# ***Random Variables:***

- $0 \leq P(X = x) \leq 1$
- $\sum P(X = x) = 1$
- $E(X) = \sum x\, P(X = x)$

***Question 1:*** ***Given the following discrete distribution:***

| *x* | *-1* | *0* | *1* | *2* | *3* | *4* |
|---|---|---|---|---|---|---|
| *P(X=x)* | *0.15* | *0.30* | *M* | *0.15* | *0.10* | *0.10* |

1. *The value of M is equal to*

$$\boxed{M = 1 - (0.15 + 0.30 + 0.15 + 0.10 + 0.10) = 1 - 0.80 = 0.20}$$

| | | | |
|---|---|---|---|
| *(A) 0.20* | *(B) 0.0* | *(C) 0.10* | *(D) 0.25* |

2. *P(X ≤0.5) =* $\boxed{0.15 + 0.30 = 0.45}$

| | | | |
|---|---|---|---|
| *(A) 0.0* | *(B) 0.50* | *(C) 0.45* | *(D) 1.0* |

3. *P(X=0) =*

| | | | |
|---|---|---|---|
| *(A) 0* | *(B)0.30* | *(C) 0.80* | *(D) 1.0* |

4. *The expected (mean ) value E[X] is equal to*

$$\boxed{E(X) = (-1 \times 0.15) + (0 \times 0.30) + (1 \times 0.20) + (2 \times 0.15) + (3 \times 0.10) + (4 \times 0.10) = 1.05}$$

| | | | |
|---|---|---|---|
| *(A) 0.0* | *(B) 1.35* | *(C) 1.05* | *(D) 1.20* |

***Question 2:***

***Let X be a discrete random variable with probability mass function:***

$f(x) = cx$ ; $x = 1,2,3,4$ ***What is the value of c ?***

| $x$ | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| $P(X = x)$ | $c$ | $2c$ | $3c$ | $4c$ |

$$c + 2c + 3c + 4c = 1 \quad \Rightarrow c = \frac{1}{10}$$

*Then probability mass function:*

| $x$ | *1* | *2* | *3* | *4* |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{10}$ | $\frac{2}{10}$ | $\frac{3}{10}$ | $\frac{4}{10}$ |

## ***Question 2:***

*The average length of stay in a hospital is useful for planning purposes. Suppose that the following is the probability distribution of the length of stay (X) in a hospital after a minor operation:*

| *Length of stay (days)* | *3* | *4* | *5* | *6* |
|---|---|---|---|---|
| *Probability* | *0.4* | *0.2* | *0.1* | *k* |

*(1) The value of k is*

$$\boxed{k = 1 - (0.4 + 0.2 + 0.1) = 1 - 0.7 = 0.3}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *A* | *0.0* | *B* | *1* | *C* | *0.3* | *D* | *6* |

*(2)* $P(X < 0) =$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *A* | *0.0* | *B* | *0.5* | *C* | *1* | *D* | *0.75* |

*(3)* $P(0 < X \le 5) =$

$$\boxed{0.4 + 0.2 + 0.1 = 0.7}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *A* | *0.32* | *B* | *0.5* | *C* | *0.7* | *D* | *0.1* |

*(4)* $P(X \le 5.5) =$

$$\boxed{0.4 + 0.2 + 0.1 = 0.7}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *A* | *0.7* | *B* | *0.6* | *C* | *0* | *D* | *0.1* |

*(5) The probability that the patient will stay at most 4 days in a hospital after a minor operation is equal to*

$$\boxed{0.4 + 0.2 = 0.6}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *A* | *0.4* | *B* | *0.1* | *C* | *0.2* | *D* | *0.6* |

*(6) The average length of stay in a hospital is*

$$\boxed{E(X) = (3 \times 0.4) + (4 \times 0.2) + (5 \times 0.1) + (6 \times 0.3) = 4.3}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *A* | *2.3* | *B* | *0.7* | *C* | *1* | *D* | *4.3* |

# ***Binomial Distribution:***

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \quad ; \quad x = 0,1 \ldots.., n$$

$$* \, E(X) = np \qquad * \, Var(X) = npq$$

$$q = 1 - p$$

## ***Question 1:***

*Suppose that 25% of the people in a certain large population have high blood pressure. A Sample of 7 people is selected at random from this population. Let X be the number of people in the sample who have high blood pressure, follows a binomial distribution then*

