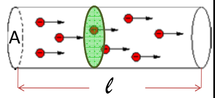
CHAPTER 17 DIRECT CURRENTS

17-1 electric current

- Whenever electric charges of like signs move under the influence of an applied of electric field, an electric current is said to exist
- The current is the rate at which the charge moves in the wire.



Definition of the current:

- The average current that passes any point in a conductor during a time Δt is defined as $\bar{I} = \frac{\Delta Q}{\Delta t}$ (17.1)
- where ΔQ is the amount of charge crosses the shaded area in a time Δt .

The instantaneous current is $I = \frac{dQ}{dt}$ (17.2)

The S.I . Current unit is the ampere (A) .often it is convenient to use the milliampere (mA)

1mA = 10^{-3} A Also 1µ A =10^{-6} A

1n A =10^{-9} A 1 pA = 10^{-12} A

One ampere of current is one coulomb per second: $1A = \frac{1C}{1s}.$

- In a metallic conductor, the current is due to the motion of electrons, so the direction of the current will the opposite to the direction of flow of the electrons.
- Charges flow only if there is a potential difference. Something that provides a potential difference is known as a voltage source

Example

A 3.8×10^{21} electrons pass through a point in a wire in 4 minutes. What was the average current?

$$I_{av} = \frac{\Delta Q}{\Delta t} = \frac{Ne}{\Delta t}$$

$$I_{av} = \frac{(3.8\times10^{21})(1.6\times10^{-19})}{(4\times60)}$$

$$I_{av} = 2.53A$$

Example 17.1

An electrochemical cell consists of two silver electrodes placed in an aqueous solution of silver nitrate .A constant 0.5 A current is passed through the cell for one hour ,

A) Find the total charge transported through the cell in coulombs and in multiples of the electronic charge .

B) Each electrons reaching the cell discharges one positively charged silver ion, which is then deposited on the negative electrode (cathode) . What is the total mass of the deposited silver?(the atomic mass of silver is 107.9 u., 1u = 1.66 10^{-27} kg)

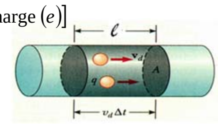
Solved in the course's text book

The relation between the current in a wire with the density of conduction electrons and their drift velocity

- Let n be the number of mobile charge carriers per unit volume (the density of conduction electrons) its unit is electron/m^3,
- The total number of charge carriers in the volume (V) of the conductor is N = n .Volume = n.A .ℓ ➡ where the Volume = A . ℓ

Charge of one electron (the electronic charge) = **e** = 1.6 x 10^{-19} C

$$\Delta Q = [number\ of\ charges(N)] \times [electronic\ \text{charge}\ (e)]$$



$$\Delta Q = ne\ \ell A$$

$$I_{av} = \frac{\Delta Q}{\Delta t} = ne\frac{\ell}{\Delta t}A$$

The drift speed, v_d, is the speed at which the carriers move

$$v_d = \frac{\ell}{\Delta t} \qquad I_{av} = nev_d A \qquad (17.3)$$

- The current is the product of the electronic charge (e) , the density of conduction electrons (n) , the area (A) and the drift velocity (v_d)
- The current also depends on the resistance that the conductor offers to the flow of charge which is called electric resistance

Example 17-2

Number 12 copper wire is often used to wire house- hold electrical outlets . Its radius is 1mm =10^{-3} m,if it carries a current of 10 A ,what is the drift velocity of the electrons ? (metallic copper has one conduction electron per atom , the atomic mass of copper is 64 u , and the density of copper is 8900 Kg m^{-3}) , 1u =1.66 x 10^{-27} Kg

The number of atoms per unit volume(n) x the mass of one atom (M)= the mass of a unit volume of copper =The density (d)

$$d = n \times M$$

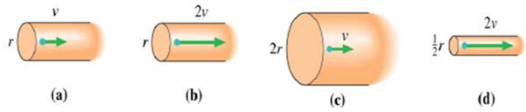
$$n = \frac{d}{M} = \frac{8900\ Kg\ m^{-3}}{(64\ u)(1.66\ \times 10^{-27}\ Kg\ u^{-1})} = 8.38 \times 10^{28}\ electron\ / m^{-3}$$

$$A = \pi r^2 = 3.14 \times (10^{-3})^2 = 3.14 \times 10^{-6}\ \text{m}^2$$

$$v_d = \frac{I}{nqA} = \frac{10\ A}{(8.38 \times 10^{28}\ m^{-3})(1.60 \times 10^{-19}\ C)(3.14 \times 10^{-6}\ m^2)} = 2.37 \times 10^{-4}\ m/s$$

Conceptual questions

These four wires are made of the same metal. Rank in order, from largest to smallest, the electron currents I_a to I_d .



A. $I_d > I_a > I_b > I_c$
B. $I_b = I_d > I_a = I_c$
⇨ C. $I_c > I_b > I_a > I_d$
D. $I_c > I_a = I_b > I_d$
E. $I_b = I_c > I_a = I_d$

Conceptual questions

- Suppose a current-carrying wire has a cross-sectional area that gradually becomes smaller along the wire, so that the wire has the shape of a very long cone. How does the drift speed vary along the wire?

(a) It slows down as the cross section becomes smaller.
(b) It speeds up as the cross section becomes smaller.
(c) It doesn't change.
(d) More information is needed.

