بسم الله الرحمن الرحيم

STAT 109

BIOSTATISTICS

Prepared by
Abdulrahman Alfaifi
King Saud University
Department of Statistics and Operation Research
alfaifi@ksu.edu.sa
@AlfaifiStat
A Alfaifi

ملاحظة : ليس بالضرورة ان تكون المذكرة شاملة للمقرر

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Chapter 6 Estimation and Confidence Interval

Estimation and Confidence Interval

Single Mean	Two Means
$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ σ known	$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ σ_1 and σ_2 known
$\bar{X} \pm t_{1-\frac{\alpha}{2},(n-1)} \frac{S}{\sqrt{n}}$ σ unknown	$(\bar{X}_1 - \bar{X}_2) \pm t_{1-\frac{\alpha}{2},\ (n_1+n_2-2)}\ S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ σ_1 and σ_2 unknown
Single Proportion	**Two Proportions**
For large sample size ($n \geq 30,\ np > 5,\ nq > 5$) $\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$	For large sample size ($n_1 \geq 30,\ n_1p_1 > 5,\ n_1q_1 > 5$) ($n_2 \geq 30,\ n_2p_2 > 5,\ n_2q_2 > 5$) $(\hat{p}_1 - \hat{p}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

$$\bar{X} \pm \underbrace{\left(\overbrace{Z_{1-\frac{\alpha}{2}}}^{\text{Reliability coefficeint}} \frac{\sigma}{\sqrt{n}} \right)}_{\substack{\text{margin of error} \\ \text{precision of the estimate}}}$$

Question 1:

Suppose we are interested in making some statistical inference about the mean μ, of a normal population with standard deviation $\sigma = 2$. Suppose that a random sample of size $n = 49$ from this population gave a sample mean $\bar{X} = 4.5$.

a. Find the upper limit of 95% of the confident interval for μ

$\sigma = 2 \quad \bar{X} = 4.5 \quad n = 49$

$$95\% \rightarrow \alpha = 0.05 \qquad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 4.5 + \left(1.96 \times \frac{2}{7}\right) = 5.06$$

b. Find the lower limit of 95% of the confident interval for μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 4.5 - \left(1.96 \times \frac{2}{7}\right) = 3.94$$

Question 2:

A researcher wants to estimate the mean of a life span a certain bulb. Suppose that the distribution is normal with standard deviation 5 hours. Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.

$\sigma = 5 \quad , \quad \bar{X} = 390 \quad , \quad n = 49$

a. find $Z_{0.975}$:

$$Z_{0.975} = 1.96$$

b. find a point estimate for μ

$$E(\bar{X}) = \hat{\mu} = \bar{X} = 390$$

c. Find the upper limit of 95% of the confident interval for μ

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 390 + \left(1.96 \times \frac{5}{\sqrt{49}}\right) = 391.4$$

d. Find the lower limit of 95% of the confident interval for μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}\right) = 390 - \left(1.96 \times \frac{5}{\sqrt{49}}\right) = 388.6$$

Question 3:

A sample of 16 college students were asked about time they spent doing their homework. It was found that the average to be 4.5 hours. Assuming normal population with standard deviation 0.5 hours.

$\sigma = 0.5 \quad \bar{X} = 4.5 \quad n = 16$

1. The point estimate for μ is:

A	0 hours	B	10 hours	C	0.5 hours	D	4.5 hours

2. The standard error of $\overline{X}$ is:

$$S.E(\overline{X}) = \frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{16}}$$

A	0.125 hours	B	0.266 hours	C	0.206 hours	D	0.245hours

3. The correct formula for calculating 100 $(1-\alpha)$% confidence interval for μ is:

A	$\overline{X} \pm t_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$	B	$\overline{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$	C	$\overline{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$	D	$\overline{X} \pm t_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$

4. The upper limit of 95% confidence interval for μ is:

A	4.745	B	4.531	C	4.832	D	4.891

5. The lower limit of 95% confidence interval for μ is:

A	5.531	B	7.469	C	3.632	D	4.255

6. The length of the 95% confidence interval for μ is:

$$\text{Length} = 4.745 - 4.255 = 0.49$$

A	4.74	B	0.49	C	0.83	D	0.89

<u>Question 4:</u>
Let us consider a hypothetical study on the height of women in their adulthood. A sample of 24 women is drown from a normal distribution with population mean μ and variance σ^2. The sample mean and variance of height of the selected women are 151 cm and 18.65 cm^2 respectively. Using given data, we want to constract a 99% confidentce interval for the mean height of the adult women in the populatopn from which the sample was drown randomly.

$$\bar{X} = 151 \quad ; \quad n = 24 \quad ; \quad S^2 = 18.65 \Longrightarrow S = 4.32$$

a. Point estimate for μ

$$\hat{\mu} = \bar{X} = 151$$

b. Find the upper limit of 99% of the confident interval for μ

$$\bar{X} + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S}{\sqrt{n}}\right)$$

$$= 151 + \left(2.807 \times \frac{4.32}{\sqrt{24}}\right) = 153.4753$$

$$99\% \rightarrow \alpha = 0.01$$

$$t_{1-\frac{\alpha}{2},n-1} = t_{1-\frac{0.01}{2},24-1}$$

$$= t_{0.995,23} = 2.807$$

c. Find the lower limit of 99% of the confident interval for μ

$$\bar{X} - \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S}{\sqrt{n}}\right)$$

$$= 151 - \left(2.807 \times \frac{4.32}{\sqrt{24}}\right) = 148.5247$$

Estimation and Confidence Interval: Two Means

1- $(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$

2- $(\bar{X}_1 - \bar{X}_2) \pm \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2}\, Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

Question 5:

The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kg. A sample of 20 pieces of the thread has an average tensile strength of 72.8 kg. Another type of thread (type II) is approximately followed normal distribution with standard deviation 6.8 kg. A sample of 25 pieces of the thread has an average tensile strength pf 64.4 kg. then for 98% confidence interval of the difference in tensile strength means between type I and type II, we have:

$$\text{Theard 1}: \ n_1 = 20, \bar{X}_1 = 72.8, \sigma_1 = 6.8$$
$$\text{Thread 2}: \ n_2 = 25, \bar{X}_2 = 64.4, \sigma_2 = 6.8$$

$$98\% \rightarrow \alpha = 0.02 \rightarrow \quad Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.325$$

$$(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$$
$$(72.8 - 64.4) \pm \left(2.325 \times \sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}} \right)$$

$$(3.657, 13.143)$$

(1): The lower limit $= 3.657$
(2): The upper limit $= 13.143$

Question 6:

	First sample	Second sample
Sample size (n)	12	14
Sample mean ($\bar{X}$)	10.5	10
Sample variance (S^2)	4	5

1. Estimate the difference $\mu_1 - \mu_2$:

$$\hat{\mu}_1 - \hat{\mu}_2 = \bar{X}_1 - \bar{X}_2 = 10.5 - 10 = 0.5$$

2. Find the pooled standard deviation estimator Sp :

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2} = \frac{4(11) + 5(13)}{24} = 4.54 \implies \boxed{Sp = 2.13}$$

3. The upper limit of 95% confidence interval for μ is:

$$95\% \rightarrow \alpha = 0.05 \rightarrow t_{1-\frac{\alpha}{2}, n_1+n_2-2} = t_{0.975, 24} = 2.064,$$

$$(\bar{X}_1 - \bar{X}_2) + \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$(0.5) + \left(2.064 \times 2.13\sqrt{\frac{1}{12} + \frac{1}{14}}\right) = 2.23$$

4. The lower limit of 95% confidence interval for μ is:

$$(\bar{X}_1 - \bar{X}_2) - \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$(0.5) - \left(2.064 \times 2.13\sqrt{\frac{1}{12} + \frac{1}{14}}\right) = -1.23$$

Question 7:

A researcher was interested in comparing the mean score of female students μ_1, with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	Male
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\overline{X}_1 = 82.63$	$\overline{X}_2 = 80.04$
Variance	$S_1^2 = 15.05$	$S_2^2 = 20.79$

1. The point estimate of $\mu_1 - \mu_2$ is:

A	2.63	B	-2.37	C	2.59	D	0.59

2. The estimate of the pooled variance S_p^2 is:

A	17.994	B	18.494	C	17.794	D	18.094

3. The upper limit of the 95% confidence interval for $\mu_1 - \mu_2$ is :

A	26.717	B	7.525	C	7.153	D	8.2

4. The lower limit of the 95% confidence interval for $\mu_1 - \mu_2$ is :

A	-21.54	B	-2.345	C	-3.02	D	-1.973

Estimation and Confidence Interval: Single Proportion

For large sample size ($n \geq 30,\ np > 5,\ nq > 5$)

$$* \text{ Point estimate for } P \text{ is: } \frac{x}{n}$$

$$* \text{ Interval estimate for } P \text{ is: } \hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$$

Question 7:

A random sample of 200 students from a certain school showed that 15 student smoke. Let p be the proportion of smokers in the school.