***1)*** *The values of the parameters of the distribution are:*

$$\boxed{p = 0.25 \quad , \quad n = 7}$$

| | | | |
|---|---|---|---|
| *(A) 7, 0.75* | *(B) <u>7, 0.25</u>* | *(C) 0.25, 0.75* | *(D) 25, 7* |

***2)*** *The probability that we find exactly one person with high blood pressure, is:*

$$\boxed{P(X = 1) = \binom{7}{1} (0.25)^1 (0.75)^6 = 0.31146}$$

| | | | |
|---|---|---|---|
| *(A) <u>0.31146</u>* | *(B) 0.143* | *(C) 0.125* | *(D) 0.25* |

***3)*** *The probability that there will be at most one person with high blood pressure, is:*

$$\boxed{P(X \leq 1) = \binom{7}{0} (0.25)^0 (0.75)^7 + \binom{7}{1} (0.25)^1 (0.75)^6 = 0.4449}$$

| | | | |
|---|---|---|---|
| *(A) 0.311* | *(B) 0.25* | *(C) <u>0.4449</u>* | *(D) 0.5551* |

***4)*** *The probability that we find more than one person with high blood pressure, is:*

$$\boxed{P(X > 1) = 1 - P(X \leq 1) = 1 - 0.4449 = 0.5551}$$

| | | | |
|---|---|---|---|
| *(A) 0.689* | *(B) 0.857* | *(C) 0.4449* | *(D) <u>0.5551</u>* |

## ***Question 2:***

*In some population it was found that the percentage of adults who have hypertension is 24 percent. Suppose we select a simple random sample of* __*five*__ *adults from this population. Then the probability that the number of people who have hypertension in this sample, will be:*

$$\boxed{p = 0.24 \quad , \quad n = 5}$$

1. *Zero:*

$$\boxed{P(X = 0) = \binom{5}{0}(0.24)^0(0.76)^5 = 0.2536}$$

2. *Exactly one*

$$\boxed{P(X = 1) = \binom{5}{1}(0.24)^1(0.76)^4 = 0.4003}$$

3. *Between one and three, inclusive*

$$\boxed{P(1 \le X \le 3) = \binom{5}{1}(0.24)^1(0.76)^4 + \binom{5}{2}(0.24)^2(0.76)^3 + \binom{5}{3}(0.24)^3(0.76)^2 = 0.7330}$$

4. *Two or fewer (at most two):*

$$\boxed{P(X \le 2) = \binom{5}{0}(0.24)^0(0.76)^5 + \binom{5}{1}(0.24)^1(0.76)^4 + \binom{5}{2}(0.24)^2(0.76)^3 = 0.9067}$$

5. *Five:*

$$\boxed{P(X = 5) = \binom{5}{5}(0.24)^5(0.76)^0 = 0.0008}$$

6. *The mean of the number of people who have hypertension is equal to:*

$$\boxed{E(X) = np = 5 \times 0.24 = 1.2}$$

7. *The variance of the number of people who have hypertension is equal to:*

$$\boxed{Var(X) = npq = 5 \times 0.24 \times 0.76 = 0.912}$$

## ***Poisson distribution:***

$$P(X = x) = \frac{e^{-\lambda}\ \lambda^{x}}{x!} \quad ; \quad x = 0,1,2,\ldots$$

$$E(X) = Var(X) = \lambda$$

### ***Question 1:***

*The number of serious cases coming to a hospital during a night follows a Poisson distribution with an average of 10 persons per night, then:*

1) *The probability that 12 serious cases coming in the next night, is:*

$$\boxed{\begin{gathered}\lambda_{one\ night} = 10 \\ P(X = 12) = \frac{e^{-10}\ 10^{12}}{12!} = 0.09478\end{gathered}}$$

| | | | |
|---|---|---|---|
| *(A) <u>0.09478</u>* | *(B) 0.3456* | *(C) 12* | *(D) 0.5* |

