- ***Two Samples Test for Paired Observation***

Q1.The following contains the calcium levels of eleven test subjects at zero hours and three hours after taking a multi-vitamin containing calcium.

Pair	0 hour (X_i)	3 hours (Y_i)	Difference $D_i = X_i - Y_i$
1	17.0	17.0	0.0
2	13.2	12.9	0.3
3	35.3	35.4	-0.1
4	13.6	13.2	0.4
5	32.7	32.5	0.2
6	18.4	18.1	0.3
7	22.5	22.5	0.0
8	26.8	26.7	0.1
9	15.1	15.0	0.1

The sample mean and sample standard deviation of the differences D are 0.144 and 0.167, respectively. To test whether the data provide sufficient evidence to indicate a difference in mean calcium levels ($H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$) with $\alpha = 0.10$ we have: $\boxed{\overline{D} = 0.144 \ , \quad S_d = 0.167 \quad , \quad n = 9}$

[1]. the reliability coefficient (the tabulated value) is:

$$t_{1-\frac{\alpha}{2}, n-1} = t_{1-\frac{0.1}{2}, 9-1} = t_{0.95,8} = \boxed{1.860}$$

[2]. the value of the test statistic is:

$$\boxed{\begin{matrix} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{matrix}} \Longrightarrow \boxed{\begin{matrix} H_0: \mu_1 - \mu_2 = 0 \\ H_1: \mu_1 - \mu_2 \neq 0 \end{matrix}} \Longrightarrow \boxed{\begin{matrix} H_0: \mu_D = 0 \\ H_1: \mu_D \neq 0 \end{matrix}}$$

$$T = \frac{\overline{D} - \mu_D}{S_d / \sqrt{n}} = \frac{0.144 - 0}{0.167/\sqrt{9}} = \boxed{2.5868}$$

[3]. the decision is:

$$T = 2.5868 \notin AR: (-1.86\ , 1.86), then\ we\ \boxed{Reject\ H_0}$$

Q2. Scientists and engineers frequently wish to compare two different techniques for measuring or determining the value of a variable. Reports the accompanying data on amount of milk ingested by each of 14 randomly selected infants.

Pair	*DD method* (X_i)	*TW method* (Y_i)	*Difference* $D_i = X_i - Y_i$
1	1509	1498	11
2	1418	1254	164
3	1561	1336	225
4	1556	1565	-9
5	2169	2000	169
6	1760	1318	442
7	1098	1410	-312
8	1198	1129	69
9	1479	1342	137
10	1281	1124	157
11	1414	1468	-54
12	1954	1604	350
13	2174	1722	452
14	2058	1518	540

1. ***The sample mean of the differences*** $\overline{D}$ ***is:***

$$\overline{D} = \frac{11+164+225-9+169+442-312+\cdots+540}{14} = 167.21$$

(A) 167.21	(B) 0.71	(C) 0.61	(D) 0.31

2. ***The sample standard deviation of the differences*** S_D ***is:***

$$S_D = \sqrt{\frac{(D_i - \overline{D})^2}{n-1}} = 228.21$$

(A) 3.15	(B) -0.71	(C) 71.53	(D) 228.21

3. ***The reliability coefficient to construct 90% confidence interval for the true average difference between intake values measured by the two methods μ_D is:***

$$\text{The reliability coefficient} = t_{1-\frac{\alpha}{2},n-1} = t_{0.95,13} = 1.771$$

(A) 1.96	(B) 1.771	(C) 2.58	(D) 1.372

4. ***The 90% lower limit for μ_D is:***

$$= \bar{D} - \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 167.21 - \left(1.771 \times \frac{228.12}{\sqrt{14}}\right) = 59.19$$

(A)24.92	(B) 22.55	(C) 59.19	(D) 44.96

5. ***The 90% upper limit for μ_D is:***

$$= \bar{D} + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 167.21 + \left(1.771 \times \frac{228.12}{\sqrt{14}}\right) = 275.23$$

(A)224.92	(B) 322.55	(C) 275.23	(D) 24.96

To test $H_0: \mu_D = 0$ versus $H_A: \mu_D \neq 0$, $\alpha = 0.10$ as a level of significance we have:

6. ***The value of the test statistic is:***

$$T = \frac{\bar{D} - \mu_D}{S_d/\sqrt{n}} = \frac{167.21 - 0}{228.12/\sqrt{14}} = \boxed{2.74}$$

(A)2.74	(B) -0.7135	(C) -7.1530	(D) -0.3157

7. ***The decision is:***

$$2.74 \notin AR: (-1.771 \quad , \quad 1.771)$$

(A) Reject H_0 (B) Not reject H_0

Q3. In a study of a surgical procedure used to decrease the amount of food that person can eat. A sample of 10 persons measures their weights before and after one year of the surgery, we obtain the following data:

Before surgery (X)	*148*	*154*	*107*	*119*	*102*	*137*	*122*	*140*	*140*	*117*
After surgery (Y)	*78*	*133*	*80*	*70*	*70*	*63*	*81*	*60*	*85*	*120*
$D_i = X_i - Y_i$	*70*	*21*	*27*	*49*	*32*	*74*	*41*	*80*	*55*	*-3*

We assume that the data comes from normal distribution.

- ***For 90% confidence interval for μ_D, where μ_D is the difference in the average weight before and after surgery.***

1. ***The sample mean of the differences $\overline{D}$ is:***

$$\overline{D} = \frac{70+21+27+\cdots 55-3}{10} = 44.6$$

2. ***The sample standard deviation of the differences S_D is:***

$$S_D = \sqrt{\frac{(D_i - \overline{D})^2}{n-1}} = 26.2$$

3. ***The 90% upper limit of the confidence interval for μ_D is:***

$$t_{1-\frac{\alpha}{2},n-1} = t_{0.95,9} = 1.833$$

$$= \overline{D} \quad + \left(t_{1-\frac{\alpha}{2},n-1} \times \frac{S_D}{\sqrt{n}}\right)$$

$$= 44.6 + \left(1.833 \quad \times \frac{26.2}{\sqrt{10}}\right) = 59.38$$

- ***To test $H_0: \mu_D \geq 43$ versus $H_A: \mu_D < 43$, with $\alpha = 0.10$ as a level of significance, we have:***

4. ***The value of the test statistic is:***

$$T = \frac{\overline{D} - \mu_D}{S_d/\sqrt{n}} = \frac{44.6-43}{26.2/\sqrt{10}} = \boxed{0.19}$$

5. ***The decision is:***

$$-t_{1-\alpha,n-1} = -t_{0.90,9} = -1.383 \quad \Longrightarrow \quad \boxed{0.19 \notin RR: (-\infty \;\;,\;\; -1.383)}$$

(A) Reject H_0 *(B) Do not reject H_0* *(C) None of them*