1. Find a point estimate for p.

$$n = 200 \quad \& \quad x = 15$$

$$\hat{p} = \frac{x}{n} = \frac{15}{200} = 0.075 \rightarrow \hat{q} = 0.925$$

2. Find 95% confidence interval for p.

$$95\% \rightarrow \alpha = 0.05 \quad \rightarrow \quad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right) = 0.075 \pm \left(1.96 \times \sqrt{\frac{0.075 \times 0.925}{200}} \right)$$

The 95% confidence interval is: $(0.038,\ 0.112)$

Question 8:

A researcher's group has perfected a new treatment of a disease which they claim is very efficient. As evidence, they say that they have used the new treatment on 50 patients with the disease and cured 25 of them. To calculate a 95% confidence interval for the proportion of the cured.

1. The point estimate of p is equal to:

A	0.25	B	0.50	C	0.01	D	0.33

2. The reliability coefficient $\left(z_{1-\frac{\alpha}{2}}\right)$ is equal is:

A	1.96	B	1.645	C	2.02	D	1.35

3. The 95% confidence interval is equal to:

A	(0.1114,0.3886)	B	(0.3837,0.6163)	C	(0.1614,0.6386)	D	(0.3614,0.6386)

Estimation and Confidence Interval: Two Proportions

For large sample size $(n_1 \geq 30,\ n_1p_1 > 5,\ n_1q_1 > 5)$
$(n_2 \geq 30,\ n_2p_2 > 5,\ n_2q_2 > 5)$

$$* \textit{ Point estimate for } P_1 - P_2 = \hat{p}_1 - \hat{p}_2 = \frac{x_1}{n_1} - \frac{x_2}{n_2}$$

$$* \textit{ Interval estimate for } P_1 - P_2 \textit{ is: } (\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \right)$$

Question 9:

A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smoker in school "A" and let p_2 be the proportion of smoker in school "B".

1. Find a point estimate for $P_1 - P_2$.

$$n_1 = 100, x_1 = 15 \rightarrow \hat{p}_1 = \frac{15}{100} = \boxed{0.15} \Rightarrow \hat{q}_1 = 1 - 0.15 = \boxed{0.85}$$

$$n_2 = 200, x_2 = 20 \rightarrow \hat{p}_2 = \frac{20}{200} = \boxed{0.10} \Rightarrow \hat{q}_2 = 1 - 0.10 = \boxed{0.90}$$

$$\boxed{\hat{p}_1 - \hat{p}_2 = 0.15 - 0.1 = 0.05}$$

2. Find 95% confidence interval for $P_1 - P_2$.

$$95\% \rightarrow \alpha = 0.05 \quad \rightarrow \quad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$(\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \right)$$

$$= (0.05) \quad \pm \left(1.96 \times \sqrt{\frac{(0.15)(0.85)}{100} + \frac{(0.1)(0.9)}{200}} \right)$$

$$= 0.05 \pm \left(1.96 \times \sqrt{0.001725}\right)$$

The 95% confidence interval is: $(-0.031,\ 0.131)$

Question 10:

a first sample of 100 store customers, 43 used a MasterCard. In a second sample of 100 store customers, 58 used a Visa card. To find the 95% confidence interval for difference in the proportion $(P_1 - P_2)$ of people who use each type of credit card?

1. The value of α is:

A	0.95	B	0.50	C	0.05	D	0.025

2. The upper limit of 95% confidence interval for the proportion difference is:

$$n_1 = 100 \ , \ x_1 = 43 \ \rightarrow \hat{p}_1 = \frac{43}{100} = \boxed{0.43} \ \Longrightarrow \hat{q}_1 = 1 - 0.43 = \boxed{0.57}$$

$$n_2 = 100 \ , \ x_2 = 58 \ \rightarrow \hat{p}_2 = \frac{58}{100} = \boxed{0.58} \ \Longrightarrow \hat{q}_2 = 1 - 0.58 = \boxed{0.42}$$

$$(\hat{p}_1 - \hat{p}_2) + \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.43 - 0.58) + \left(1.96 \times \sqrt{\frac{(0.43)(0.57)}{100} + \frac{(0.58)(0.42)}{100}}\right) = -0.013$$

3. The lower limit of 95% confidence interval for the proportion difference is:

$$(\hat{p}_1 - \hat{p}_2) - \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}\right)$$

$$= (0.43 - 0.58) - \left(1.96 \times \sqrt{\frac{(0.43)(0.57)}{100} + \frac{(0.58)(0.42)}{100}}\right) = -0.287$$

Question from previous midterms and finals:

- In procedure of construction $(1-\alpha)100\%$ confidence interval for the population mean (μ) of a normal population with a known standard deviation (σ) based on a random sample of size n.

1. The width of $(1-\alpha)100\%$ confidence interval for (μ) is:

A	$2\, Z_{1-\alpha} \frac{\sigma^2}{n}$	B	$2\, Z_{1-\alpha} \frac{\sigma}{\sqrt{n}}$	C	$2\, Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$	D	$2\, Z_{1-\alpha} \frac{\sigma^2}{\sqrt{n}}$

2. For $n = 70$ and $\sigma = 4$ the width of a 95% confidence interval for (μ) is:

A	3.1458	B	1.5153	C	6.1601	D	1.8741

3. For $\overline{X} = 60$ and a 95% confidence interval for μ is $(57\ ,\ k)$, then the value of the upper confidence limit k is:

A	64.5	B	66	C	61.5	D	63

4. When comparing the width of the 95% confidence interval (C.I.) for μ with that of 90% C.I., we found that:

	95% C.I. is shorter	B	95% C.I. is wider	C	They have the same width	D	We can't decide

5. When the sample size n increase, the width of the C.I. will:

A	Decrease	B	Increase	C	Not be change	D	We can't decide

6. The most typical form of a calculated confidence interval is:

A	Point estimate $\pm$ standard error
B	Population parameter $\pm$ margin of error
C	Population parameter $\pm$ standard error
D	Point estimate $\pm$ margin of error

7. Confidence intervals are useful when trying to estimate .... parameter:

A	Sample	B	Statistics	C	Population	D	None of these

8. The following C.I. is obtained for a population proportion $(0.505, 0.545)$, then the margin of error equals (let $\hat{p} = 0.525$)

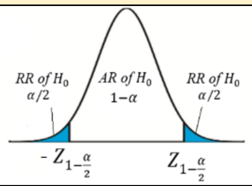
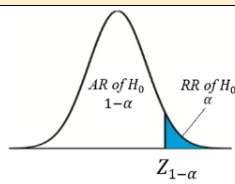
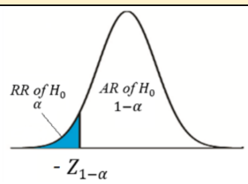
A	0.01	B	0.04	C	0.03	D	0.02