2) *The average number of serious cases in a two nights period is:*

$$\boxed{\lambda_{two\ nights} = 20}$$

| | | | |
|---|---|---|---|
| *(A) 10.5* | *(B) <u>20</u>* | *(C) 0.2065* | *(D) 0.0867* |

3) *The probability that 20 serious cases coming in next two nights is:*

$$\boxed{\begin{gathered}\lambda_{two\ nights} = 20 \\ P(X = 20) = \frac{e^{-20}\ 20^{20}}{20!} = 0.0888\end{gathered}}$$

| | | | |
|---|---|---|---|
| *(A) 10.5* | *(B) 0.7694* | *(C) 0.20* | *(D) <u>0.0888</u>* |

***Question 2:***

*Given the mean number of serious accidents per year in a large factory is five. If the number of accidents follows a Poisson distribution, then the probability that in the next year there will be:*

1. *Exactly seven accidents:*

$$\lambda_{one\ year} = 5$$

$$P(X = 7) = \frac{e^{-5}\ 5^7}{7!} = 0.1044$$

2. *No accidents*

$$P(X = 0) = \frac{e^{-5}\ 5^0}{0!} = 0.0067$$

3. *one or more accidents*

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X = 0) \\ &= 1 - 0.0067 = 0.9933 \end{aligned}$$

4. *The expected number (mean) of serious accidents in the next two years is equal to*

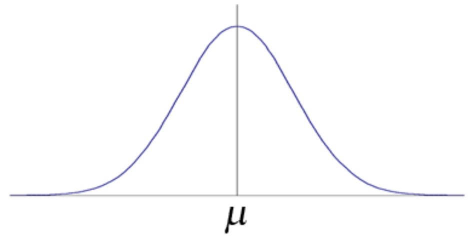
$$\lambda_{two\ years} = 10$$

5. *The probability that in the next two years there will be three accidents*

$$\lambda_{two\ years} = 10$$

$$P(X = 3) = \frac{e^{-10}\ 10^3}{3!} = 0.0076$$

# ***The Normal Distribution:***




*Normal distribution* $X \sim N(\mu, \sigma^2)$

*Standard normal* $Z \sim N(0, 1)$

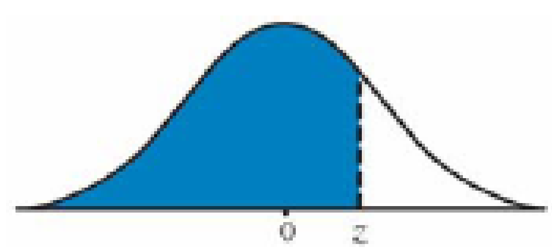
## ***Question 1:***

*Given the standard normal distribution, Z~N (0, 1), find:*

**1.** $P(Z < 1.43) = 0.92364$

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 |
|---|---|---|---|---|---|
| 0.00 | 0.50000 | 0.50399 | 0.50798 | [illegible] | 0.51595 |
| 0.10 | 0.53983 | 0.54380 | 0.54776 | [illegible] | 0.55567 |
| 0.20 | 0.57926 | 0.58317 | 0.58706 | [illegible] | 0.59483 |
| 0.30 | 0.61791 | 0.62172 | 0.62552 | [illegible] | 0.63307 |
| 0.40 | 0.65542 | 0.65910 | 0.66276 | [illegible] | 0.67003 |
| 0.50 | 0.69146 | 0.69497 | 0.69847 | [illegible] | 0.70540 |
| 0.60 | 0.72575 | 0.72907 | 0.73237 | [illegible] | 0.73891 |
| 0.70 | 0.75804 | 0.76115 | 0.76424 | [illegible] | 0.77035 |
| 0.80 | 0.78814 | 0.79103 | 0.79389 | [illegible] | 0.79955 |
| 0.90 | 0.81594 | 0.81859 | 0.82121 | [illegible] | 0.82639 |
| 1.00 | 0.84134 | 0.84375 | 0.84614 | [illegible] | 0.85083 |
| 1.10 | 0.86433 | 0.86650 | 0.86864 | [illegible] | 0.87286 |
| 1.20 | 0.88493 | 0.88686 | 0.88877 | [illegible] | 0.89251 |
| 1.30 | 0.90320 | 0.90490 | 0.90658 | [illegible] | 0.90988 |
| 1.40 | [illegible] | [illegible] | [illegible] | 0.92364 | 0.92507 |
| 1.50 | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 |
| 1.60 | 0.94520 | 0.94630 | 0.94738 | 0.94845 | 0.94950 |
| 1.70 | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 |
| 1.80 | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

