



4.1

- (5)  $\int \left( \frac{1}{z^3} - \frac{3}{z^2} \right) dz$       (7)  $\int \left( 7\sqrt{u} + \frac{1}{\sqrt{u}} \right) du$   
 (11)  $\int (3x-1)^2 dx$       (14)  $\int (2x-5)(3x+1) dx$   
 (15)  $\int \frac{8x-5}{\sqrt[3]{x}} dx$       (17)  $\int \frac{x^3-1}{x-1} dx$   
 (23)  $\int \frac{7}{\csc x} dx$       (27)  $\int \frac{\sec t}{\cos t} dt$   
 (29)  $\int \csc v \cot v \sec v dv$       (35)  $\int \frac{1}{dx} (\sqrt{x^2+4}) dx$

4.2

- (9)  $\int \sqrt{3x-2} dx$       (16)  $\int v \sqrt{9-v^2} dv$   
 (20)  $\int (3+s^3)^2 ds$       (21)  $\int \frac{(\sqrt{x}+3)^4}{\sqrt{x}} dx$   
 (27)  $\int \cos(4x-3) dx$       (37)  $\int \frac{\sin x}{\cos^4 x} dx$   
 (32)  $\int \frac{\sin(2x)}{\sqrt{1-\cos(2x)}} dx$

4.3

Express the sum in terms of  $n$ .

- (9)  $\sum_{k=1}^n (k^2 + 3k + 5)$       (12)  $\sum_{k=1}^{55} (3k^3 + k)$

(2) Approximate the area under the graph of  $f(x) = 3-x$  from  $a = -2$  to  $b = 2$ . Approximate  $A$  by dividing  $[a, b]$  into subintervals of equal length  $\Delta x = 1$  and using (a) AIP and (b) ACP  
 10      14

4.4

- (11) Find the Riemann Sum  $R_p$  for  $f(x) = x^2$  on the interval  $[-2, 6]$  with a regular partition  $p$  of size  $n = 32$  by choosing on each subinterval of  $p$  (a) the left-hand endpoint (b) the right-hand endpoint (c) the mid point  
 292.5      398.5      319.75



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(18)  $f(x) = \sqrt{x}$ ,  $P = \{1, 3, 4, 9, 12, 16\}$ ,  $n = 5$

(16) Use def of definite integral to express the limit  $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n \pi \frac{w_k}{k} \Delta x_k$  as a definite integral on the interval  $[0, 4]$

Given  $\int_0^4 \sqrt{x} dx = \frac{4}{3}$

(19) Evaluate  $\int_4^1 \sqrt{x} dx$

(20)  $\int_1^4 \sqrt{5} ds$

Evaluate the definite integral by regarding it as the area under the graph of a function.

25 (31)  $\int_{-3}^2 (2x+6) dx$  (32)  $\int_0^{2.5} |x-1| dx$

12+2R (37)  $\int_{-2}^2 (3 + \sqrt{4-x^2}) dx$

4.5

(15) Verify the inequality without evaluating the integral.  $\int_0^{2\pi} (1 + \sin x) dx \geq 0$

(22)  $\int_c^m f(x) dx = \int_c^m f(x) dx$  as one integral

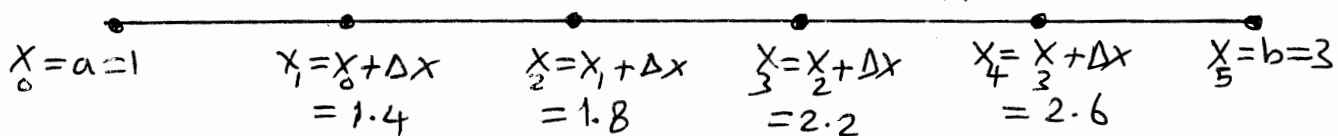
(25) (a) Rind 2 MVT (1)

(b) Find average value of  $f$  on  $[-3, 1]$

~~Rind~~  $\int_{-2}^1 (x^2 + 1) dx = 6$  (2)

1) Approximate the integral  $\int_1^3 \sqrt{1+x^3} dx$  for  $n=5$ , using the Trapezoidal rule.

