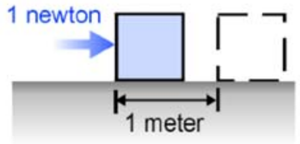
Work, energy and power

Work

In physics, work has a very specific meaning.

In physics, work represents a measurable change in a system, caused by a force.

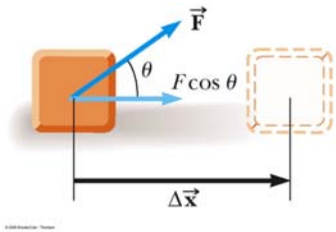
If you push a box with a force of one newton for a distance of one meter, you have done exactly one joule of work.



Definition of Work W

- The work, W, done by a constant force on an object is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement

$$W \equiv (F\cos\theta)\Delta x$$



- F is the magnitude of the force
- Δ x is the magnitude of the object's displacement
- θ is the angle between $\vec{F}$ and $\Delta\vec{x}$

Example: When Work is Zero

A man carries a bucket of water horizontally at constant velocity.

The force does no work on the bucket

Displacement is horizontal

Force is vertical

cos 90° = 0

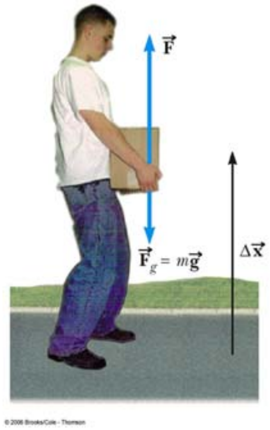
$$W \equiv (F\cos\theta)\Delta x$$

Example: Work Can Be Positive or Negative

Work is positive when lifting the box

Work would be negative if lowering the box.

The force would still be upward, but the displacement would be downward



Work and Dissipative Forces

Work can be done by friction

The energy lost to friction by an object goes into heating both the object and its environment

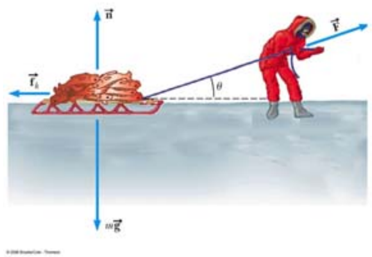
Some energy may be converted into sound

For now, the phrase “Work done by friction” will denote the effect of the friction processes on mechanical energy alone

Work and Force

- An Eskimo returning pulls a sled as shown. The total mass of the sled is 50.0 kg, and he exerts a force of 1.20×10^2 N on the sled by pulling on the rope. How much work does he do on the sled if θ = 30° and he pulls the sled 5.0 m ?

$$W = (F\cos\theta)\Delta x$$
$$= (1.20\times10^2\,N)(\cos 30^\circ)(5.0m)$$
$$= 5.2\times10^2\,J$$



Work Unit

- This gives no information about
  - the time it took for the displacement to occur
  - the velocity or acceleration of the object
- Work is a scalar quantity
- SI Unit

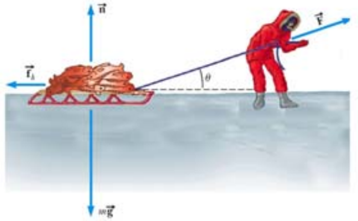
$$\frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2 = (F\cos\theta)\Delta x$$

  - Newton • meter = Joule
- N • m = J
- J = kg • m^2 / s^2 = (kg • m / s^2) • m

$$W \equiv (F\cos\theta)\Delta x$$

Work and Multiple Forces

- Suppose μ_k = 0.200, How much work done on the sled by friction, and the net work if θ = 30° and he pulls the sled 5.0 m ?



$$F_{net,y} = N - mg + F\sin\theta = 0$$
$$N = mg - F\sin\theta$$

$$\begin{aligned}W_{fric} &= (f_k\cos 180^\circ)\Delta x = -f_k\Delta x\\ &= -\mu_k N\Delta x = -\mu_k(mg - F\sin\theta)\Delta x\\ &= -(0.200)(50.0kg \cdot 9.8m/s^2\\ &\quad -1.2\times10^2 N\sin 30^\circ)(5.0m)\\ &= -4.3\times10^2 J\end{aligned}$$

$$\begin{aligned}W_{net} &= W_F + W_{fric} + W_N + W_g\\ &= 5.2\times10^2 J - 4.3\times10^2 J + 0 + 0\\ &= 90.0 J\end{aligned}$$

Work (force is parallel to distance

Force (N)

Work (joules) → **W = F . d**

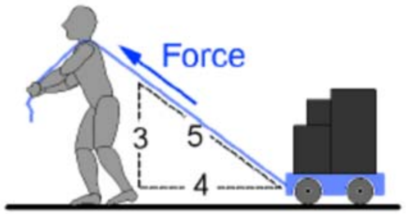
Distance (m)

Work (force at angle to distance)

Force (

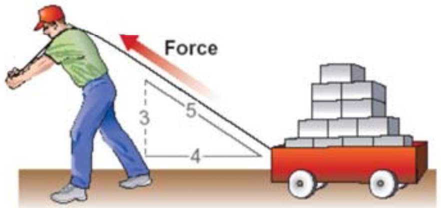
Work (joules → **W = Fd cos (q)** ← Angle

Distance (m)



Force at an Angle to the Distance

PROBLEM



ANALYSIS

SOLUTION

$$W = Fd \times \left(\frac{4}{5}\right) = Fd\cos 37^\circ$$

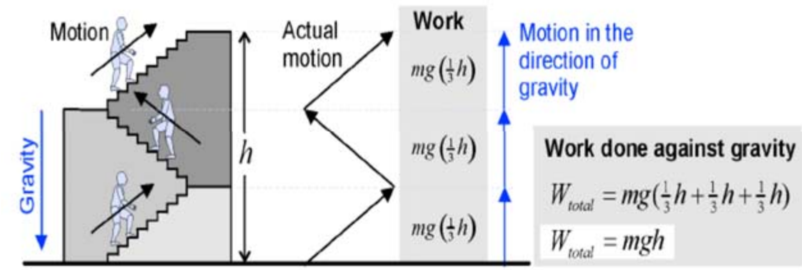
Work done against gravity

Mass (g)

Work (joules) → **$W = mgh$** ← Height object raised (m)

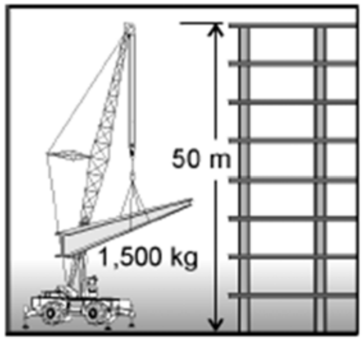
Gravity (m/sec²)

Why the path doesn't matter



Calculate work

A crane lifts a steel beam with a mass of 1,500 kg.
Calculate how much work is done against gravity if the beam is lifted 50 meters in the air.
How much time does it take to lift the beam if the motor of the crane can do 10,000 joules of work per second?