Chapter 7 Hypotheses Testing

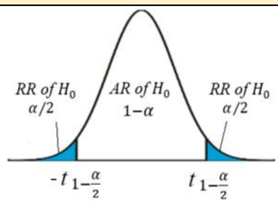
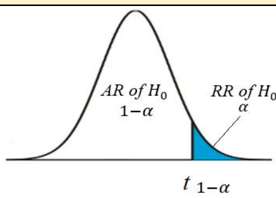
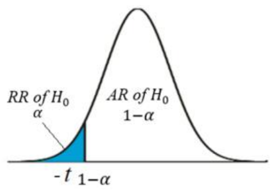
Hypotheses Testing

1-Single Mean

(if σ known):

Hypothesis Null H_0 Alternative (Research) H_A	$H_0: \mu = \mu_o$ $H_A: \mu \neq \mu_o$	$H_0: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_0: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
Test Statistics (TS)	$Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$		
Rejection Region (RR) of H_0 Acceptance Region (AR) of H_0			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ or $Z > Z_{1-(\alpha/2)}$ Two sides test	$Z > Z_{1-\alpha}$ One side test	$Z < -Z_{1-\alpha}$ One side test

(if σ unknown):

Hypothesis Null H_0 Alternative (Research) H_A	$H_0: \mu = \mu_o$ $H_A: \mu \neq \mu_o$	$H_0: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_0: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
Test Statistics (TS)	$t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$		
Rejection Region (RR) of H_0 Acceptance Region (AR) of H_0			
Decision	We reject H_0 at the significance level α if		
	$t < -t_{1-(\alpha/2)}$ or $t > t_{1-(\alpha/2)}$ Two sides test	$t > t_{1-\alpha}$ One side test	$t < -t_{1-\alpha}$ One side test

Question 1:

Suppose that we are interested in estimating the true average time in seconds it takes an adult to open a new type of tamper-resistant aspirin bottle. It is known that the population standard deviation is $\sigma = 5.71$seconds. A random sample of 40 adults gave a mean of 20.6 seconds. Let μ be the population mean, then, to test if the meanμ is 21 seconds at level of significant 0.05 ($H_0: \mu = 21$ vs $H_A: \mu \neq 21$) then:

(1) The value of the test statistic is:

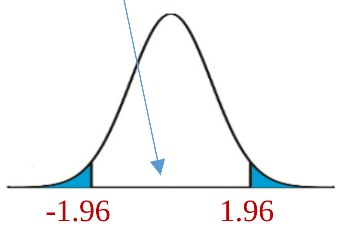
$$\boxed{\sigma = 5.71 \quad n = 40 \quad \bar{X} = 20.6}$$

$$Z = \frac{\bar{X} - \mu_o}{\sigma/\sqrt{n}} = \frac{20.6 - 21}{5.71/\sqrt{40}} = \boxed{-0.443}$$

A	0.443	B	-0.012	C	-0.443	D	0.012

(2) The acceptance area is:

$$Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{0.975} = 1.96$$



A	$(-1.96, 1.96)$	B	$(1.96, \infty)$	C	$(-\infty, 1.96)$	D	$(-\infty, 1.645)$

(3) The decision is:

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

$$\boxed{P - value = 2 \times P(Z < -0.443) \quad = 2 \times 0.32997 \quad = 0.66 > 0.05} \leftarrow \alpha$$

or

$$\boxed{P - value = 2 \times P(Z > |-0.443|) = 2 \times P(Z > 0.443) = 0.66 > 0.05} \leftarrow \alpha$$

Question 2:

If the hemoglobin level of pregnant women (امرأه حامل) is normally distributed, and if the mean and standard deviation of a sample of 25 pregnant women were $\bar{X} = 13$ (g/dl), $s = 2$ (g/dl). Using $\alpha = 0.05$, to test if the average hemoglobin level for the pregnant women is greater than 10 (g/dl) [$H_0: \mu \leq 10$, $H_A: \mu > 10$].

$$s = 2 \;\;,\;\; n = 25 \;\;,\;\; \bar{X} = 13$$

1. The test statistic is:

A	$Z = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$	B	$Z = \frac{\bar{X}-10}{S/\sqrt{n}}$	C	$t = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$	D	$t = \frac{\bar{X}-10}{S/\sqrt{n}}$

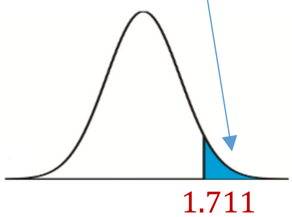
2. The value of the test statistic is:

$$t = \frac{\bar{X} - \mu_o}{S/\sqrt{n}} = \frac{13 - 10}{2/\sqrt{25}} = \boxed{7.5}$$

A	10	B	1.5	C	7.5	D	37.5

3. The rejection of H_0 is:

$$t_{1-\alpha,n-1} = t_{0.95,24} = 1.711$$



A	$Z < -1.645$	B	$Z > 1.645$	C	$t < -1.711$	D	$t > 1.711$

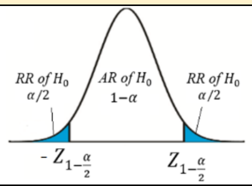
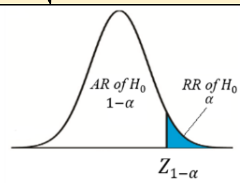
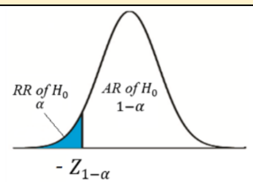
4. The decision is:

A	Reject H_0
B	Do not reject (Accept) H_0.
C	Accept both H_0 and H_A.
D	Reject both H_0 and H_A.

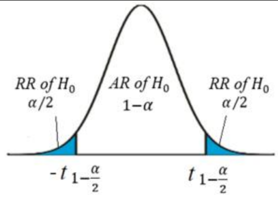
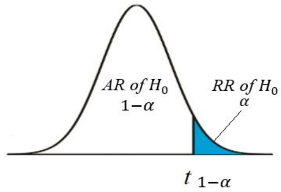
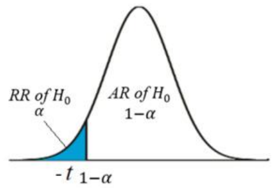
10 Feb 2023

2-Two Means:

$(\, if\ \sigma_1\ and\ \sigma_2\ known\,):$

Hypothesis *Null* H_0 *Alternative (Research)* H_A	$H_0: \mu_1 - \mu_2 = d$ $H_A: \mu_1 - \mu_2 \neq d$	$H_0: \mu_1 - \mu_2 \leq d$ $H_A: \mu_1 - \mu_2 > d$	$H_0: \mu_1 - \mu_2 \geq d$ $H_A: \mu_1 - \mu_2 < d$
Test Statistics *(TS)*	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$		
Rejection Region *(RR) of* H_0 *Acceptance Region* *(AR) of* H_0			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ *or* $Z > Z_{1-(\alpha/2)}$ *Two sides test*	$Z > Z_{1-\alpha}$ *One side test*	$Z < -Z_{1-\alpha}$ *One side test*

$(\, if\ \sigma_1\ and\ \sigma_2\ unknown\,):$

Hypothesis *Null* H_0 *Alternative (Research)* H_A	$H_0: \mu_1 - \mu_2 = d$ $H_A: \mu_1 - \mu_2 \neq d$	$H_0: \mu_1 - \mu_2 \leq d$ $H_A: \mu_1 - \mu_2 > d$	$H_0: \mu_1 - \mu_2 \geq d$ $H_A: \mu_1 - \mu_2 < d$
Test Statistics *(TS)*	$t = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$		
Rejection Region *(RR) of* H_0 *Acceptance Region* *(AR) of* H_0			
Decision	We reject H_0 at the significance level α if		
	$t < -t_{1-(\alpha/2)}$ *or* $t > t_{1-(\alpha/2)}$ *Two sides test*	$t > t_{1-\alpha}$ *One side test*	$t < -t_{1-\alpha}$ *One side test*

$$S_p^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

Question 3:

