

## Work \& Energy

So far: Motion analysis with forces.
NOW: An alternative analysis using the concepts of Work \& Energy.
Easier?

## Conservation of Energy:

NOT a new law! Just Newton's Laws in a different language.

- One of the most important concepts in physics
> Alternative approach to mechanics
- Many applications beyond mechanics
$>$ Thermodynamics (movement of heat)
> Quantum mechanics...
- Very useful tools
$>$ You will learn new (sometimes much easier) ways to solve problems


## Definition of Work:

Work, W, is energy transferred to or from an object by means of a force acting on the object.
We can find an expression for work by considering an object that moves due to a force applied to it....

## Work Done by Constant Force

$>$ Work in physics is done only when a sum of forces exerted on an object made a motion to the object.
$>$ For an object moving under a Constant Force, Work done $(W)=$ product of magnitude of displacement $(\mathrm{d}) \times$ component of force parallel to displacement $\left(F_{\|}\right)$:


## Work can be positive or negative

- Man does positive work lifting box
- Man does negative work lowering box
- Gravity does positive work when box lowers
- Gravity does negative work when box is raised

$$
W=F_{11} d=F d \cos \theta
$$

- Can exert a force \& do no work!

$$
\text { Could have } \mathrm{d}=0 \Rightarrow \mathrm{~W}=0
$$

Could have $F \perp d$

$$
\Rightarrow \theta=90^{\circ}, \cos \theta=0
$$

$$
\Rightarrow \mathbf{w}=\mathbf{0}
$$

Example, walking at constant v with grocery bag:



What is the work done by

- the gravitational force $=0$
- the normal force $=0$
- the force $\mathrm{F}=\mathrm{F} \cos \theta$
when the block is displaced along the horizontal.



## Example

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F=50.0 \mathrm{~N}$ at an angle of $30^{\circ}$ with the horizontal.

Calculate the work done by the force on the vacuum cleaner as its displaced 3.00 m to the right .

(b)

Example: On a frictionless plane, the block is subject to three forces: gravity, the normal force and tension $\mathbf{T}$. If it moves up the incline through displacement d:


Work done by $\mathbf{T}: \mathrm{W}_{\mathbf{T}}=\mathrm{T} \cdot \mathrm{d} \quad$ "effective"
Work done by $\mathbf{n}: W_{\mathbf{n}}=0 \cdot d=0 \quad$ "ineffective"
Work done by $\mathrm{mg}: \mathrm{W}_{\mathrm{mg}}=(-\mathrm{mg} \sin \phi) \cdot \mathrm{d}=-\mathrm{mg}$ dsin $\phi$ "effective",

Net work $\Sigma \mathrm{W}=\mathrm{W}_{\mathrm{T}}+\mathrm{W}_{\mathrm{n}}+\mathrm{W}_{\mathrm{mg}}=\mathrm{Td}-\mathrm{mg}$ dsin $\phi$


## Work due to friction

If friction is involved in moving objects, work has to be done against the kinetic frictional force.

This work is:


$$
W_{f}=f_{k} \cdot d=f_{k} d \cos 180^{\circ}=-f_{k} d
$$



## Example

$2.40 \times 10^{2} \mathrm{~N}$ force is pulling an $85.0-\mathrm{kg}$ refrigerator across a horizontal surface. The force acts at an angle of $20.0^{\circ}$ above the surface. The coefficient of kinetic friction is 0.200 , and the refrigerator moves a distance of 8.00 m . Find
(a) the work done by the pulling force, and
(b) the work done by the kinetic frictional force.
(a) $\mathrm{W}=\mathrm{F} \cos \theta \mathrm{d}=1.8 \times 10^{3} \mathrm{~J}$
(b) $\mathrm{W}_{f}=f_{k} \mathrm{~d} \cos \theta$
$\boldsymbol{f}_{\mathrm{k}}=\mu_{\mathrm{k}}(\mathrm{mg}-\mathrm{F} \sin \theta)$ $=1.5 \times 10^{2} \mathrm{~N}$
so $\mathrm{W}_{f}=-1.2 \times 10^{3} \mathrm{~J}$


## Work \& Kinetic Energy الشغل والطاقة الحركية

## A constant net Force



$$
\text { Work done by } \vec{F} \Rightarrow W_{\text {net }}=\vec{F} \cdot \overrightarrow{\boldsymbol{d}}=F d
$$