Work: + or -?

Work can be positive, negative, or zero. The sign of the work depends on the direction of the force relative to the displacement

Work positive: $W > 0$ if $90° > \theta > 0°$
Work negative: $W < 0$ if $180° > \theta > 90°$
Work zero: $W = 0$ if $\theta = 90°$
Work minimum if $\theta = 180°$
Work maximum if $\theta = 0°$

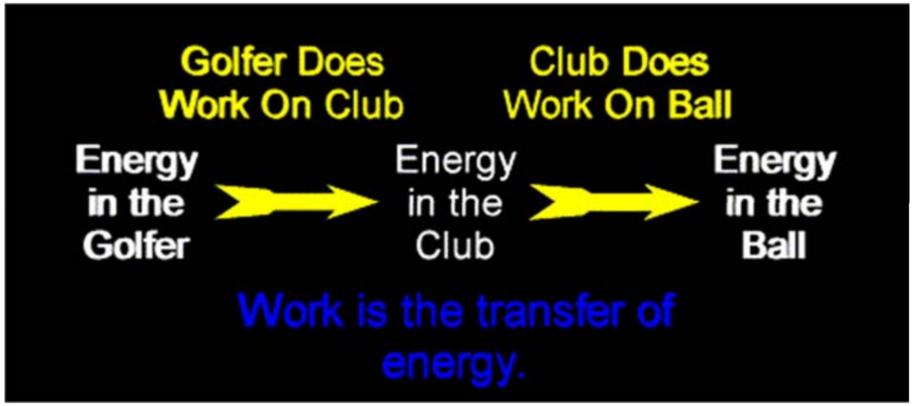
Work & Energy

Work is the ***transfer*** of ***energy***

Work ***is done on an object*** when you transfer energy to that object.

Example

a golfer uses a club and gets a stationary • golf ball moving when he or she hits the ball. The club does work on the golf ball as it strikes the ball. Energy leaves the club and enters the ball. This is a transfer of energy. Thus, we say that the club did work on the ball . And, before the ball was struck, the golfer did work on the club. The club was initially standing still, and the golfer got it moving when he or she swung the club.

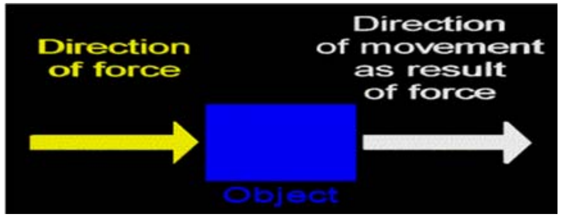


Work equation

$W = F.\ d\cos\theta$

Where F is the force, d is the displacement,θ the angle (theta) is defined as the angle between the force and displacement vector.

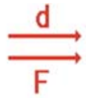
At first we will consider only forces that are aimed in the same direction as the displacement. For example, we will imagine an object being pushed horizontally to the right, and the object will be moving horizontally to the right as a result of this applied force



an idea of the meaning of theta angle, consider the following three scenarios:

Scenario A

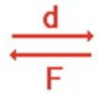
A force acts rightward upon an object as it is • displaced rightward. In such an instance, the force vector and the displacement vector are in the same direction. Thus, the angle between the F and d is 0 degrees.



$\Theta = 0$ degrees

scenario B

A force acts leftward upon an object which is displaced rightward. In such an instance, the force vector and displacement vector are in the opposite direction .thus , the angle between F and d is 180 degree



$\Theta = 180$ degrees

Scenario C

A force act upward on an object and it is • displaced rightward. In such an instance, the force vector and the displacement vector are at right angles to each other. Thus, the angle between F and d is 90 degrees.

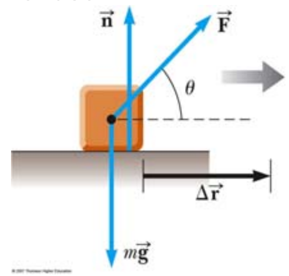
Work Done by Multiple Forces

If more than one force acts on an object, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

Remember work is a scalar, so this is the algebraic sum

$$W_{net} = W_g + W_N + W_F = (F\cos\theta)\Delta r$$



Work done by a Gravitational Force

Gravitational Force

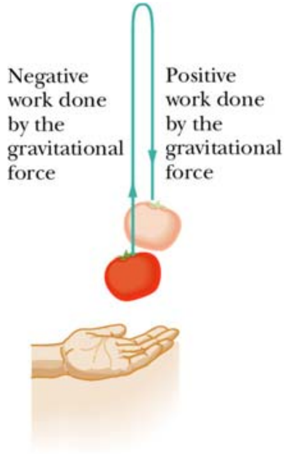
Magnitude: mg

Direction: downwards to the Earth's center

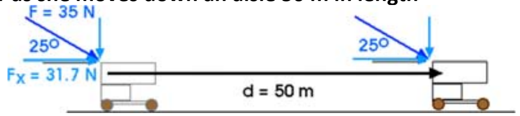
Work done by Gravitational Force

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}$$

$$W_g = mg\Delta r\cos\theta$$



.example **A shopper in a supermarket pushes a cart with a force of 35 N directed at an angle of 25° downward from the horizontal. Find the work done by the shopper as she moves down an aisle 50 m in length**



W = F d cos

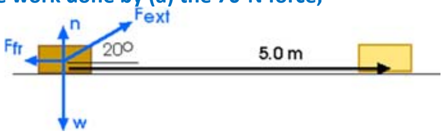
W) = 35 N)(50 m) cos 25°

W) = 35 N)(50 m)(0,906

W = 1586N-m = 1586 J

It is entirely equivalent to think of this as

2)A 15.0-kg block is dragges over a rough, horizontal surface by a 70.0-N force acting at 20° above the horizontal. The block is displaced 5.0 m and the coefficient of kinetic friction is 0.30. Find the work done by (a) the 70-N force,
(b) the normal force, and
(c) the force of gravity.
(d) What is the energy loss due to friction?