A standardized chemistry test was given to 50 girls and 75 boys. The girls made an average of 84, while the boys made an average grade of 82. Assume the population standard deviations are 6 and 8 for girls and boys respectively. To test the null hypothesis

$$H_0: \mu_1 - \mu_2 \leq 0 \;\; vs \; H_A: \mu_1 - \mu_2 > 0 \quad use \; \alpha = 0.05$$

(1) The standard error of $(\bar{X}_1 - \bar{X}_2)$ is:

$$girls: \quad n_1 = 50, \; \bar{X}_1 = 84, \; \sigma_1 = 6$$
$$boys: \quad n_2 = 75, \; \bar{X}_2 = 82, \; \sigma_2 = 8$$

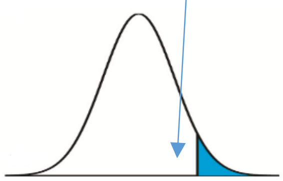
$$S.E(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{6^2}{50} + \frac{8^2}{75}} = 1.2543$$

(2) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(84-82)}{\sqrt{\frac{6^2}{50} + \frac{8^2}{75}}} = \frac{2}{1.2543} = \boxed{1.5945}$$

(3) The rejection region (RR) of H_0 is:

$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$



A	$(1.645, \infty)$	B	$(-\infty, -1.645)$	C	$(1.96, \infty)$	D	$(-\infty, -1.96)$

(4) The decision is:

A	Reject H_0
B	Do not reject (Accept) H_0.
C	Accept both H_0 and H_A.
D	Reject both H_0 and H_A.

$$P - value = P(Z > 1.59) = 1 - P(Z < 1.59) = 0.056 > 0.05$$

α

Question 4:

Cortisol level determinations were made on two samples of women at childbirth. Group 1 subjects underwent emergency cesarean section (عملية قيصرية) following induced labor. Group 2 subjects natural childbirth route following spontaneous labor (الولادة الطبيعية). The random sample sizes, mean cortisol levels, and standard deviations were $(n_1 = 40, \bar{x}_1 = 575, \sigma_1 = 70)$, $(n_2 = 44, \bar{x}_2 = 610, \sigma_2 = 80)$.

If we are interested to test if the mean Cortisol level of group 1 (μ_1) is less than that of group 2 (μ_2) at level 0.05 (or $H_0: \mu_1 \geq \mu_2$ vs $H_1: \mu_1 < \mu_2$), then:

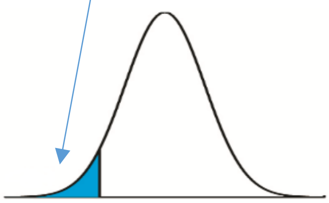
(1) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(575-610)}{\sqrt{\frac{70^2}{40} + \frac{80^2}{44}}} = \boxed{-2.138}$$

A	-1.326	B	-2.138	C	-2.576	D	-1.432

(2) Reject H_0 if :

$$Z_{1-\alpha} = Z_{0.95} = 1.645$$



-1.645

A	$Z > 1.645$	B	$T > 1.98$	C	$Z < -1.645$	D	$T < -1.98$

(3) The decision is:

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

$$P - value = P(Z < -2.138) = 0.01618 < 0.05$$

α

Question 5:

An experiment was conducted to compare time length (duration time in minutes) of two types of surgeries (A) and (B). 10 surgeries of type (A) and 8 surgeries of type (B) were performed. The data for both samples is shown below.

Surgery type	A	B
Sample size	10	8
Sample mean	14.2	12.8
Sample standard deviation	1.6	2.5

Assume that the two random samples were independently selected from two normal populations with equal variances. If μ_A and μ_B are the population means of the time length of surgeries of type (A) and type (B), then, to test if μ_A is greater than μ_B at level of significant 0.05 ($H_0: \mu_A \leq \mu_B$ vs $H_A: \mu_A > \mu_B$) then:

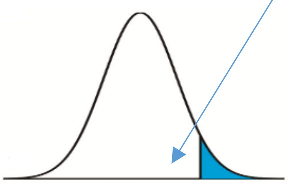
1. The value of the test statistic is:

$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{1.6^2(10-1)+2.5^2(8-1)}{10+8-2} = 4.174$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{(14.2-12.8)}{\sqrt{4.174}\sqrt{\frac{1}{10}+\frac{1}{8}}} = \boxed{1.44}$$

2. Reject H_0 if:

$$t_{1-\alpha,n_1+n_2-2} = t_{0.95,10+8-2} = t_{0.95,16} = 1.746$$



1.746

A	$Z > 1.645$	B	$Z < -1.645$	C	$T > 1.746$	D	$T < -1.746$

3. The decision is:

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

Question 6:

A researcher was interested in comparing the mean score of female students μ_1 ,with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	Male
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\overline{X}_1 = 82.63$	$\overline{X}_2 = 80.04$
Variance	$S_1^2 = 15.05$	$S_2^2 = 20.79$

Test that is there is a difference between the mean score of female students and the mean score of male students.

1. The hypotheses are:

A	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 \neq \mu_2$	B	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 < \mu_2$	C	$H_0: \mu_1 < \mu_2$ $H_A: \mu_1 > \mu_2$	D	$H_0: \mu_1 \leq \mu_2$ $H_A: \mu_1 > \mu_2$

2. The value of the test statistic is:

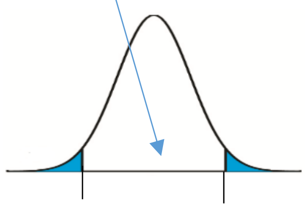
$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{15.05(4)+20.79(6)}{5+7-2} = 18.494$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{82.63-80.04}{\sqrt{18.494}\sqrt{\frac{1}{5}+\frac{1}{7}}} = \boxed{1.029}$$

A	1.3	B	1.029	C	0.46	D	0.93

3. The acceptance region (AR) of H_0 is:

$$t_{1-\frac{\alpha}{2}, n_1+n_2-2}$$
$$= t_{1-\frac{0.05}{2}, 5+7-2}$$
$$= t_{0.975,10}$$
$$= 2.228$$



-2.228 2.228

A	$(2.228, \infty)$	B	$(-\infty, -2.228)$	C	(-2.228, 2.228)	D	(-1.96, 1.96)

Question 7:

A nurse researcher wished to know if graduates of baccalaureate (بكالوريس) nursing program and graduate of associate degree (الزمالة) nursing program differ with respect to mean scores on personality inventory at α = 0.02. A sample of 50 associate degree graduates (sample A) and a sample of 60 baccalaureate graduates (sample B) yielded the following means and standard deviations:

$$\bar{X}_A = 88.12,\ \ S_A = 10.5,\ \ n_A = 50$$
$$\bar{X}_B = 83.25,\ \ S_B = 11.2,\ \ n_B = 60$$

1) The hypothesis is:

A	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 \neq \mu_2$	B	$H_0: \mu_1 \leq \mu_2$ $H_A: \mu_1 > \mu_2$	C	$H_0: \mu_1 \geq \mu_2$ $H_A: \mu_1 < \mu_2$	D	None of these

2) The test statistic is:

A	Z	B	t	C	F	D	None of these

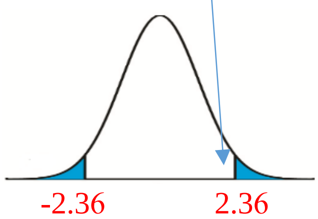
3) The computed value of the test statistic is:

$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{10.5^2(50-1)+11.2^2(60-1)}{50+60-2} = 118.55$$

$$t = \frac{(\bar{X}_1-\bar{X}_2)}{Sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{88.12-83.25}{\sqrt{118.55}\sqrt{\frac{1}{50}+\frac{1}{60}}} = \boxed{2.34}$$