Solution:  $f(x) = \sqrt{1+x^3}$ ,  $a=1$ ,  $b=3$ ,  $\Delta x = \frac{b-a}{n} = 0.4$



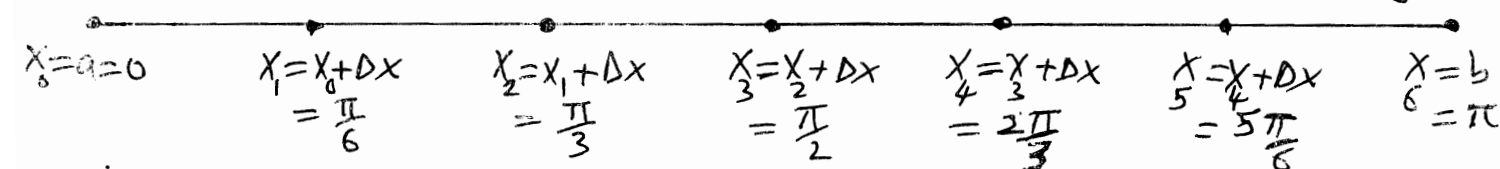
$i$	$x_i$	$m_i$	$f(x_i)$	$m_i f(x_i)$
0	1	1	1.41421356237	1.41421356237
1	1.4	2	1.93494185959	3.86988371918
2	1.8	2	2.61380948043	5.22761896086
3	2.2	2	3.41291664123	6.82583328246
4	2.6	2	4.30998839906	8.61997679812
5	3	1	5.29150262213	5.29150262213

$$\int_1^3 \sqrt{1+x^3} dx \approx \frac{\Delta x}{2} [\text{sum}] = 0.2 [31.24902894515] = 6.24980578903$$

$\text{sum} = \sum_{i=0}^5 m_i f(x_i) = 31.24902894515$

2) Approximate the integral  $\int_0^\pi \cos(\sin x) dx$  for  $n=6$ , using the Simpson's rule.

Solution:  $f(x) = \cos(\sin x)$ ,  $a=0$ ,  $b=\pi$ ,  $\Delta x = \frac{b-a}{n} = \frac{\pi}{6}$



$i$	$x_i$	$m_i$	$f(x_i)$	$m_i f(x_i)$
0	0	1	$\cos(0) = 1$	1
1	$\frac{\pi}{6}$	4	$\cos(\frac{1}{2}) = 0.999962$	3.999848
2	$\frac{\pi}{3}$	2	$\cos(\frac{\sqrt{3}}{2}) = 0.433013$	0.866026
3	$\frac{\pi}{2}$	4	$\cos(1) = 0.998477$	3.993908
4	$\frac{2\pi}{3}$	2	$\cos(\frac{\sqrt{3}}{2}) = 0.433013$	0.866026
5	$\frac{5\pi}{6}$	4	$\cos(\frac{1}{2}) = 0.999962$	3.999848
6	$\pi$	1	$\cos(0) = 1$	1

$$\text{sum} = \sum_{i=0}^6 m_i f(x_i) = 15.725656$$

$$\int_0^\pi \cos(\sin x) dx \approx \frac{\Delta x}{3} [\text{sum}] = \frac{\pi}{18} [15.725656] = 2.74644742$$



**4.6**

(17)  $\int (4x^2 - 5) dx$       (21)  $\int_{-3}^6 |x - 4| dx$        $\frac{53}{2}$

Find the derivative without integrating

(45)  $\frac{d}{dx} \int_0^3 \sqrt{x^2 + 16} dx$       (47)  $\frac{d}{dx} \int_0^x \frac{1}{t+1} dt$

**4.7**

(15) Approximate  $\int_0^3 \sqrt{1+x^3} dx$  using trapezoidal rule for  $n = 5, 10, 20,$  and  $40$   
6.249806

(17) Approximate  $\int_0^\pi \cos(\sin x) dx$  using Simpson's rule for  $n = 2, 6, 18,$  and  $54$   
2.398752962      2.4039394306

**6.2**

(11) Find  $f'(x)$ .  $f(x) = \ln \sqrt{7-2x^3}$

(35) Use implicit diff to find  $y'$ :  $3y - x^2 + \ln(x,y) = 2$

(41) Use logarithmic diff to find  $\frac{dy}{dx}$ :  
 $y = \sqrt{4x+7} (x-5)^3$