θ

W = F d cos

$W_{70} = F_{ext}$ d cos 20° **W_{70} = (70 N)(5 m)(0.9397)**

W_{70} = 328.9 J

$W_{fr} = F_{fr}$ d cos 180° **What is the force of friction F_{fr}? We do know that**

F_{fr} = 0.30 n

$F_{net,y} = n + F_{ext,y} - w = 0$ **n + (70 N)(sin 20°) - (15 kg)(9.8 m/s²) = 0**

n + (70 N)(0.342) - (15)(9.8) N = 0

n + 23.9 N - 147 N = 0

n = 123.1 N

With the value of the normal force n now known, we can calculate the friction force

F_{fr} = 0.30 (123.1 N) **F_{fr} = 36.9 N**

Now we can calculate the work done by the friction force,

$W_{fr} = F_{fr}$ d cos 180° **W_{fr} = - (36.9 N)(5.0 m**

W_{fr} = - 184.6 J

What does the negative sign mean. The force of friction decreases the energy. This is the energy loss which shows up as heat.

Work done by a Gravitational Force

- For a rising tomato
  - Work done by Gravitational Force

$$W_g = mg\Delta y \cos\theta$$

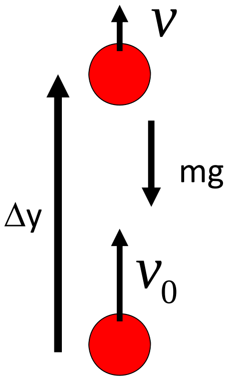
  - cos180° = -1
  - Work done by a Gravitational Force

$$W_g = -mg\Delta y < 0$$

  - Speed will decrease if work is negative

$$W_{net} = KE_f - KE_i = \Delta KE$$

$$W_g = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2}$$



Work done by a Gravitational Force

- For a falling tomato
  - Work done by Gravitational Force

$$W_g = mg\Delta y \cos\theta$$

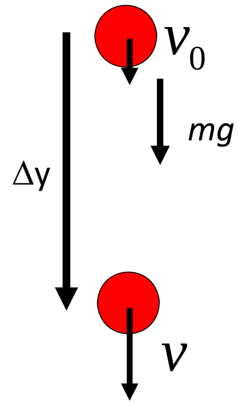
  - cos0° = +1

$$W_g = +mg\Delta y > 0$$

  - Speed will increase if work is positive

$$W_{net} = KE_f - KE_i = \Delta KE$$

$$W_g = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2}$$



1. Potential Energy

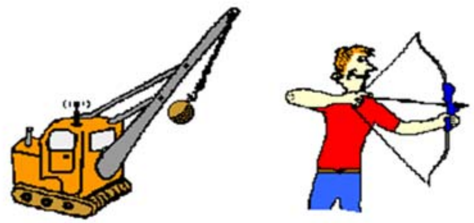
An object can store energy as the result of its position

Energy

The ability to do work

There are three forms of Energy

- the heavy ball of a demolition machine is storing energy when it is held at an elevated position. This stored energy of position is referred to as potential energy. Similarly, a drawn bow is able to store energy as the result of its position. When assuming its *usual position* (i.e., when not drawn), there is no energy stored in the bow. Yet when its position is altered from its usual equilibrium position, the bow is able to store energy by virtue of its position. This stored energy of position is referred to as potential energy. **Potential energy** is the stored energy of position possessed by an object.



The massive ball of a demolition machine and the stretched bow possesses stored energy of position - potential energy.

A. *Gravitational Potential Energy*

Gravitational potential energy is the energy stored in an object as the result of its vertical position or height. The energy is stored as the result of the gravitational attraction of the Earth for the object. The gravitational potential energy of the massive ball of a demolition machine is dependent on two variables - the mass of the ball and the height to which it is raised. There is a direct relation between gravitational potential energy and the mass of an object. More massive objects have greater gravitational potential energy. There is also a direct relation between gravitational potential energy and the height of an object. The higher that an object is elevated, the greater the gravitational potential energy.

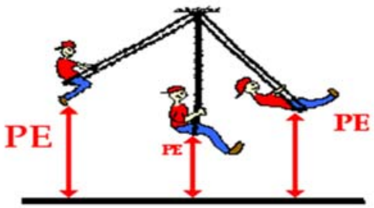
These relationships are expressed by the following equation

$$PE_{grav} = \text{mass} \times g \times \text{height} = m \times g \times h$$

Where, **m** represents the mass of the object.
h represents the height of the object.
g represents the acceleration of gravity.

Example

a pendulum bob swinging to and from above the table top has a potential energy which can be measured based on its height above the tabletop. By measuring the mass of the bob and the height of the bob above the tabletop, the potential energy of the bob can be determined



B. *Elastic Potential Energy*

Elastic potential energy is the energy stored in elastic materials as the result of their stretching or compressing. Elastic potential energy can be stored in rubber bands, bungee chords, trampolines, springs, an arrow drawn into a bow, etc. The amount of elastic potential energy stored in such a device is related to the amount of stretch of the device - the more stretch, the more stored energy.

The compressed springs of a dart gun store elastic potential energy. When the trigger is pulled, the springs apply a force to do work on the dart.

To summarize, potential energy is the energy which is stored in an object due to its position relative to some zero position. An object possesses gravitational potential energy if it is positioned at a height above (or below) the zero height. An object possesses elastic potential energy if it is at a position on an elastic medium other than the equilibrium position.

2. *Kinetic Energy*

Kinetic energy is the energy of motion. An object which has motion - whether it be vertical or horizontal motion - has kinetic energy. There are many forms of kinetic energy - vibrational (the energy due to vibrational motion), rotational (the energy due to rotational motion), and translational (the energy due to motion from one location to another). To keep matters simple, we will focus upon translational kinetic energy. The amount of translational kinetic energy (from here on, the phrase kinetic energy will refer to translational kinetic energy) which an object has depends upon two variables: the mass (m) of the object and the speed (v) of the object. The following equation is used to represent the kinetic energy (KE) of an object.

Kinetic Energy

- Kinetic energy associated with the motion of an object

$$KE = \frac{1}{2}mv^2$$

- Scalar quantity with the same unit as work
- Work is related to kinetic energy

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\ 2} = F_{net}\Delta x$$

$$W_{net} = KE_f - KE_i = \Delta KE$$

Since the gravitational potential energy of an object is directly proportional to its height above the zero position, a *doubling* of the height will result in a *doubling* of the gravitational potential energy. A *tripling* of the height will result in a *tripling* of the gravitational potential energy.

Kinetic Energy

- Kinetic Energy is energy associated with the state of motion of an object
- For an object moving with a speed of v

$$KE = \frac{1}{2} m v^2$$

- SI unit: joule (J)

1 joule = 1 J = 1 kg m^2/s^2

Special case: Constant Acceleration

Remember result eliminating t:

$$\boxed{v^2 - v_0^2 = 2a(x - x_0)}$$

Multiply by ½ m:

$$\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = ma(x - x_0)$$

$$= ma\Delta x$$

But $F = ma$!

$$\Delta\left(\frac{1}{2} m v^2\right) = F\Delta x$$

The following equation is used to represent the kinetic energy (KE) of an object.

$$KE = \frac{1}{2} * m * v^2$$

Where **m** = mass of object
and **v** = speed of object

1) **Can the kinetic energy of an object be negative?**
No. Kinetic energy is given by KE = (1/2) m v2. The mass m is always positive and v^2 is positive so KE must always be positive (of course, if v = 0 then KE = 0). KE can not be negative.