4) The critical region (rejection area) is:

$$t_{1-\frac{\alpha}{2}, n_1+n_2-2}$$
$$= t_{1-\frac{0.02}{2},\ 50+60-2}$$
$$= t_{0.99,108} = 2.36$$

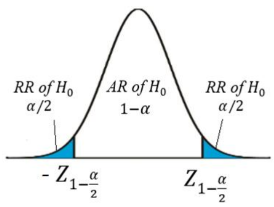
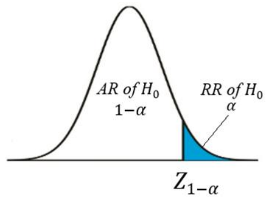
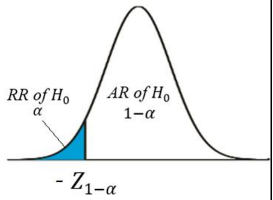


A	2.60 or − 2.60	B	2.06 or − 2.06	C	2.36 or − 2.36	D	2.58

5) Your decision is:

A	Reject H_0	B	Accept H_0	C	Accept H_A	D	No decision

3- Single proportion:

Hypothesis *Null* H_0 *Alternative (Research)* H_A	$H_0: p = p_o$ $H_A: p \neq p_o$	$H_0: p \leq p_o$ $H_A: p > p_o$	$H_0: p \geq p_o$ $H_A: p < p_o$
Test Statistics *(TS)*	$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} \sim N(0,1)$		
Rejection Region *(RR) of* H_0 *Acceptance Region* *(AR) of* H_0			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ *or* $Z > Z_{1-(\alpha/2)}$ *Two sides test*	$Z > Z_{1-\alpha}$ *One side test*	$Z < -Z_{1-\alpha}$ *One side test*

Question 8:

Toothpaste (معجون الأسنان) company claims that more than 75% of the dentists recommend their product to the patients. Suppose that 161 out of 200 dental patients reported receiving a recommendation for this toothpaste from their dentist. Do you suspect that the proportion is actually morethan75%. If we use 0.05 level of significance to test $H_0: P \le 0.75$, $H_A: P > 0.75$, then:

(1) The sample proportion $\hat{p}$ is:

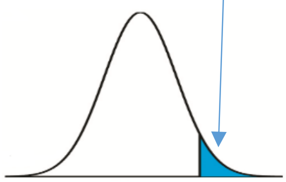
$$n = 200, \ \hat{p} = \frac{161}{200} = 0.8050$$

(2) The value of the test statistic is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.805 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{200}}} = \boxed{1.7963}$$

(3) The decision is:

$$\alpha = 0.05$$
$$Z_{1-\alpha} = Z_{0.95} = 1.645$$



A	Reject H_0 (agree with the claim)
B	Do not reject (Accept) H_0
C	Accept both H_0 and H_A
D	Reject both H_0 and H_A

$$\boxed{P-value = P(Z > 1.7963) = 1 - P(Z < 1.7963) = 1 - 0.96407 = 0.03593 < 0.05}$$

α

Question 9:

A researcher was interested in studying the obesity (السمنة) disease in a certain population. A random sample of 400 people was taken from this population. It was found that 152 people in this sample have the obesity disease. If p is the population proportion of people who are obese. Then, to test if p is greater than 0.34 at level 0.05 ($H_0: p \leq 0.34$ vs $H_A: p > 0.34$) then:

(1) The value of the test statistic is:

$$n = 400, \quad \hat{p} = \frac{152}{400} = 0.38$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.38 - 0.34}{\sqrt{\frac{0.34 \times 0.66}{400}}} = \boxed{1.69}$$

A	0.023	B	1.96	C	2.50	D	1.69

(2) The P-value is

$$P - value = P(Z > 1.69) = 1 - P(Z < 1.69) = 1 - 0.9545 = 0.0455$$

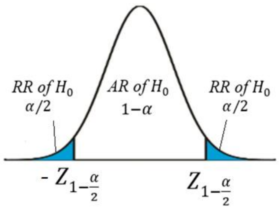
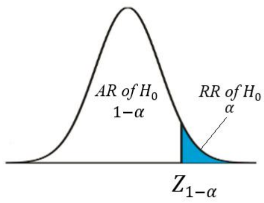
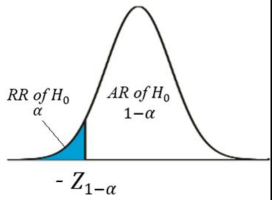
A	0.9545	B	0.0910	C	0.0455	D	1.909

(3) The decision is:

$$P - value = 0.0455 < 0.05$$

A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

4-Two proportions:

Hypothesis *Null* H_0 *Alternative (Research)* H_A	$H_0: p_1 - p_2 = d$ $H_A: p_1 - p_2 \neq d$	$H_0: p_1 - p_2 \leq d$ $H_A: p_1 - p_2 > d$	$H_0: p_1 - p_2 \geq d$ $H_A: p_1 - p_2 < d$
Test Statistics (TS)	$Z = \frac{(\hat{p}_1 - \hat{p}_2) - d}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} = \frac{(\hat{p}_1 - \hat{p}_2) - d}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$		
Rejection Region (RR) of H_0 *Acceptance Region (AR) of* H_0			
Decision	We reject H_0 at the significance level α if		
	$Z < -Z_{1-(\alpha/2)}$ *or* $Z > Z_{1-(\alpha/2)}$ *Two sides test*	$Z > Z_{1-\alpha}$ *One side test*	$Z < -Z_{1-\alpha}$ *One side test*

Question 10:

In a first sample of 200 men, 130 said they used seat belts and a second sample of 300 women, 150 said they used seat belts. To test the claim that men are more safety-conscious than women ($H_0: p_1 - p_2 \leq 0, H_1: p_1 - p_2 > 0$), at 0.05 level of significant:

(1) The value of the test statistic is:

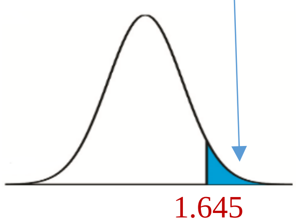
$$n_1 = 200,\ \ \hat{p}_1 = \frac{130}{200} = 0.65 \qquad n_2 = 300,\ \ \hat{p}_2 = \frac{150}{300} = 0.5$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{130+150}{200+300} = 0.56$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.65-0.5)}{\sqrt{(0.56)(0.44)\left(\frac{1}{200} + \frac{1}{300}\right)}} = \boxed{3.31}$$

(2) The decision is:

$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$



A	Reject H_0
B	Do not reject (Accept) H_0
C	Accept both H_0 and H_A
D	Reject both H_0 and H_A

$$P - value = P(Z > 3.31) = 1 - P(Z < 3.31) = 1 - 0.99953 = 0.00047 < 0.05$$

α

Question 11:

In a study of diabetes, the following results were obtained from samples of males and females between the ages of 20 and 75. Male sample size is 300 of whom 129 are diabetes patients, and female sample size is 200 of whom 50 are diabetes patients. If P_M, P_F are the diabetes proportions in both populations and $\hat{p}_M, \hat{p}_F$ are the sample proportions, then:
A researcher claims that the Proportion of diabetes patients is found to be more in males than in female $(H_0\colon\ P_M - P_F \leq 0 \text{ vs } H_A\colon P_M - P_F > 0)$. Do you agree with his claim, take $\alpha = 0.10$

$$n_m = 300 \quad , \quad x_m = 129 \quad \Longrightarrow \hat{p}_1 = \frac{129}{300} = 0.43$$

$$n_f = 200 \quad , \quad x_f = 50 \quad \Longrightarrow \hat{p}_2 = \frac{50}{200} = 0.25$$