$$P\left(Z < \boxed{\text{أطراف الجدول}}\right) = \boxed{\text{داخل الجدول}}$$




**2.** $P(Z > 1.67) = 1 - P(Z < 1.67) = 1 - 0.9525 = 0.0475$

3.

$$P(-2.16 < Z < -0.65)$$

$$= P(Z < -0.65) - P(Z < -2.16)$$

$$= \quad 0.2578 \quad - \quad 0.0154 \quad = 0.2424$$

## Question 2:

*Given the standard normal distribution, Z~N(0,1), find:*

1. $P(Z > 2.71) = 1 - P(Z < 2.71) = 1 - 0.9966 = 0.0034$

| | | | |
|---|---|---|---|
| (A) 0.7088 | (B)<u>0.0034</u> | (C) 0.3645 | (D) 0.1875 |

2. $P(-1.96 < Z < 1.96)$

$$= P(Z < 1.96) - P(Z < -1.96)$$
$$= \quad 0.9750 \quad - \quad 0.0250 \quad = 0.9500$$

| | | | |
|---|---|---|---|
| (A) 0.0746 | (B) 0.9950 | (C) <u>0.9500</u> | (D) 0.9750 |

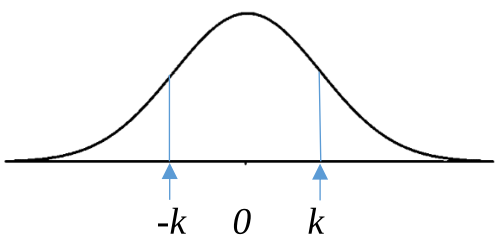
3. *If P(Z < a) = 0.9929, then the value of a =*

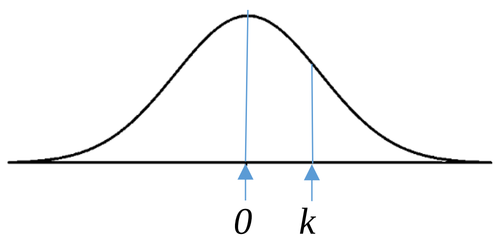
| | | | |
|---|---|---|---|
| (A) -2.54 | (B) 0 | (C) 1.64 | (D) <u>2.45</u> |

4. *P(Z=1.33)=*

| | | | |
|---|---|---|---|
| (A) 0.1220 | (B) 0.1660 | (C) 0.1550 | (D) <u>0.0</u> |

5. *If P(-k < Z < k)= 0.8132, then the value of k =*




$$\Longrightarrow 2 \times P(0 < Z < k) = 0.8132$$
$$\Longrightarrow P(0 < Z < k) = 0.4066$$
$$\Longrightarrow P(Z < k) - P(Z < 0) = 0.4066$$
$$\Longrightarrow P(Z < k) - \quad 0.5 \quad = 0.4066$$
$$\Longrightarrow P(Z < k) = 0.9066$$

| | | | |
|---|---|---|---|
| (A) 2.54 | (B) 2.31 | (C) <u>1.32</u> | (D) 0.5 |

**_Question 3:_**

*Given the standard normal distribution, then:*

1) $P(-1.1 < X < 1.1)$ *is:*

$$\Longrightarrow P(Z < 1.1) - P(Z < -1.1)$$

$$0.86433 - 0.11702 = 0.74731$$

| | | | |
|---|---|---|---|
| (A) 0.3254 | (B) 0.8691 | (C) <u>0.7473</u> | (D) 0.1475 |

2) $P(Z > -0.15)$ *is:*

$$P(Z > -0.15) = 1 - P(Z < -0.15) = 1 - 0.44038 = 0.55962$$

| | | | |
|---|---|---|---|
| ( A) <u>0.5596</u> | ( B) 0.9394 | ( C) 0.0606 | ( D) 0.4404 |