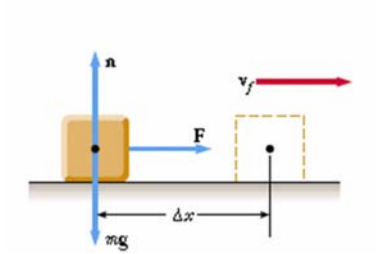
2)A -If the speed of a particle is doubled, what happens to its kinetic energy?
KE = (1/2) m v^2 **so doubling the speed v means the Kinetic Energy is increased fourfold**.
(b) if the net work done on a particle is zero, what can be said about the speed?
The net work done on a particle is equal to the **change** in its Kinetic Energy. So if the net work is **zero** the Kinetic Energy remains **constant** and that means the **speed**, too, remains **constant.**

3)When a punter kicks a football, is he doing any work on the ball while his toe is in contact with it? Yes, a force is being exerted on the ball as it moves through a distance. That's the definition of work.
Is he doing any work on the ball after it loses contact with his toe?
No, once his toe looses contact with the ball, the force his toe exerts is **zero.**
Are any forces doing work on the ball while it is in flight?
Yes, **gravity** and **air resistance** continue to do work on the ball.

Example 3 :

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

W =F dcos 180 =12x 3x-1
=-36j
Δk = w
0.5mV =W =-36j

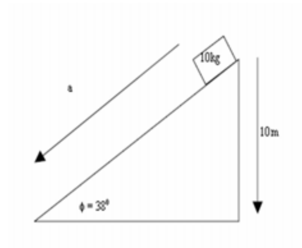


Example 4.

What acceleration is required to stop a 1000kg car traveling 28 m/s in a distance of 100 meters?

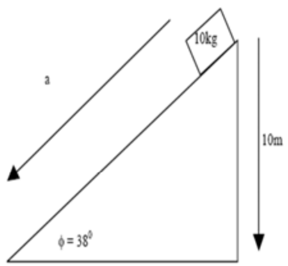
Example 5.

Consider the following. If the surface is smooth what is the speed of the block at the bottom of the incline?



example

If the surface is smooth what is the speed of the block at the bottom of the incline?



$$\Delta KE = W \Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = Fs\cos\theta$$

$$\frac{1}{2}mv_f^2 - 0 = (mg\sin\phi)(s)(\cos\theta)$$

$$\frac{1}{2}v_f^2 = (g\sin\phi)(s)$$

Notice that $(sin\phi)(s)$ = 10 meters, or the initial height of the block above the ground.

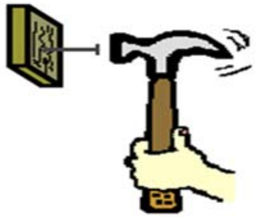
$$v_f = \sqrt{2gh} = \sqrt{2(9.8m\cdot s^{-2})(10m)} = 14m\cdot s^{-1}$$

3. *Mechanical Energy*

When the work is done upon the object, that object gains energy. The energy acquired by the objects upon which work is done is known as **mechanical energy**

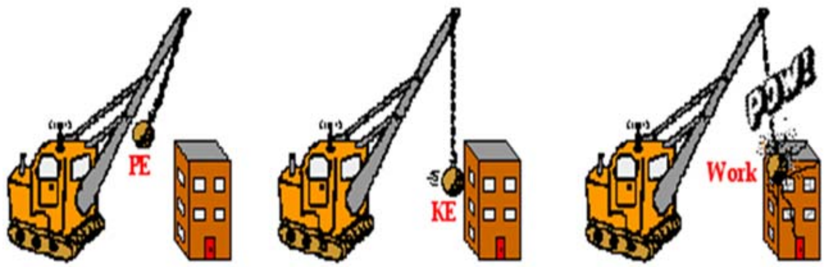
Mechanical Energy as the Ability to Do Work

An object which possesses mechanical energy is able to do work. In fact, mechanical energy is often defined as the ability to do work. Any object which possesses mechanical energy - whether it be in the form of potential energy or kinetic energy - is able to do work. That is, its mechanical energy enables that object to apply a force to another object in order to cause it to be displaced



Another Example

A hammer is a tool which utilizes mechanical energy to do work. The mechanical energy of a hammer gives the hammer its ability to apply a force to a nail in order to cause it to be displaced. Because the hammer has mechanical energy (in the form of kinetic energy), it is able to do work on the nail. Mechanical energy is the ability to do work



The massive ball of a demolition machine possesses mechanical energy - the ability to do work. When held at a height, it possesses mechanical energy in the form of potential energy. As it falls, it exhibits mechanical energy in the form of kinetic energy. As it strikes the structure to be demolished, it applies a force to displace the structure - i.e., it does work upon the structure.

Example

The wrecking ball is a massive object which is swung backwards to a high position and allowed to swing forward into building structure or other object in order to demolish it. Upon hitting the structure, the wrecking ball applies a force to it in order to cause the wall of the structure to be displaced. The diagram below depicts the process by which the mechanical energy of a wrecking ball can be used to do work.

$E = P_E + K_E$

Where is, E=Total Mechanical Energy
PE=Potential Energy
KE=Kinetic Energy

The relation between **Work** *&* **Energy**

product of a force applied to a body and the displacement of the body in the direction of the applied force. While work is done on a body, there is a transfer of energy to the body, and so work can be said to be energy in transit. The units of work are identical to those of energy.

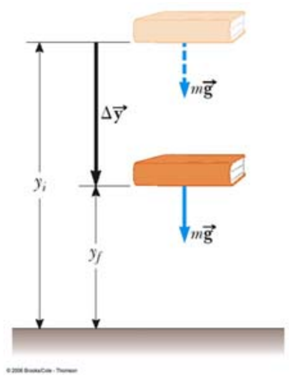
The Total Mechanical Energy

As already mentioned, the mechanical energy of an object can be the result of its motion (i.e., kinetic energy) and/or the result of its stored energy of position (i.e., potential energy). The total amount of mechanical energy is merely the sum of the potential energy and the kinetic energy. This sum is simply referred to as the total mechanical energy (abbreviated TME).