(1) The pooled proportion is:

$$\bar{p} = \frac{x_m + x_f}{n_m + n_f} = \frac{129 + 50}{300 + 200} = 0.358$$

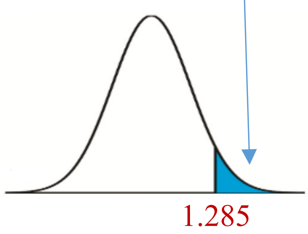
A	0.43	B	0.18	C	0.358	D	0.68

(2) The value of the test statistic is:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.43 - 0.25)}{\sqrt{(0.358)(1 - 0.358)\left(\frac{1}{300} + \frac{1}{200}\right)}} = \boxed{4.11}$$

(3) The decision is:

$$Z_{1-\alpha} = Z_{1-0.10} = Z_{0.90} = 1.285$$

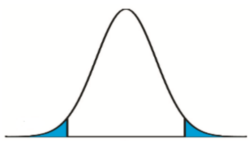
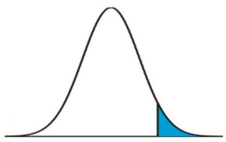
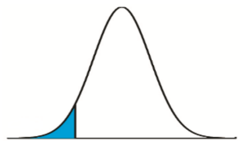


A	Agree with the claim (Reject H_0)
B	do not agree with the claim
C	Can't say

$$P - value = P(Z > 4.11) = 1 - P(Z < 4.11) = 1 - 1 = 0 < 0.10$$

α

- *P – value:*

Hypothesis	$H_0: \mu = \mu_o$ $H_A: \mu \neq \mu_o$	$H_0: \mu \leq \mu_o$ $H_A: \mu > \mu_o$	$H_0: \mu \geq \mu_o$ $H_A: \mu < \mu_o$
RR			
P-value	$2 \times P(Z > \lvert TS \rvert)$	$P(Z > TS)$	$P(Z < TS)$

$2 \times P(Z > TS)$ *If* $TS > 0$	$2 \times P(Z < TS)$ *If* $TS < 0$

	population normal or not normal n large ($n \geq 30$)		population normal n small ($n < 30$)	
	σ known	σ unknown	σ known	σ unknown
Testing	$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$	$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$	$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$

- **Two Samples Test for Paired Observation**

<u>Question 1:</u>

The following contains the calcium levels of eleven test subjects at zero hours and three hours after taking a multi-vitamin containing calcium.

Pair	0 hour (X_i)	3 hours (Y_i)	Difference $D_i = X_i - Y_i$
1	17.0	17.0	0.0
2	13.2	12.9	0.3
3	35.3	35.4	-0.1
4	13.6	13.2	0.4
5	32.7	32.5	0.2
6	18.4	18.1	0.3
7	22.5	22.5	0.0
8	26.8	26.7	0.1
9	15.1	15.0	0.1

The sample mean and sample standard deviation of the differences D are 0.144 and 0.167, respectively. To test whether the data provide sufficient evidence to indicate a difference in mean calcium levels ($H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$) with $\alpha = 0.10$ we have: $\boxed{\bar{D} = 0.144 \;,\; S_d = 0.167 \;,\; n = 9}$

[1]. The reliability coefficient (the tabulated value) is:

$$t_{1-\frac{\alpha}{2},n-1} = t_{1-\frac{0.1}{2},9-1} = t_{0.95,8} = \boxed{1.860}$$

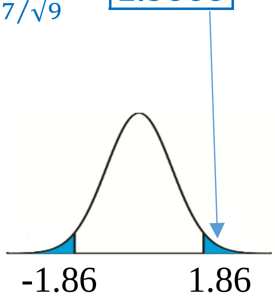
[2]. The value of the test statistic is:

$$\boxed{\begin{matrix} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{matrix}} \Longrightarrow \boxed{\begin{matrix} H_0: \mu_1 - \mu_2 = 0 \\ H_1: \mu_1 - \mu_2 \neq 0 \end{matrix}} \Longrightarrow \boxed{\begin{matrix} H_0: \mu_D = 0 \\ H_1: \mu_D \neq 0 \end{matrix}}$$

$$T = \frac{\bar{D} - \mu_D}{S_d/\sqrt{n}} = \frac{0.144 - 0}{0.167/\sqrt{9}} = \boxed{2.5868}$$

[3]. The decision is:

We reject H_0



Question 2:

Scientists and engineers frequently wish to compare two different techniques for measuring or determining the value of a variable. Reports the accompanying data on amount of milk ingested by each of 14 randomly selected infants.

Pair	DD method (X_i)	TW method (Y_i)	Difference $D_i = X_i - Y_i$
1	1509	1498	11
2	1418	1254	164
3	1561	1336	225
4	1556	1565	-9
5	2169	2000	169
6	1760	1318	442
7	1098	1410	-312
8	1198	1129	69
9	1479	1342	137
10	1281	1124	157
11	1414	1468	-54
12	1954	1604	350
13	2174	1722	452
14	2058	1518	540

1. The sample mean of the differences $\bar{D}$ is:

$$\bar{D} = \frac{11+164+225-9+169+442-312+\cdots+540}{14} = 167.21$$

A	167.21	B	0.71	C	0.61	D	0.31

2. The sample standard deviation of the differences S_D is:

$$S_D = \sqrt{\frac{\sum(D_i-\bar{D})^2}{n-1}} = 228.21$$

A	3.15	B	−0.71	C	71.53	D	228.21

3. The reliability coefficient to construct 90% confidence interval for the true average difference between intake values measured by the two methods μ_D is:

$$\text{The reliability coefficient} = t_{1-\frac{\alpha}{2},n-1} = t_{0.95,13} = 1.771$$

A	1.96	B	1.771	C	2.58	D	1.372

4. The 90% lower limit for μ_D is:

$$= \bar{D} - \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 167.21 - \left(1.771 \times \frac{228.12}{\sqrt{14}}\right) = 59.19$$

A	24.92	B	22.55	C	59.19	D	44.96

5. The 90% upper limit for μ_D is:

$$= \bar{D} + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 167.21 + \left(1.771 \times \frac{228.12}{\sqrt{14}}\right) = 275.23$$

A	224.92	B	322.55	C	275.23	D	24.96

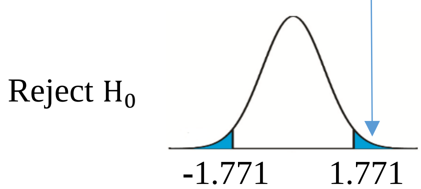
To test $H_0: \mu_D = 0$ versus $H_A: \mu_D \neq 0$, $\alpha = 0.10$ as a level of significance we have:

6. The value of the test statistic is:

$$T = \frac{\bar{D} - \mu_D}{S_d/\sqrt{n}} = \frac{167.21 - 0}{228.12/\sqrt{14}} = \boxed{2.74}$$

A	2.74	B	−0.7135	C	−7.153	D	−0.3157

7. The decision is:



A	Reject H_0	B	Accept H_0	C	No decision	D	None of these

10 Feb 2023

Question 3:

In a study of a surgical procedure used to decrease the amount of food that person can eat. A sample of 10 persons measures their weights before and after one year of the surgery, we obtain the following data:

Before surgery (X)	148	154	107	119	102	137	122	140	140	117
After surgery (Y)	78	133	80	70	70	63	81	60	85	120
$D_i = X_i - Y_i$	70	21	27	49	32	74	41	80	55	-3

We assume that the data comes from normal distribution.
For 90% confidence interval for μ_D, where μ_D is the difference in the average weight before and after surgery.

1. The sample mean of the differences $\overline{D}$ is:

$$\overline{D} = \frac{70+21+27+49+32+74+41+80+55-3}{10} = 44.6$$

2. The sample standard deviation of the differences S_D is:

$$S_D = \sqrt{\frac{\sum(D_i - \overline{D})^2}{n-1}} = 26.2$$

3. The 90% upper limit of the confidence interval for μ_D is:

$$t_{1-\frac{\alpha}{2},n-1} = t_{0.95,9} = 1.833$$

$$= \overline{D} + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 44.6 + \left(1.833 \times \frac{26.2}{\sqrt{10}}\right) = 59.79$$

4. To test $H_0: \mu_D \geq 43$ versus $H_A: \mu_D < 43$, with $\alpha = 0.10$ as a level of significance, the value of the test statistic is:

$$T = \frac{\overline{D} - \mu_D}{S_d/\sqrt{n}} = \frac{44.6-43}{26.2/\sqrt{10}} = \boxed{0.19}$$

5. The decision is:

$$-t_{1-\alpha,n-1} = -t_{0.90,9} = -1.383 \Longrightarrow \boxed{0.19 \notin RR: (-\infty, -1.383)}$$

A	Reject H_0	B	Do not reject H_0	C	No decision	D	None of these

Questions 4:

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water.