**6.3**

(3) Find  $f'(x)$ .  $f(x) = e^{3x^2}$

Use implicit diff to find  $y'$ :

(31)  $e^{xy} - x^3 + 3y^2 = 1$       (33)  $e^x \cot y = xe^{2y}$

**6.4**

(11)  $\int \frac{x-2}{x^2-4x+9} dx$

(15)  $\int \frac{\ln x}{x} dx$

(18)  $\int \frac{\sqrt{x}}{e^{\sqrt{x}}} dx$

(31)  $\int \frac{\cos^4 x}{\sin x} dx$

**6.5**

Find  $f'(x)$ :

- (4)  $7^x$       (5)  $\log(x^4 + 3x^2 + 1)$       (15)  $\log(\ln x)$
- (17)  $x^e + e^x$       (23) (e)  $x^{x^2}$



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$$(29) \int 7^x dx \quad (37) \int \frac{2^x}{2^x + 1} dx$$

$$(39) \int \frac{1}{x \log x} dx \quad (42) \int \frac{5^{\tan x}}{\cos^2 x} dx$$

6.7

$$(53) \int_0^1 \frac{e^x}{1+e^{2x}} dx \quad (56) \int \frac{\cos x}{\sqrt{9-\sin^2 x}} dx$$

$$(60) \int \frac{dx}{x\sqrt{x^6-4}} \quad (61) \int \frac{1}{\sqrt{e^{2x}-25}} dx$$

$$(62) \int \frac{1}{x\sqrt{x-1}} dx$$

6.8

$$(20) \int \frac{1}{\operatorname{sech} 7x} dx \quad (21) \int \frac{\sinh \sqrt{x}}{\sqrt{x}} dx$$

$$(28) \int \sinh x \operatorname{sech}^2 x dx \quad (29) \int \cosh x \operatorname{csch}^2 x dx$$

Find  $f'(x)$

$$(61) \sinh^{-1}(x) \quad (63) \cosh^{-1} \sqrt{x} \quad (65) \tanh^{-1}(-4x) \quad (3) \operatorname{sech}^{-1}(x^2)$$

$$(73) \int \frac{1}{\sqrt{81+16x^2}} dx \quad (74) \int \frac{1}{\sqrt{16x^2-9}} dx$$

$$(75) \int \frac{dx}{49-4x^2} \quad (79) \int \frac{1}{x\sqrt{9-x^4}} dx$$

$$(80) \int \frac{1}{\sqrt{5-e^{2x}}} dx$$

6.9

$$(51) \lim_{x \rightarrow \infty} (x^2-1)e^{-x^2} \quad (49) \lim_{x \rightarrow 0^+} x \ln x$$

$$(57) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{5x} \quad (59) \lim_{x \rightarrow 0^+} (e^x - 1)^x$$

$$(64) \lim_{x \rightarrow \infty} \left( \frac{x^2}{x-1} - \frac{x^2}{x+1} \right)$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\left(\frac{u}{a}\right) / \int \frac{du}{\sqrt{u^2+a^2}} = \frac{1}{a} \operatorname{arcsinh}\left(\frac{u}{a}\right)$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = -\frac{1}{a} \operatorname{sech}^{-1}\left(\frac{u}{a}\right) / \int \frac{du}{u\sqrt{u^2-a^2}} = -\frac{1}{a} \operatorname{arcsch}\left(\frac{u}{a}\right)$$



$u = \text{LIATE}$

7.1

(2)  $\int x \sin x dx$     (11)  $\int \tan^{-1} x dx$     ~~(12)  $\int \sqrt{x} \ln x dx$~~   
 (16)  $\int x \tan^{-1} x dx$     (17)  $\int e^{-x} \sin x dx$   
 (31)  $\int (\ln x)^2 dx$

7.2

(4)  $\int \cos^3 x dx$     (3)  $\int \sin^2 x \cos^2 x dx$   
 (6)  $\int \sin^3 x \cos^2 x dx$     (7)  $\int \sin^6 x dx$   
 (11)  $\int \tan^3 x \sec^4 x dx$     (13)  $\int \tan^6 x dx$   
 (5)  $\int \sqrt{\sin x} \cos^3 x dx$