Potential Energy

- **Potential energy is associated with the position of the object**
- **Gravitational Potential Energy is the energy associated with the relative position of an object in space near the Earth's surface**
- **The gravitational potential energy**

$$\boxed{PE \equiv mgy}$$



- m **is the mass of an object**
- g **is the acceleration of gravity**
- y **is the vertical position of the mass relative the surface of the Earth**
- **SI unit: joule** (J)

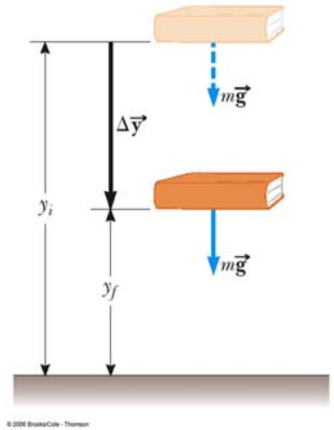
Reference Levels

- A location where the gravitational potential energy is zero must be chosen for each problem
  - The choice is arbitrary since the change in the potential energy is the important quantity
  - Choose a convenient location for the zero reference height
    - often the Earth's surface
    - may be some other point suggested by the problem
  - Once the position is chosen, it must remain fixed for the entire problem

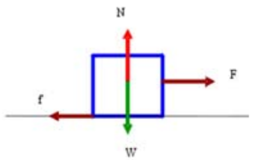
Work and Gravitational Potential Energy

- PE = mgy
- $$W_g = F\Delta y\cos\theta = mg(y_i - y_f)\cos 0$$
  $$= -mg(y_f - y_i)$$
- Units of Potential Energy are the same as those of Work and Kinetic Energy

$$W_{gravity} = PE_i - PE_f$$



Example 2:
A horizontal force F pulls a 10 kg carton across the floor at constant speed. If the coefficient of sliding friction between the carton and the floor is 0.30, how much work is done by F in moving the carton by 5m?



The carton moves with constant speed. Thus, the carton is in horizontal equilibrium.
F = f = μN = μmg.
Thus F = 0.3 x 10 x 9.8
= 29.4 N
Therefore work done W = FS
=(29.4 Cos 0°)%
=147 J

1) A 10kg mass is hanging 20 m from the ground. What is it's potential energy? How much kinetic energy will it have when it hits the ground

PE = mgh = 1960 J... which will become KE as it falls... so it will have 1960 J at the instant of impact .

2) A 1500 kg car is traveling at 22 m/s (regular size car at ~ 50 mi/hr) when it hits the brakes causing the car to skid. If the friction force during the skid is 12,000 N... how far will the car go before it stops

$W_{done} = PE_{top} = KE_{bottom}$

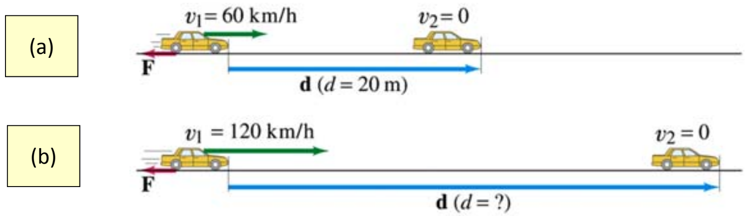
Plug in values... m x 9.8m/s/s x 10m = 1/2 m v^2 since the mass is on both sides it cancels out and you can solve for v

v = 14 m/s

3) A car increases it's velocity from 10m/s to 30m/s in 10 m at which point it has 700,000 J of KE. How much force did the car apply during those 10 m?

Example

A car traveling 60.0 km/h to can brake to a stop within a distance of 20.0 m. If the car is going twice as fast, 120 km/h, what is its stopping distance ?



(1) $W_{net} = F\,d_{(a)} \cos 180°$

$= -F\,d_{(a)} = 0 - m\,v_{(a)}^2 / 2$

→ $-F \times (20.0\text{ m}) = -m\,(16.7\text{ m/s})^2 / 2$

(2) $W_{net} = F\,d_{(b)} \cos 180°$

$= -F\,d_{(b)} = 0 - m\,v_{(b)}^2 / 2$

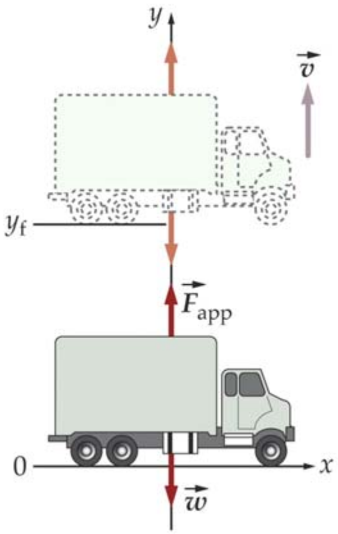
→ $-F \times (d) = -m\,(33.3\text{ m/s})^2 / 2$

(3) F & m are common. Thus, d = 80.0 m

Example – Lifting a truck

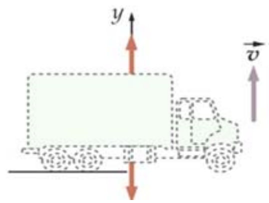
A truck of mass 3000 kg is to be loaded onto a ship using a crane that exerts a force of 31 kN over a displacement of 2m.

Find the upward speed of truck after its displacement.



Example

A truck of mass **3000 kg** is to be loaded onto a ship using a crane that exerts a force of **31 kN** over a displacement of **2m**.
Find the upward speed of truck after its displacement.



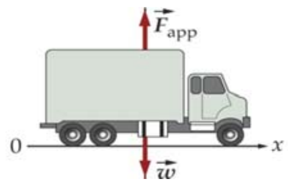
Work done ***on*** truck ***by*** gravity

$$W_g = \vec{w} \cdot \Delta\vec{y} = mg\Delta y \cos 180^o$$
$$= (3000\text{ kg})(9.81\text{ N/kg})(2\text{ m})(-1)$$
$$= -58.9\text{ kJ}$$

Work done ***on*** truck ***by*** crane

Work done ***on*** truck ***by*** crane

$$W_{app} = \vec{F}_{app} \cdot \Delta\vec{y} = F_{app}\Delta y \cos 0^o$$
$$= (31\text{ kN})(2\text{ m})(1)$$
$$= 62.0\text{ kJ}$$



From the work-kinetic energy theorem

$$\boxed{W_g + W_{\text{app}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2}$$

we obtain:

$$v_2^2 = v_1^2 + \frac{2(W_g + W_{\text{app}})}{m}$$

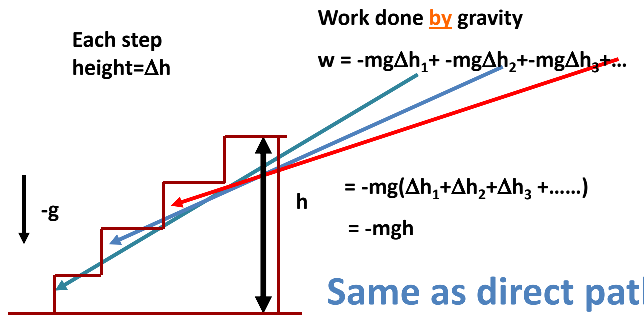
$$= 0 + \frac{2(-58{,}900\ \text{J} + 62{,}000\ \text{J})}{3000\ \text{kg}}$$

$$= 2.09\ \text{m}^2/\text{s}^2$$

$$v_2 = 1.45\ \text{m/s}$$

Conservative Forces

A force is conservative if the work done by it on a particle that moves between two points is the same for all paths connecting these points: otherwise the force is non-conservative.