The Data is given below:

	zinc concentration in Bottom water	zinc concentration in Surface water	Difference
1	0.43	0.415	0.015
2	0.266	0.238	0.028
3	0.567	0.39	0.177
4	0.531	0.41	0.121
5	0.707	0.605	0.102
6	0.716	0.609	0.107
7	0.651	0.632	0.019
8	0.589	0.523	0.066
9	0.469	0.411	0.058
10	0.723	0.612	0.111

Note that the mean and the standard deviation of the difference are given respectively by $\overline{D}$ = 0.0804 and S_D = 0.0523 We want to determine the 95 % confidence interval for $\mu_1 - \mu_2$, where μ_1 and μ_2 represent the true mean zinc concentration in Bottom water and surface water respectively. Assume the distribution of the differences to be approximately normal.

1. The 95% lower limit for $\mu_1 - \mu_2$ equals to:

A	0.02628	B	0.13452	C	0.04299	D	0.11781

2. The 95% upper limit for $\mu_1 - \mu_2$ equals to:

A	0.02628	B	0.13452	C	0.04299	D	0.11781

	Estimation	Testing
Single mean	$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ σ known	$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$ σ known
	$\bar{X} \pm t_{1-\frac{\alpha}{2},(n-1)} \frac{S}{\sqrt{n}}$ σ unknown	$T = \frac{\bar{X}-\mu_0}{S/\sqrt{n}}$ σ unknown
Two means	$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ σ_1 and σ_2 known	$Z = \frac{(\bar{X}_1-\bar{X}_2)-d}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$ σ_1 and σ_2 known
	$(\bar{X}_1 - \bar{X}_2) \pm t_{1-\frac{\alpha}{2},\ (n_1+n_2-2)}\ S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ σ_1 and σ_2 unknown	$T = \frac{(\bar{X}_1-\bar{X}_2)-d}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ σ_1 and σ_2 unknown
Single proportion	$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$	$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}}$
Two proportions	$(\hat{p}_1 - \hat{p}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$	$Z = \frac{(\hat{p}_1-\hat{p}_2)-d}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

$$S_p^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}$$

	H_0 is true	H_0 is false
Accepting H_0	Correct decision ✓	Type II error (β)
Rejecting H_0	Type I error (α)	Correct decision ✓

Type I error = Rejecting H_0 when H_0 is true P(Type I error) = P(Rejecting $H_0 \mid H_0$ is true) = α	Type II error = Accepting H_0 when H_0 is false P(Type II error) = P(Accepting $H_0 \mid H_0$ is false) = β

Question from previous midterms and finals:

Q1. In the procedure of testing the statistical hypotheses H_0 against H_A using a significance level α

1. The type I error occur if we:

A	Rejecting H_0 when H_0 is true	B	Rejecting H_0 when H_0 is false	C	Accepting H_0 when H_0 is true	D	Accepting H_0 when H_0 is false

2. The probability of type I error is:

A	β	B	α	C	$1-\beta$	D	$1-\alpha$

3. When we use P-value method, we reject H_0 if

A	P- value > α	B	P- value < α	C	P- value < β	D	P- value > β

4. To determine the rejection region for H_0, it depends on:

A	α and H_A	B	H_0	C	α and H_0	D	β

5. Which one is an example of two-tailed test:

A	$H_A: \mu = 0$	B	$H_A: \mu \neq 0$	C	$H_A: \mu < 0$	D	$H_A: \mu > 0$

Q2. To compare the mean times spent waiting for a heart transplant for two age groups, you randomly select several people in each age group who have had a heart transplant. The result is shown below. Assume both population is are normally distributed with equal variance.

Sample statistics for heart transplant		
Age group	18-34	35-49
Mean	171 days	169 days
Standard deviation	8.5 days	11.5 days
Sample size	20	17

Do this data provide sufficient evident to indicate a difference among the population means at $\alpha = 0.05$

1. The alternative hypothesis is:

A	$H_A: \mu_1 \neq \mu_2$	B	$H_A: \mu_1 \leq \mu_2$	C	$H_A: \mu_1 > \mu_2$	D	$H_A: \mu_1 = \mu_2$

2. The pooled estimator of the common variance S_p^2 is:

A	9935.82	B	105.5214	C	10.4429	D	99.6786

3. The appropriate test statistics is:

A	$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$	B	$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	C	$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	D	$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$

4. The 95% confidence interval for the different in mean times spent waiting for heart transplant for the two age groups:

A	$(-3.548, 7.565)$	B	$(-0.1306, 4.1306)$	C	$(-4.6862, 8.6862)$	D	$(-4.8519, 8.8519)$

5. Base on the 95% C.I. in the above question, it can be concluded that:

A	$\bar{X}_1 = \bar{X}_2$	B	$\mu_1 \neq \mu_2$	C	$\mu_1 = \mu_2$	D	None of these

Chapter 6 and Chapter 7

		chapter 7	chapter 6
	Assumptions	Test statistics (T.S.) $T.S. = \frac{\text{Estimator} - \text{hypothesized parameter}}{(\text{standard error})}$	Confidence Interval (C.I.) Estimator $\pm$ (reliability cofficent)(standard error) Estimator $\pm$ *margin error*
Single population mean (μ)	• Normal + σ^2 Known • Non-normal + σ^2 Known + $n \geq 30$ (large)	$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$	$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
	• Normal + σ^2 Unknown	$T = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$	$\bar{X} \pm t_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$ $df = n - 1$
	• Non-normal + σ^2 Unknown + $n \geq 30$ (large)	$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$	
Difference between Two Population Means $(\mu_1 - \mu_2)$	• Normal + σ_1^2 and σ_2^2 Known • Non-normal + σ_1^2 and σ_2^2 Known + $n_1, n_2 \geq 30$ (large)	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
	• Normal + $\sigma_1^2 = \sigma_2^2 = \sigma^2$ Unknown , equal	$T = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_0}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}}$ $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$	$(\bar{X}_1 - \bar{X}_2) \pm t_{1-\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$ $df = n_1 + n_2 - 2$ $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$
	Related population	$T = \frac{\bar{D}}{S_D / \sqrt{n}}$	$\bar{D} \pm t_{1-\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}$, $df = n - 1$

	Assumptions	chapter 7	chapter 6
Population Proportion (P)	$n \geq 30$ (large)	$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ $q_0 = 1 - p_0$ $\hat{p} = \frac{x}{n}$	$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\hat{p} = \frac{x}{n}$
Difference between Two Population Proportion $P_1 - P_2$	$n_1 \geq 30$ (large) $n_2 \geq 30$ (large)	$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$ $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$	$(\hat{P}_1 - \hat{P}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}$ $\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{p}_2 = \frac{x_2}{n_2}$

	Sample	Population
Size	n	N
Mean	$\bar{x}$	μ
Variance	S^2	σ^2
Standard deviation	S	σ
Proportion	$\hat{p}$	P

Chapter 6

Parameters : measure computed from population

μ σ^2 σ P

Statistical Inferences

Estimation

تقدير بقيمة حقيقة للمعلمة المجهولة

Estimating the actual value of unknown parameters

Hypothesis test

Ch 7

Point Estimate

single value used to estimate the corresponding population parameter.

