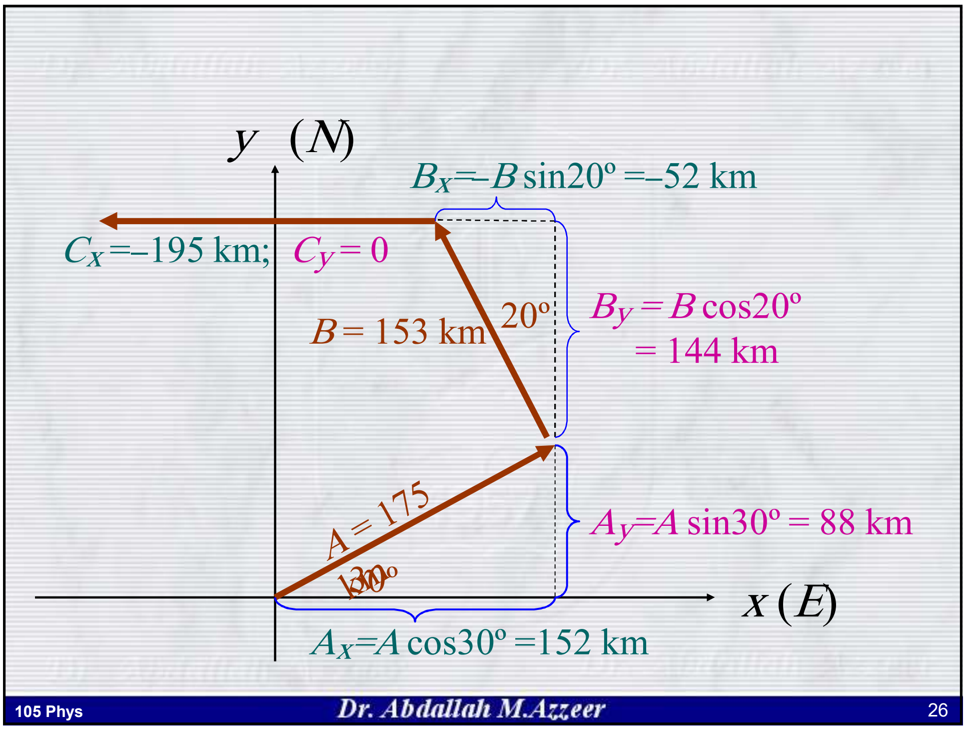
25


y (N)
Bx=–B sin20° =–52 km
Cx=–195 km; Cy= 0
B= 153 km
20°
By = B cos20° = 144 km
A = 175 km
30°
Ay=A sin30° = 88 km
x (E)
Ax=A cos30° =152 km
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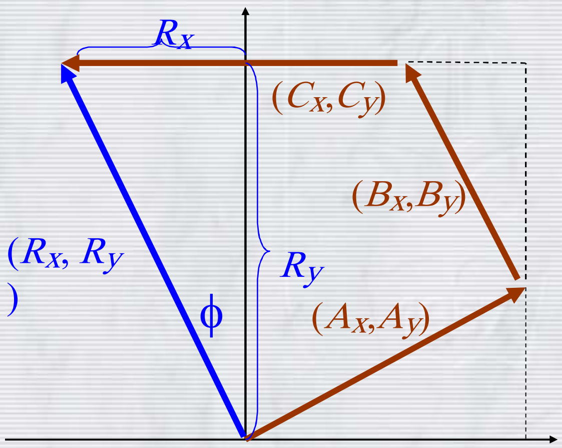
26

$\mathbf{R} = (R_x, R_y) = (A_x, A_y) + (B_x, B_y) + (C_x, C_y) = (-95, 232)$ km



$$R = \sqrt{R_x^2 + R_y^2}$$
$$= 251 \text{ km}$$

$$\tan\phi = \frac{|R_x|}{|R_y|} = \frac{95}{232}$$
$$= 0.41$$
$$\phi = 22.3°$$

West of north

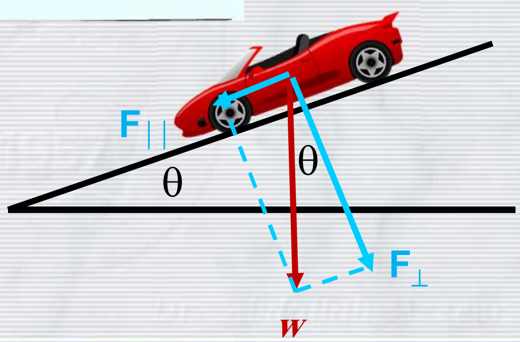
EXAMPLE 2.6

A car weighing 12.0 kN (2700 lb) is parked on a driveway that is at a 15° angle with the horizontal. Find the components of the car's weight parallel and perpendicular to the driveway.

SOLUTION The weight **w** of anything is the gravitational force the earth exerts on it, a force that acts vertically downward. Because **w** is vertical and $\mathbf{F}_\perp$ is perpendicular to the road, the angle θ between **w** and $\mathbf{F}_\perp$ is equal to the angle $\theta = 15°$ between the road and the horizontal. Hence

$$F_\parallel = w \sin\theta = (12.0 \text{ kN})(\sin 15°) = 3.1 \text{ kN}$$
$$F_\perp = w \cos\theta = (12.0 \text{ kN})(\cos 15°) = 11.6 \text{ kN}$$

◆



READ AND SOLVE THE EXAMPLES ON THIS CHAPTER also don’t forget the assigned PROBLEMS