Same as direct path (-mgh)

Doing Work to Decrease Energy

- When you catch a ball, its kinetic energy is reduced (or absorbed) by the negative work you do on it
- Your muscles do negative work on your limbs and absorb energy when you land from a jump or fall
- Average force you must exert to absorb energy in catching a ball or landing from a jump or fall depends on how much energy must be absorbed and the displacement over which the force is absorbed

Law of Conservation of Energy

- As energy takes different forms and changes things by doing work, nature keeps perfect track of the total.
- No new energy is created and no existing energy is destroyed.

Conservation of Energy

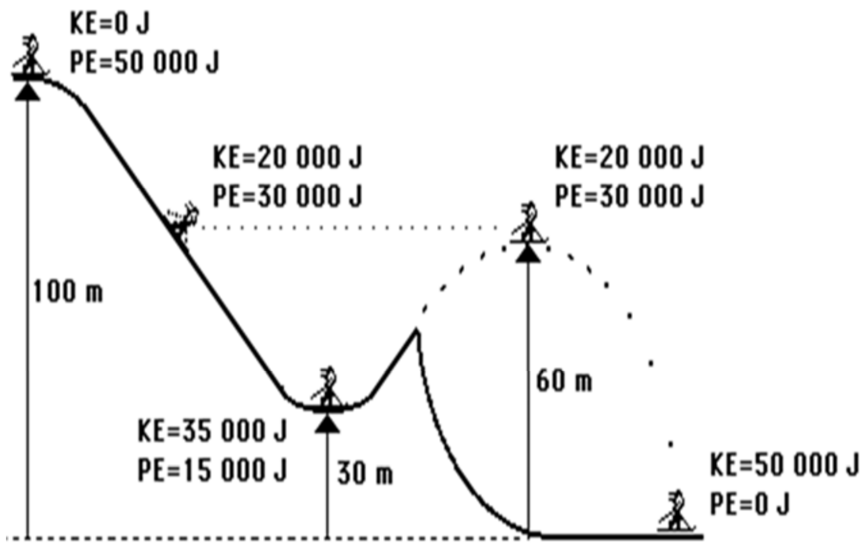
- Where there is no air resistance or friction, $\boxed{\Delta KE = -\Delta PE}$

$$KE_f - KE_i = -(PE_f - PE_i)$$

$$\boxed{PE_i + KE_i = PE_f + KE_f}$$

- This is the principle of the conservation of energy

Weight =500 N



example

A 0.400-kg bead slides on a curved wire, starting from rest at point **A** in Figure . If the wire is frictionless, find the speed of the bead (a) at **B** and (b) at **C**.

At B

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

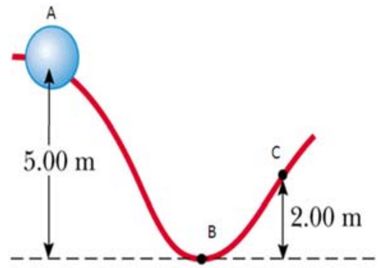
$$mgh_i + 0 = 0 + \frac{1}{2}mv_f^2$$

$$(2 \times 9.8 \times 5)^{1/2} = v$$

At C

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$0 + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

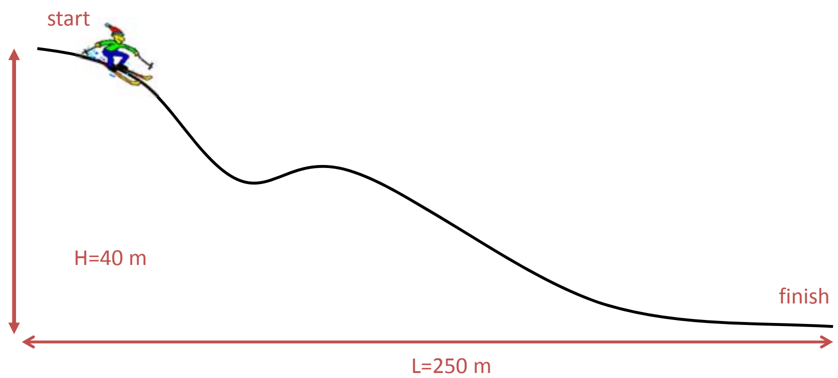


$$v_i^2 - 2gh_f = v_f^2$$

$$(v_i^2 - 2gh_f)^{1/2} = v_f$$

Example 5.3

A skier slides down the frictionless slope as shown. What is the skier's speed at the bottom?



28.0 m/s

Work done on a system by external force

$$\Delta K_{sys} + \Delta U_{sys} = W_{ext}$$

Positive external work done by the environment on the system carries energy into the system, thereby increasing its total energy; vice versa.

The external work represents a transfer of energy between the system and the environment.

Law of conservation of mechanical energy:

$$K_f + U_f = K_i + U_i - f_k d = K_i + U_i - \mu N d$$

"In a system in which only conservative forces do work, the total mechanical energy remains constant"

Work – Energy Principle & Mechanical Energy Conservation

- If we ignore non conservative forces (friction and the such), the implication is that no non-mechanical energies are present (heat, sound, light, etc) therefore…

$$\Delta KE + \Delta PE = 0$$

Law of conservation of mechanical energy:

$$K_i + U_i = K_f + U_f$$

"In a system in which only conservative forces do work, the total mechanical energy remains constant"

$$E_1 = E_2$$

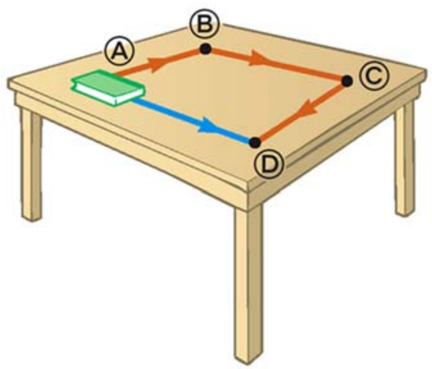
- As energy takes different forms and changes things by doing work, nature keeps perfect track of the total.
- No new energy is created and no existing energy is destroyed.