$\because F=m a$,

$$
\begin{aligned}
& \overrightarrow{v_{i}} \xrightarrow[v_{f}]{l} v_{f}^{2}=v_{i}^{2}+2 a d \Rightarrow d=\frac{1}{2 a}\left(v_{f}^{2}-v_{i}^{2}\right) \\
& \Rightarrow W_{n e t}=m a \cdot \frac{1}{2 a}\left(v_{f}^{2}-v_{i}^{2}\right) \\
& W_{n e t}=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2} \quad K \equiv \frac{1}{2} m v^{2} \quad W_{n e t}=K_{f}-K_{i}=\Delta K
\end{aligned}
$$

## Kinetic Energy; Work-Energy Principle

- Energy $\equiv$ The ability to do work
- Kinetic Energy $\equiv$ The energy of motion
- "Kinetic" $\equiv$ Greek word for motion
- An object in motion has the ability to do work
- Net work an object = Change in KE.

$$
\mathbf{W}_{\mathrm{net}}=\Delta K
$$

## The Work-Energy Principle

* Note: $\mathbf{W}_{\text {net }}=$ work done by the net (total) force.
* $W_{\text {net }}$ is a scalar.
* $W_{\text {net }}$ can be positive or negative (because $\Delta K E$ can be both $+\&-$ )

業 Units are Joules for both work \& KE.

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Kinetic Energies for Various Objects

| Object | Mass (kg) | Speed (m/s) | Kinetic Energy (J) |
| :--- | :---: | :--- | :--- |
| Earth orbiting the Sun | $5.98 \times 10^{24}$ | $2.98 \times 10^{4}$ | $2.65 \times 10^{33}$ |
| Moon orbiting the Earth | $7.35 \times 10^{22}$ | $1.02 \times 10^{3}$ | $3.82 \times 10^{28}$ |
| Rocket moving at escape speed ${ }^{\mathrm{a}}$ | 500 | $1.12 \times 10^{4}$ | $3.14 \times 10^{10}$ |
| Automobile at $65 \mathrm{mi} / \mathrm{h}$ | 2000 | 29 | $8.4 \times 10^{5}$ |
| Running athlete | 70 | 10 | 3500 |
| Stone dropped from 10 m | 1.0 | 14 | 98 |
| Golf ball at terminal speed | 0.046 | 44 | 45 |
| Raindrop at terminal speed | $3.5 \times 10^{-5}$ | 9.0 | $1.4 \times 10^{-3}$ |
| Oxygen molecule in air | $5.3 \times 10^{-26}$ | 500 | $6.6 \times 10^{-21}$ |

a Escape speed is the minimum speed an object must reach near the Earth's surface in order to move infinitely far away from the Earth.

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## COSERVATION OF ENERGY

A book sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left. The initial velocity of the book is $v_{p}$ and its final velocity is $v_{f}$ The normal force and the gravitational force are not included in the diagram


## We will be back to this point later



(b)

## A Question

- A box is pulled up a rough ( $\mu>0$ ) incline by a rope-pulley-weight arrangement as shown below.
How many forces are doing work on the box?
(a) 2
(b) 3
(c) 4


## Solution

Draw FBD of box: Consider direction of motion of the box Any force not perpendicular to the motion will do work: N does no work ( $\perp$ v) Tdoes positive work f does negative work mg does negative work



## Example

A rescue helicopter lifts a $79-\mathrm{kg}$ person straight up by means of a cable. The person has an upward acceleration of $0.70 \mathrm{~m} / \mathrm{s}^{2}$ and is lifted from rest through a distance of 11 m .
(a) What is the tension in the cable?
(b) How much work is done by the tension in the cable
(c) How much work is done by the person's weight?
(d) Use the work-energy theorem and find the final speed of the person.
a) $\mathrm{T}-\mathrm{mg}=\mathrm{ma}, \mathrm{T}=8.3 \times 10^{2} \mathrm{~N}$

b) $\mathrm{W}_{\mathrm{T}}=\mathrm{Td}=9.13 \times 10^{3} \mathrm{~J}$
c) $W_{w}=-m g d=-8.5 \times 10^{3} \mathrm{~J}$
d) $\mathrm{W}_{\mathrm{T}}+\mathrm{W}_{\mathrm{W}}=1 / 2 \mathrm{mv}_{\mathrm{f}}{ }^{2}-1 / 2 \mathrm{mv}_{\mathrm{o}}{ }^{2}$ $v_{f}=3.92 \mathrm{~m} / \mathrm{s}$

## Example

Two blocks have masses $m_{1}$ and $m_{2}$, where $m_{1}>m_{2}$. They are sliding on a frictionless floor and have the same kinetic energy when they encounter a long rough stretch (i.e. $\mu>$ 0 ) which slows them down to a stop. Which one will go farther before stopping?