3) *The k value that has an area of 0.883 to its right, is:*

| Left | Right |
|---|---|
| < | > |

$$P(Z > k) = 0.883$$
$$1 - P(Z < k) = 0.883$$
$$P(Z < k) = 0.117$$

| | | | |
|---|---|---|---|
| (A) -0.811 | (B) 1.19 | (C) 0.811 | (D) -1.19 |

***Question:***

Q2. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Then,

1) the proportion of rings that will have inside diameter less than 12.05 centimeters is:

   (A) 0.0475 (B) 0.9525 (C) 0.7257 (D) 0.8413

2) the proportion of rings that will have inside diameter exceeding 11.97 centimeters is:

   (A) 0.0475 (B) 0.8413 (C) 0.1587 (D) 0.4514

3) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:

   (A) 0.905 (B) −0.905 (C) 0.4514 (D) 0.7257

*Solution:*

$$X \sim N(\mu, \sigma^2)$$

$$Z \sim N(12, 0.03^2)$$

*(1):*

$$P(X < 12.05) = P\left(Z < \frac{12.05 - \mu}{\sigma}\right)$$

$$= P\left(Z < \frac{12.05 - 12}{0.03}\right) = P(Z < 1.67) = 0.9525$$

*(2):*

$$P(X > 11.97) = P\left(Z > \frac{11.97 - \mu}{\sigma}\right)$$

$$= P\left(Z > \frac{11.97 - 12}{0.03}\right) = P(Z > -1)$$

$$= 1 - P(Z < -1) = 1 - 0.1587 = 0.8413$$

*(3):*

$$P(11.95 < X < 12.05) = P\left(\frac{11.95 - 12}{0.03} < Z < \frac{12.05 - 12}{0.03}\right)$$

$$= P(-1.67 < Z < 1.67)$$

$$= P(Z < 1.67) - P(Z < -1.67)$$

$$0.9525 - 0.0475 = 0.905$$

## ***Question:***

Q6. The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

(1) The percentage of fat persons with weights at most 110 kg is
(A) 0.09 % (B) 90.3 % (C) 99.82 % (D) 2.28 %

(2) The percentage of fat persons with weights more than 149 kg is
(A) 0.09 % (B) 0.99 % (C) 9.7 % (D) 99.82 %

(3) The weight x above which 86% of those persons will be
(A) 118.28 (B) 128.28 (C) 154.82 (D) 81.28

(4) The weight x below which 50% of those persons will be
(A) 101.18 (B) 128 (C) 154.82 (D) 81

*Solution:*

$$X \sim N(\mu, \sigma^2)$$

$$X \sim N(128, 9^2)$$

*(1):*

$$P(X \leq 110) = P\left(Z < \frac{110 - 128}{9}\right) = P(Z < -2) = 0.0228$$

*(2):*

$$P(X > 149) = P\left(Z > \frac{149 - 128}{9}\right)$$

$$= 1 - P(Z < 2.33) = 1 - 0.9901 = 0.0099$$

*(3):*

$$P(X > x) = 0.86 \Rightarrow P(X < x) = 0.14 \Rightarrow P\left(Z < \frac{x - 128}{9}\right) = 0.14$$

*by searching inside the table for 0.14, and transforming X to Z, we got:*

$$\frac{x - 128}{9} = -1.08 \Rightarrow x = 118.28$$

*(4):*

$P(X < x) = 0.5$, *by searching inside the table for 0.5, and transforming X to Z*

$$\frac{x - 128}{9} = 0 \Rightarrow x = 128$$

## ***Questions:***

Q8. If the random variable X has a normal distribution with the mean $\mu$ and the variance $\sigma^2$, then $P(X<\mu+2\sigma)$ equals to

(A) 0.8772 (B) 0.4772 (C) 0.5772 (D) 0.7772 (E) 0.9772

Q9. If the random variable X has a normal distribution with the mean $\mu$ and the variance 1, and if $P(X<3)=0.877$, then $\mu$ equals to

(A) 3.84 (B) 2.84 (C) 1.84 (D) 4.84 (E) 8.84

Q10. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 70 and the variance 25. If it is known that 33% of the student failed the exam, then the passing mark x is