Non conservative Forces

- **A force is non conservative if the work it does on an object depends on the path taken by the object between its final and starting points.**
- **Examples of non conservative forces**
  - **kinetic friction**

Friction Depends on the Path

- The blue path is shorter than the red path
- The work required is less on the blue path than on the red path
- Friction depends on the path and so is a non-conservative force



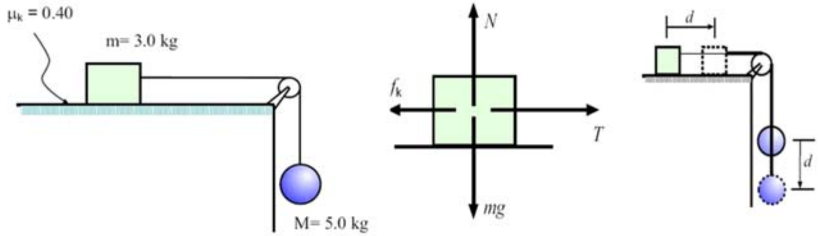
Mechanical Energy Conservation with energy lost

$$E_1 = E_2 + E_{lost}$$

$$E_1 = E_2 + W_{friction}$$

example

The coefficient of friction between the 3.0 kg mass and surface in the Fig. is 0.40. The system starts from rest. What is the speed of the 5.0 kg mass when it has fallen 1.5 m?



When the system starts to move, both masses accelerate; because the masses are connected by a string, *they always have the same speed*. The block (m) slides on the rough surface, and friction does work on it. Since its height does not change, its potential energy does not change, but its kinetic energy increases. The hanging mass (M) drops freely; its potential energy decreases but its kinetic energy increases.

the force of kinetic friction on m has magnitude $\mu_k N = \mu_k mg$.

the work done by friction is

$$W_{\text{fric}} = f_k d \cos\phi = (\mu_k mg)(d)(-1) = -(0.40)(3.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2})(1.5\,\text{m}) = -17.6\,\text{J}\,.$$

Mass m's change in kinetic energy is $\Delta K = \frac{1}{2}(3.0\,\text{kg})v^2 - 0 = (1.5\,\text{kg})v^2$

Mass M's change in kinetic energy is $\Delta K = \frac{1}{2}(5.0\,\text{kg})v^2 - 0 = (2.5\,\text{kg})v^2$

it has a *change in height* given by $-d$, its change in (gravitational) potential energy

$$\Delta U = Mg\Delta y = (5.0\,\text{kg})(9.80\,\tfrac{\text{m}}{\text{s}^2})(-1.5\,\text{m}) = -73.5\,\text{J}$$

Adding up the changes for both masses, the total change in mechanical energy of this system is

$$\begin{aligned} \Delta E &= (1.5\,\text{kg})v^2 + (2.5\,\text{kg})v^2 - 73.5\,\text{J} \\ &= (4.0\,\text{kg})v^2 - 73.5\,\text{J} \end{aligned}$$

$\Delta E = W_{\text{fric}}$ and get:

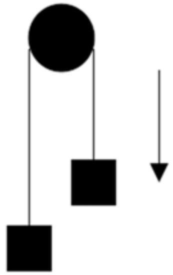
$$(4.0\,\text{kg})v^2 - 73.5\,\text{J} = -17.6\,\text{J}$$

$$(4.0\,\text{kg})v^2 = 55.9\,\text{J} \quad \Longrightarrow \quad v^2 = \frac{55.9\,\text{J}}{4.0\,\text{kg}} = 14.0\,\frac{\text{m}^2}{\text{s}^2}$$

$$v = 3.74\,\frac{\text{m}}{\text{s}}$$

example Use the *work-energy principle* to determine the final speed of a 5 kg mass (m_1) attached via a light cord over a massless, frictionless pulley to another mass of 3.5 kg (m_2), when the 5 kg mass has fallen (starting from rest) a distance of 2.5 meters.

$$W = \Delta KE_{system} = KE_{f_{sys}} - KE_{i_{sys}} = \sum \vec{F} \cdot \vec{s}$$



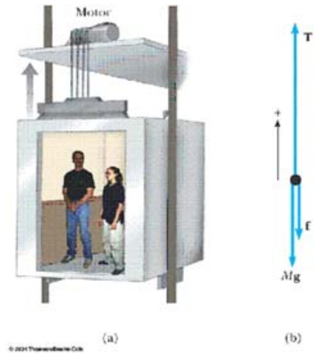
$$\sum \vec{F} \cdot \vec{s} = KE_{f_{sys}} - 0 = \frac{1}{2}(m_1 + m_2)v_f^{\,2} - 0$$

$$v_f = \sqrt{\frac{2\left(\sum \vec{F} \cdot \vec{s}\right)}{m_1 + m_2}} = \sqrt{\frac{2\left[(m_1 g - m_2 g) \cdot s\right]}{m_1 + m_2}} = \sqrt{\frac{2\left[(m_1 - m_2)g \cdot s\right]}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2\left[(5kg - 3.5kg)(9.8m \cdot s^{-2}) \cdot 2.5m\right]}{8.5kg}} = 2.94 m \cdot s^{-1}$$

Example:

An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4000 N retards its motion upward, as shown below a constant speed of 3.00 m/s ?



solution

Since the speed is constant,

$\Sigma F_y = T - F_f - Mg = 0$

where M = total mass of the system

Solving for T,

$T = F_f + Mg$

$T = (4.00 \times 10^3\ N) + (1.80 \times 10^3\ kg)(9.80 m/s^2)$

$T = 2.16 \times 10^4\ N$

Since P = Fv,

$P = (2.16 \times 10^4\ N)(3.00\ m/s)$

P = $\underline{6.48 \times 10^4\ W}$

example

In a rifle barrel, a 15.0 g bullet is accelerated from rest to a speed of 780 m/s.

a)Find the work that is done on the bullet.?

Using Work-Kinetic Energy Theorem,

W = ΔKE

$W = KE_f - KE_i$

$W = 1/2(0.015\ kg)(780\ m/s)^2 - 0$

W = 4.56 kJ

b) If the rifle barrel is 72.0 cm long, find the magnitude of the average total force that acted on the bullet

Using W = Fcosθd,

F = (4.56 kJ) / (0.72 m)

F = 6.33 kN

A car increases it's velocity from 10m/s to 30m/s in 10 m at which point it has 700,000 J of KE. How much force did the car apply during those 10 m? Similar concept as #4... the car has more KE because WORK was done to it.... So W_{done} = ΔKE... how much work must be done to increase the kinetic energy as much as it was??? First solve for the amount the KE increased... this is equal to the amount of WORK done to the car. WORK = Force * dist... so now plug in distance and solve for the amount of force.