7.3

(5)  $\int \frac{1}{x^2 \sqrt{x^2-25}} dx$     ~~(2)  $\int \frac{(4+x^2)^2}{x} dx$~~     (10)  $\int \frac{dx}{\sqrt{4x^2-25}}$   
 (3)  $\int \frac{dx}{x \sqrt{9+x^2}}$     (1)  $\int \frac{dx}{x \sqrt{4-x^2}}$

7.4

(9)  $\int \frac{5x^2-10x-8}{x^3-4x} dx$     (16)  $\int \frac{2x^2-25x-33}{(x+1)^2(x-5)} dx$   
 (25)  $\int \frac{x^6-x^3+1}{x^4+9x^3} dx$

7.5

(5)  $\int \frac{1}{\sqrt{4x-x^2}} dx$     (10)  $\int \frac{1}{(x^2-6x+34)^{3/2}} dx$   
 9.337 (16)  $\int_0^{25} \frac{1}{\sqrt{4+\sqrt{x}}} dx$     (27)  $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx$   
 (32)  $\int \frac{x^{3/2}+1}{x^{3/2}-1} dx$     (48)  $\int \frac{1}{1+\sin x + \cos x} dx$   
 (50)  $\int \frac{1}{\tan x + \sin x} dx$

7.7

(15)  $\int_{-\infty}^{\infty} x e^{-x^2} dx$     49



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5.1

Sketch and area

(10)  $y = x^3$  ;  $y = x^2$

(14)  $x = y^2$  ;  $x - y = 2$

(28)  $y = 1 - x^2$  ;  $y = x - 1$

9 (3)  $y = 6 - 3x^2$  ;  $y = 3x$  ;  $[0, 2]$

5.2 (Disk or Washer)

Sketch & volume

(6)  $y = \sqrt{x}$  ,  $x = 4$  ,  $y = 0$  ;  $x$ -axis

(9)  $y = x^2$  ,  $y = 2$  ;  $y$ -axis

(25)  $y = x^2$  ,  $y = 4$  about:

(a)  $y = 4$  (b)  $y = 5$  (c)  $x = 2$  (d)  $x = 3$   
 $\frac{512\pi}{15}$   $\frac{832\pi}{15}$   $\frac{128\pi}{3}$   $64\pi$

5.3 (Cylindrical shells)

(7)  $y = x^2$  ,  $y^2 = 8x$  ;  $y$ -axis

(13)  $x^2 = 4y$  ,  $y = 4$  ;  $x$ -axis

(17)  $y = \sqrt{x+4}$  ,  $y = 0$  ,  $x = 0$  ;  $x$ -axis

(21)  $y = x^2$  ,  $y = 4$  about:

(a)  $y = 4$  (b)  $y = 5$  (c)  $x = 2$  (d)  $x = -3$

5.5

Find the arc length of the graph of the equation from A to B

(7)  $y = 5 - \sqrt{x^3}$  ; A(1, 4) , B(4, -3)

(11)  $30xy^3 - y^8 = 15$  ; A( $\frac{8}{15}$ , 1) , B( $\frac{271}{240}$ , 2)

The graph of the equation from A to B is revolved about  $x$ -axis. Find the area of the resulting surface

(32)  $y = 2\sqrt{x+1}$  A(0, 2) , B(3, 4)

16 $\pi$  (35) If the smaller arc of the circle  $x^2 + y^2 = 2.5$  between the points (-3, 4) and (3, 4) is revolved about  $y$ -axis. Find the area of the resulting surface.

$2 \int_4^5 2\pi x \sqrt{1+(x^2)^2} dy =$



9.1

(a) Find an equation in  $x$  and  $y$  whose graph contains the points on the curve  $C$  (b) Sketch the graph of  $C$  and indicate the orientation

(3)  $x = t^2 + 1, y = t^2 - 1, -2 \leq t \leq 2$

(5)  $x = 4t^2 - 5, y = 2t + 3, t \in \mathbb{R}$

9.2

Find the slope of the tangent line and the normal line at the point on the curve that corresponds to  $t = 1$

(1)  $x = t^2 + 1, y = t^2 - 1, -2 \leq t \leq 2$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

(9) Let  $C$  be the curve with:  
 $x = -t^3, y = -6t^2 - 18t$  for  $t \in \mathbb{R}$ .