	Population parameters	Point estimation
Mean	μ	$\bar{x}$
Variance	σ^2	S^2
Standard deviation	σ	S
Proportion	P	$\hat{p}$
Difference between Two Population Means	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$
Difference Between Two Population Proportions	$P_1 - P_2$	$\hat{P}_1 - \hat{P}_2$

Confidence Interval =Interval estimate

consists of two numerical values defining a range of values that most likely includes the parameter

Parameter ∈ (L , U)

L < Parameter < U

L: Lower limit

U: Upper limit

(1-α): Confidence coefficient, Confidence Level

الطريقة العامة لكتابة الفترة التقديرية :

Estimator $\pm$ (reliability cofficent)(standard error)

Estimator $\pm$ *margin error*

Confidence Interval of μ

Interval Estimation of μ

- Normal + σ^2 Known
- Non-normal + σ^2 Known + $n \geq 30$ (large)

$$\bar{X} \pm Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$\bar{X}$ → Estimator

$Z_{1-\frac{\alpha}{2}}$ → Reliability Coefficient

$\frac{\sigma}{\sqrt{n}}$ → Standard Error of $\bar{x}$

Reliability Coefficient × Standard Error of $\bar{x}$ → margin error

Precision of the estimate

Upper limit :

$$\bar{X} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Lower limit :

$$\bar{X} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

- Normal + σ^2 Unknown

$$\bar{X} \pm t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$\bar{X}$ → Estimator

$t_{1-\frac{\alpha}{2}}$ → Reliability Coefficient

$\frac{s}{\sqrt{n}}$ → Standard Error of $\bar{x}$

Reliability Coefficient × Standard Error of $\bar{x}$ → margin error

Precision of the estimate

df = v = n-1

Upper limit :

$$\bar{X} + t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

Lower limit :

$$\bar{X} - t_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

Confidence Interval for the Difference between Two Population Means $\mu_1 - \mu_2$

Interval Estimate $\mu_1 - \mu_2$

- Normal + σ_1^2 and σ_2^2 Known
- Non-normal + σ_1^2 and σ_2^2 Known + $n_1, n_2 \geq 30$ (large)

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- $(\bar{X}_1 - \bar{X}_2)$: Estimator
- $Z_{1-\frac{\alpha}{2}}$: Reliability Coefficient
- $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$: Standard Error of $\bar{X}_1 - \bar{X}_2$

Reliability Coefficient × Standard Error = margin error

Precision of the estimate

Upper limit :

$$(\bar{X}_1 - \bar{X}_2) + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Lower limit :

$$(\bar{X}_1 - \bar{X}_2) - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- Normal + $\sigma_1^2 = \sigma_2^2 = \sigma^2$ Unknown

$$(\bar{X}_1 - \bar{X}_2) \pm t_{1-\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

- $(\bar{X}_1 - \bar{X}_2)$: Estimator
- $t_{1-\frac{\alpha}{2}}$: Reliability Coefficient
- $\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$: Standard Error of $\bar{X}_1 - \bar{X}_2$

Reliability Coefficient × Standard Error = margin error

Precision of the estimate

df = v = $n_1 + n_2 - 2$

Pooled estimate of the common variance σ^2:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Upper limit :

$$(\bar{X}_1 - \bar{X}_2) + t_{1-\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

Lower limit :

$$(\bar{X}_1 - \bar{X}_2) - t_{1-\frac{\alpha}{2}} \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

Confidence Interval for a Population Proportion (P)

Interval Estimate (P)

- $n \geq 30$ (large) , ($\hat{q} = 1 - \hat{p}$)

$$\hat{p} \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Estimator: $\hat{p}$

Reliability Coefficient: $Z_{1-\frac{\alpha}{2}}$

Standard Error: $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

margin error (Reliability Coefficient × Standard Error)

Precision of the estimate

Upper limit :

$$\hat{p} + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Lower limit :

$$\hat{p} - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence Interval for the Difference between Two Population Proportion $P_1 - P_2$

Interval Estimate $P_1 - P_2$

$n_1 \geq 30$ (large) $\quad\quad (\widehat{q_1} = 1 - \widehat{P_1})$

$n_2 \geq 30$ (large) $\quad\quad (\hat{q}_2 = 1 - \hat{P}_2)$

$$(\hat{P}_1 - \hat{P}_2) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}$$

Estimator

Reliability

Coefficient

Standard Error

margin error

Precision of the estimate

Upper limit :

$$(\hat{P}_1 - \hat{P}_2) + Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}$$

Lower limit :

$$(\hat{P}_1 - \hat{P}_2) - Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}$$

$\hat{P}_1 = \frac{x_1}{n_1} = \frac{\text{جزء}}{\text{كل}}$ $\quad\quad$ $\hat{P}_2 = \frac{x_2}{n_2} = \frac{\text{جزء}}{\text{كل}}$

How to know if σ known or unknown :

σ known

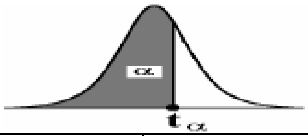
- **The population variance ………(σ^2)**
- **The population standard deviation…….. (σ)**
- **It is normal distribution with variance …..(σ^2)**
- **It is normal distribution with standard deviation …..(σ)**

σ unknown: (Use S instead)

- **Sample variance….. (S^2)**
- **Sample standard deviation ….. (S)**
- **If we have a sample of size ..(n),has mean ($\overline{X}$) with variance …(S^2)**
- **If we have a sample of size ..(n),has mean ($\overline{X}$) with standard deviation …(S)**

****

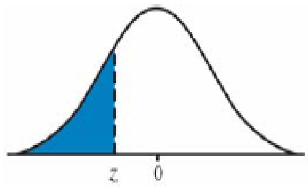
Critical Values of the t-distribution (t_α)



ν=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.2901	1.6602	1.9840	2.3642	2.6259
120	1.2886	1.6577	1.9799	2.3578	2.6174
140	1.2876	1.6558	1.9771	2.3533	2.6114
160	1.2869	1.6544	1.9749	2.3499	2.6069
180	1.2863	1.6534	1.9732	2.3472	2.6034
200	1.2858	1.6525	1.9719	2.3451	2.6006
∞	1.282	1.645	1.960	2.326	2.576

Standard Normal Table

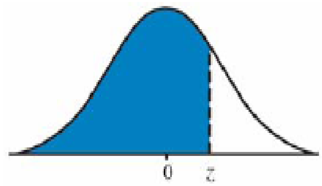
Areas Under the Standard Normal Curve



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	**-3.50**
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	**-3.40**
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	**-3.30**
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	**-3.20**
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	**-3.10**
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	**-3.00**
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	**-2.90**
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	**-2.80**
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	**-2.70**
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	**-2.60**
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	**-2.50**
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	**-2.40**
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	**-2.30**
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	**-2.20**
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	**-2.10**
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	**-2.00**
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	**-1.90**
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	**-1.80**
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	**-1.70**
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	**-1.60**
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	**-1.50**
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	**-1.40**
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	**-1.30**
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	**-1.20**
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	**-1.10**
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	**-1.00**
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	**-0.90**
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	**-0.80**
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	**-0.70**
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	**-0.60**
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	**-0.50**
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	**-0.40**
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	**-0.30**
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	**-0.20**
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	**-0.10**
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	**-0.00**

Standard Normal Table (continued)

Areas Under the Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	**0.00**
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	**0.10**
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	**0.20**
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	**0.30**
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	**0.40**
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	**0.50**
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	**0.60**
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	**0.70**
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	**0.80**
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	**0.90**
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	**1.00**
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	**1.10**
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	**1.20**
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	**1.30**
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	**1.40**
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	**1.50**
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	**1.60**
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	**1.70**
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	**1.80**
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	**1.90**
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	**2.00**
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	**2.10**
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	**2.20**
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	**2.30**
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	**2.40**
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	**2.50**
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	**2.60**
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	**2.70**
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	**2.80**
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	**2.90**
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.9990	**3.00**
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	**3.10**
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	**3.20**
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	**3.30**
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	**3.40**
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	**3.50**