Force = 62,200 N

Power

Climbing a flight of stairs requires a certain amount of work to overcome the gravitational force. This amount of work is the same whether one climbs the stairs slowly or runs up the stairs quickly. However you are more likely to be breathing heavily after running up the flight of stairs than after climbing them slowly. Why? The difference is power. The total work or energy is the same either way, but running up the stairs quickly requires more power than walking up slowly.

Mechanical systems, an engine for example, are not limited by the amount of work they can do, but rather by the rate at which they can perform the work. This quantity, the rate at which work is done, is defined as power

Equations for Power

From this very simple definition, we can come up with a simple equation for the average power of a system. If the system does an amount of work, *W*, over a period of time *t* then the average power is simply given by :

$$\overline{P} = \frac{W}{t}$$

$$\frac{dW}{dt} = P \qquad \frac{(F\cos\theta)(dx)}{dt}$$

$$\frac{dx}{dt} = v$$

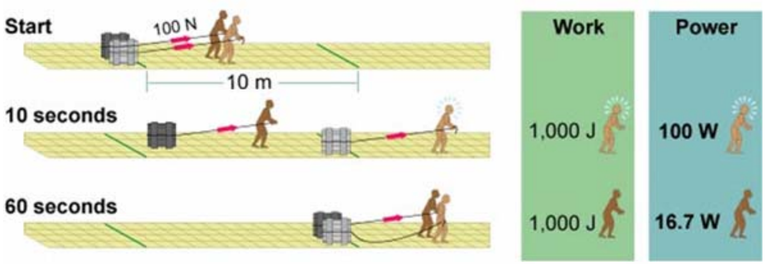
$$P = Fv\cos\vartheta$$

Though the calculus is not necessarily important to remember, the final equation is quite valuable. We now have two simple, numerical equations for both the average and instantaneous power of a system. Note, in analyzing this equation, we can see that if the force is parallel to the velocity of the particle, then the power delivered is simply $P = Fv$.

Units of Power

The unit of power is the joule per second, which is more commonly called a watt.

Power Example



$$\text{Power} = \frac{\text{Amount of work done}}{\text{time}}$$

problems

1)An elevator must lift 1000 kg a distance of 100 m at a velocity of 4 m/s. What is the average power the elevator exerts during this trip?

solution

$mgh = (1000)(9.8)(100) = 9.8\times10^5$ Joules T =25 s

P = 3.9×10^4 Watts

example

A constant force of 2kN pulls a crate along a level floor a distance of 10 m in 50s.
What is the power used?

Solution

$$\begin{aligned}\text{Work done} &= \text{force} \times \text{distance}\\ &= 2000\times10\\ &= 20000\,J\end{aligned}$$

$$\begin{aligned}\text{Power} &= \frac{\text{work done}}{\text{time taken}}\\ &= \frac{20000}{50}\\ &= 400\,W\end{aligned}$$

Alternatively we could have calculated the speed first

$$v = \frac{\text{distance}}{\text{time}} = \frac{10}{50} = 0.2 m/s$$

$$\begin{aligned} \text{Power} &= \text{Force} \times \text{Speed} \\ &= Fv \\ &= 2000 \times 0.2 \\ &= 400W \end{aligned}$$

Example

A hoist operated by an electric motor has a mass of 500 kg. It raises a load of 300 kg vertically at a steady speed of 0.2 m/s. Frictional resistance can be taken to be constant at 1200 N.
What is the power required?

Solution

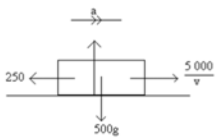
$$\begin{aligned} \text{Total mass} &= m = 800kg \\ \text{Weight} &= 800 \times 9.81 \\ &= 7848N \\ \text{Total force} &= 7848 + 1200 \\ &= 9048N \end{aligned}$$

$$\begin{aligned} \text{Power} &= \text{force} \times \text{speed} \\ &= 9048 \times 0.2 \\ &= 1810W \\ &= 1.81kW \end{aligned}$$

Example

A car of mass 500kg is travelling along a horizontal road. The engine of the car is working at a constant rate of 5kW. The total resistance to motion is constant and is 250N. What is the acceleration of the car when its speed is 5m/s?

solution



P = FV

$$\frac{5000}{v} - 250 = 500a$$

when v = 5m/s:
500a = 750N
a = 1.5m/s^{-2}
so the acceleration is 1.5ms^{-2}

Before we see how much electricity costs, we have to understand how it's measured. When you buy gas they charge you by the gallon. When you buy electricity they charge you by the *kilowatt-hour (kWh).* When you use 1000 watts for 1 hour, that's a kilowatt-hour. For example

The wattage will be printed on the device or its label. To get kilowatt-hours, take the wattage and divide by 1000 to turn it into kilowatts, and then multiply by the number of hours you're using the item.

Many of my readers get confused about the difference between *watts* and *watt-hours*. Here's the difference:
- **Watts is the *rate* of use at *this instant.***
- **Watt-hours is the *total energy used over time*.**

So there are two ways to turn watts into kilowatt-hours. Take your pick

- **Watts ÷ 1000 to get Kilowatts, then Kilowatts x Hours to get Kilowatt-Hours, -or-**
- **Watts x Hours to get Watt-Hours, then Watts ÷ 1000 to get Kilowatt-hours**

Exercise #2. Assume that the lights in your kitchen and living room together use 400 watts. How much does it cost if the lights are on 24 hours a day, for a whole month? How much per year? Assume 12H/kWh.

Solution:

- 400 watts x 24 hours/day x 30.5 days/month = 288,000 Total Watt-hours
- 288,000 Wh / 1000 Wh = 288 kWh
- 288 kWh x 12H/kWh = 3456H. =34.56 SR

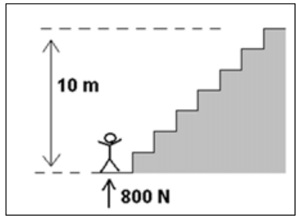
Exercise #3. Assume your window AC uses 1440 watts. How much does it cost to run it continuously for a month? How much per year? Assume 12H/kWh.

Solution:

- 1440 watts x 24 hours/day x 30.5 days/monh = 1,054,080 Total Watt-hours
- 1,054,080 Wh / 1000 Wh = 1,054 kWh
- 1,054 kWh x 12¢/kWh = $126/mo.; $1518/yr.

Example:

The minimum work required to raise a 800 N person up 10 m, is:



W = F d

W = (800 N) (10 m) = 8000 J

If this work is done in 60 sec, then what is the power?

$$P = \frac{W}{t} = \frac{8000\ J}{60\ sec} = \frac{133\ J}{sec} = 133\ watts$$