(A) 67.8 (B) 60.8 (C) 57.8 (D) 50.8 (E) 70.8

*Solution of Q8*

$$P(X<\mu+2\sigma)=P\left(Z<\frac{(\mu+2\sigma)-\mu}{\sigma}\right)=P(Z<2)=0.9772$$

*Solution of Q9*

$$Given\ that\ \ \sigma=1$$

$$P(X<3)=0.877\Rightarrow P\left(Z<\frac{3-\mu}{1}\right)=0.877$$

$$3-\mu=1.16\Rightarrow\mu=1.84$$

*Solution of Q10*

$$X\sim N(70,25)$$

$$P(X<x)=0.33\Rightarrow P\left(Z<\frac{x-70}{5}\right)=0.33$$

*by searching inside the table for 0.33, and transforming X to Z, we got:*

$$\frac{x-70}{5}=-0.44\Rightarrow x=67.8$$

***Question:***

*A nurse supervisor has found that staff nurses, on the average, complete a certain task in 10 minutes. If the times required to complete the task are approximately normally distributed with a standard deviation of 3 minutes, then:* $X \sim N(10, 3^2)$

1) *The probability that a nurse will complete the task in less than 8 minutes is:*

$$P(X < 8) = P\left(Z < \frac{8-10}{3}\right) = P(Z < -0.67) = 0.2514$$

2) *The probability that a nurse will complete the task in more than 4 minutes is:*

$$P(X > 4) = 1 - P\left(Z < \frac{4-10}{3}\right) = 1 - P(Z < -2) = 1 - 0.0228$$

*If eight nurses were assigned the task, the expected number of them who will complete it within 8 minutes is approximately equal to:*

$$n \times P(0 < X < 8) = 8 \times P\left(\frac{0-10}{3} < Z < \frac{8-10}{3}\right)$$

$$= 8 \times P(-3.33 < Z < -0.67)$$

$$= 8 \times [P(Z < -0.67) - P(Z < -3.33)]$$

$$= 8 \times [0.2514 - 0.0004] = 2$$

3) *If a certain nurse completes the task within k minutes with probability 0.6293; then k equals approximately:*

$$P(0 < X < k) = 0.6293$$

$$\Rightarrow P\left(\frac{0-10}{3} < Z < \frac{8-k}{3}\right) = 0.6293$$

$$\Rightarrow P\left(-3.33 < Z < \frac{k-10}{3}\right) = 0.6293$$

$$\Rightarrow P\left(Z < \frac{k-10}{3}\right) - P(Z < -3.33) = 0.6293$$

$$\Rightarrow P\left(Z < \frac{k-10}{3}\right) - \quad 0.0004 \quad = 0.6293$$

$$\Rightarrow P\left(Z < \frac{k-10}{3}\right) = 0.6297$$

$$\Rightarrow \frac{k-10}{3} = 0.33 \Rightarrow k = 11$$

**Question:**

*Given the normally distributed random variable X with mean 491 and standard deviation 119,*

1) *If $P(X < k) = 0.9082$, the value of k is equal to*

| | | | |
|---|---|---|---|
| (A) 649.27 | (B) 390.58 | (C) 128.90 | (D) 132.65 |

2) *If $P(292 < X < M) = o.8607$, the value of M is equal to*

| | | | |
|---|---|---|---|
| (A) 766 | (B) 649 | (C) 108 | (D) 136 |

**Question:**

*The IQ (Intelligent Quotient) of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10, then:*

**1)** *The probability that an individual picked at random will have an IQ greater than 75 is:*

| | | | |
|---|---|---|---|
| (A) 0.9332 | (B) 0.8691 | (C) 0.7286 | (D) 0.0668 |

**2)** *The probability that an individual picked at random will have an IQ between 55 and 75 is:*

| | | | |
|---|---|---|---|
| ( A) 0.3085 | ( B) 0.6915 | ( C) 0.6247 | ( D) 0.9332 |

**3)** *If the probability that an individual picked at random will have an IQ less than $k$ is 0.1587. Then the value of $k$*

| | | | |
|---|---|---|---|
| (A) 50 | (B) 45 | (C) 51 | (D) 40 |