## Solution

## for the first mass $m_{1}$

- The work-energy theorem says that for any object $W=\Delta K$
- In this example the only force that does work is friction (since both $\mathbf{N}$ and mg are perpendicular to the blocks motion).
- The net work done to stop the box is $-f d=-\mu m g d$.

This work "removes" the kinetic energy that the box had: $\boldsymbol{W}=\boldsymbol{K}_{\boldsymbol{f}}-\boldsymbol{K}_{\boldsymbol{i}}=\boldsymbol{0}-\boldsymbol{K}_{\boldsymbol{i}}$


## for the second mass $m_{2}$

- The net work done to stop a box is $-\boldsymbol{f d}=-\mu \mathrm{mg} d$.
$>$ This work "removes" the kinetic energy that the box had:
$>\boldsymbol{W}=\boldsymbol{K}_{\boldsymbol{f}}-\boldsymbol{K}_{\boldsymbol{i}}=\boldsymbol{0}-\boldsymbol{K}_{\boldsymbol{i}}$
- This is the same for both boxes (same starting kinetic energy).


$$
\mu m_{2} g d_{2}=\mu m_{1} g d_{1}
$$

$$
\square m_{2} d_{2}=m_{1} d_{1}
$$

Since $\boldsymbol{m}_{1}>\boldsymbol{m}_{2}$ we can see that $d_{2}>d_{1}$


## Example

$0.075-\mathrm{kg}$ arrow is fired horizontally. The bowstring exerts an average force of 65 N on the arrow over a distance of 0.90 m . With what speed does the arrow leave the bow?

$$
W_{n e t}==F \bullet d=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}
$$

$$
V_{f}=39 \mathrm{~m} / \mathrm{s}
$$



## Power

Power is the rate at which work is done by a force

$$
\begin{aligned}
& P_{A V G}=W / \Delta t \quad \text { Average Power } \\
& P=d W / d t \quad \text { Instantaneous Power }
\end{aligned}
$$

The unit of power is a Joule/second (J/s) which we define as a Watt (W)

$$
1 \mathrm{~W}=1 \mathrm{~J} / \mathrm{s}=1 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{3}
$$



In general, power is defined for any type of energy transfer.
$P=\boldsymbol{d} \boldsymbol{E} / \boldsymbol{d} \boldsymbol{t}$
Where $\mathrm{dE} / \mathrm{dt}$ is the rate at which energy is crossing the boundary of the system by a given transfer mechanism.


## Example 7.12

An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg . A constant friction forces of 4000 N retards its motion upward, as shown in the figure.
(a) What power deliver by the motor is required to lift the elevator car at a constant speed of $3 \mathrm{~m} / \mathrm{s}$ ?
(b) What power must the motor deliver at the instant the speed of the elevator is $v$ if the motor is designed to provide the elevator car with an upward acceleration of $1 \mathrm{~m} / \mathrm{s}^{2}$ ?

$$
\text { (a) } M_{\text {max }}=1800 \mathrm{~kg}, f=4000 \mathrm{~N}
$$

$v=3 \mathrm{~m} / \mathrm{s}($ constant $) \Rightarrow a=0$
$\sum \boldsymbol{F}=\boldsymbol{T}-\boldsymbol{f}-\boldsymbol{M g}=\mathbf{0}$
$T=f+M g=4000 N+1800 \times 9.8 N=2.16 \times 10^{4} N$
Power: $P=\vec{T} \cdot \vec{v}=T v=\left(2.16 \times 10^{4}\right)(3)=64.8 \mathrm{~kW}$
(b) Left for you to try

$\qquad$ $-$



Problem: determine the power after an elapsed time of 3.0 s

$$
\bar{P}=\frac{\mathrm{mgh}}{\Delta \mathrm{t}}=\frac{50 \mathrm{~kg} \times 9.8 \mathrm{~m} / \mathrm{s} \times 2.0 \mathrm{~m}}{3.0 \mathrm{~s}}=330 \mathrm{~W}
$$