Find the point on  $C$  at which the slope of the tangent line is  $m = 2$   $(-27, -108)$   
 $(1, 12)$

Find the length of the curve:  $L = \int_a^b \sqrt{(x')^2 + (y')^2} dt$

(23)  $x = e^t \cos t, y = e^t \sin t, 0 \leq t \leq \frac{\pi}{2}$

Find the area of the surface generated by revolving the curve about  $x$ -axis

$\frac{11\pi}{9}$  (31)  $x = t^2, y = t - \frac{1}{3}t^3, 0 \leq t \leq 1$   $S = \int_a^b 2\pi y \sqrt{(x')^2 + (y')^2} dt$

$\frac{64\pi}{3}$  (33)  $x = t - \cos t, y = t - \cos t, 0 \leq t \leq 2\pi$

Find the area of the surface generated by revolving the curve about the  $y$ -axis

$\frac{536\pi}{5}$  (35)  $x = 4t^{\frac{1}{2}}, y = \frac{1}{2}t^2 + t, 1 \leq t \leq 4$

(37)  $x = e^t \sin t, y = e^t \cos t, 0 \leq t \leq \frac{\pi}{2}$

$\frac{2}{5} \sqrt{2} \pi (e^{\pi} + 1)$

$S = \int_a^b 2\pi x \sqrt{(x')^2 + (y')^2} dt$



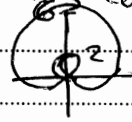


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**9.3**

Sketch the graph of the polar equation

- (1)  $r=5$  (2)  $r=2$  (3)  $\theta = -\frac{\pi}{6}$  (4)  $r=3 \cos \theta$   
 (7)  $r=4-4 \sin \theta$  (8)  $r=2+4 \sin \theta$



Find a polar equation that has the same graph as the equation in  $x$  and  $y$

- $r = -3 \sec \theta$  (27)  $x = -3$  (31)  $y = -x$   $\theta = \tan^{-1}(\frac{1}{2})$   
 (33)  $y^2 - x^2 = 4$   $r^2 = -4 \sec 2\theta$

Find an equation in  $x$  and  $y$  that has the same graph as the polar equation and use it to help sketch the graph in  $xy$ -plane

- (37)  $r \cos \theta = 5$  (38)  $r \sin \theta = -2$

Find the slope of the tangent line to the graph of the polar equation at the point corresponding to value of  $\theta$

$\frac{\sqrt{3}}{3}$

(51)  $r = 2 \cos \theta$ ,  $\theta = \frac{\pi}{3}$

(53)  $r = 4(1 - \sin \theta)$ ,  $\theta = 0$

$\frac{1}{\ln 2}$  (59)  $r = 2^{\theta}$ ,  $\theta = \pi$

$$m = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

**9.4**

Find the area of the region bounded by the graph of the polar equation

$\frac{3\pi}{2}$  (3)  $r = 1 - \cos \theta$

(19) Outside  $r = 2 + 2 \cos \theta$ , inside  $r = 3$   $2\pi + \frac{9}{2}\sqrt{3}$

(22)  $r = 1 - \sin \theta$ ,  $r = 3 \sin \theta$

(23) Inside both  $r = \sin \theta$ ,  $r = \sqrt{3} \cos \theta$   
 $\frac{5\pi}{24} - \frac{1}{4}\sqrt{3}$



Find the length of the curve

(29)  $r = e^{-\theta}$  from  $\theta = 0$  to  $\theta = 2\pi$   $\sqrt{2} (1 - e^{-2\pi})$

(30)  $r = 2^{\theta}$   $\theta = 0$  to  $\theta = \pi$   $L = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

Find the area of the surface generated by revolving the graph of the equation about the polar axis

$\frac{128\pi}{5}$  (35)  $r = 2 + 2\cos\theta$

$4\pi^2 a^2$  (37)  $r = 2a\sin\theta$

$$S = \int_{\theta_1}^{\theta_2} 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

$ds$