(b). Under steady-state conditions, the current is the same in all parts of the wire. Thus, the drift velocity, given by $v_d = I / n\, e\, A$, is inversely proportional to the cross-sectional area

17-2 Resistance

The electrical resistance R of a conductor is the potential difference V between its ends divided by the current I

$$R = \frac{V}{I} \qquad (17.4) \qquad \boxed{\text{Ohm's law}}$$

The S . I. unit of resistance is the ohm: $1\,\Omega = \frac{1\text{ V}}{1\text{ A}}.$

Resistances of kilohms and megohms are common:

$$1\text{ k}\Omega = 10^3\Omega, \quad 1\text{ M}\Omega = 10^6\Omega.$$

Ohmic and Nonohmic Materials

- For many materials , the potential difference and the current are directly proportional ,so the resistance is a constant independent of the current
The ratio R = V / I is constant
- Materials with a constant resistance are said to obey Ohm's law and are called ohmic conductors

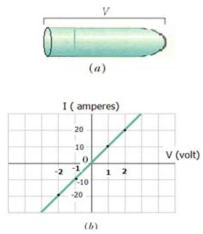


Figure 17.2. (a) copper wire has a variable potential difference V between its end (b) the current varies linearly with the potential difference in the wire ,so it is an ohmic conductor

Example 17.3

Find the resistance of the wire in figure 17.2

Solved in the course's text book

Materials that do not follow Ohm's Law are called "nonohmic" materials, and have curved I vs. V graphs.

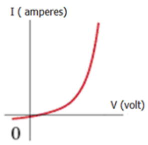


Figure 17.3 the current versus potential difference graph for a rectifying transistor which produces a large current in one direction, produces only a small current in the opposite direction. This is an example of a conductor that does not satisfy ohm's law

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature, i.e., resistivity is a property of substances

- R depends on the material type and shape
- resistance of a metal wire is directly proportional to its length, and inversely proportional to its cross-sectional area, A:
  - R – resistance (Ω)
  - ρ – resistivity (Ω m)
  - L – length (m)
  - A – cross sectional area (m^2)

$$R = \rho \frac{L}{A} \qquad (17.5)$$

$$\rho = \frac{R\,A}{\ell} \qquad (\Omega . m)$$

$$\sigma = \frac{1}{\rho} = \text{conductivity} \qquad \sigma = \frac{1}{\rho} = \frac{\ell}{R\,A} \quad (\Omega . m)^{-1} \qquad (17.6)$$

- Thick wires have less resistance than thin wires. Longer wires have more resistance than short wires.
- If the conductivity of the material the wire is made of is high. then there will be less resistance.

Table 17.1		Resistivities in Ω m at 20^0C
	substance	Resistivity { ρ } [Ω m]
conductors	Silver	1.47×10^{-8}
	Cooper	1.72×10^{-8}
	Aluminum	2.63×10^{-8}
semiconductors	Germanium	0.60
	Silicon	2300
Insulators	Glass	$10^{10} – 10^{14}$
	Sulfur	10^{15}
Ionic conductors	Body fluids	approx. 0.15

- Good conductors have low resistivities
- Good insulators have high resistivities

Example 17.4

Find the room temperature resistance of a copper wire 100 m long with a radius of 1 mm = 10^{-3} m (from table 17 .1 ρ_{copper} at room temperature(20^0C) is 1.72×10^{-8} Ω . m)

Solved in the course's text book

Example:

A small flashlight bulb draws 300 mA from its 1.5-V battery. (*a*) What is the resistance of the bulb? (*b*) If the voltage dropped to 1.2 V, how would the current change?

$$R = \frac{V}{I} = \frac{1.5}{300 \times 10^{-3}} = 5\Omega$$

$$I = \frac{V}{R} = \frac{1.2}{5} = 0.24 \ A = 240 \ mA$$

Example:

A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire?

$$A = (0.600 \ mm)^2 \left(\frac{1.00 \ m}{1000 \ mm}\right)^2 = 6.00 \times 10^{-7} \ m^2$$

$$I = \frac{\Delta V A}{\rho \ell} = \frac{(0.900 \ V)(6.00 \times 10^{-7} \ m^2)}{(5.60 \times 10^{-8} \ \Omega \cdot m)(1.50 \ m)}$$

$$I = \boxed{6.43 \ A}$$

Conceptual questions

- A cylindrical wire has a radius *r* and length L . If both *r* and L are doubled, the resistance of the wire

(a) increases
(b) decreases
(c) remains the same

Answer: (b). The doubling of the radius causes the area *A* to be four times as large, so the resistance decreases.

- A cylindrical copper rod has resistance R. It is reformed to twice its original length with no change of volume. Its new resistance is:

1. R
2. 2R ⇦
3. 4R
4. 8R
5. R/2

Conceptual question

- If a piece of wire has a certain resistance, which wire made of the same material will have a lower resistance?

A) a thicker wire أسمك
B) a longer wire أطول
C) a thinner wire أرق

Ans: A

Conceptual questions

- If the length of a wire increased, the current flow decreases because of the longer path
- If the area of a wire increased, the current flow increases because of the wider path

R = ρ L/A

- If we change to a material with better conductivity, the current flow increases because charge carriers move better

Quiz 1

If a current of 80.0 mA exists in a metal wire, how many electrons flow past a given cross section of the wire in 10.0 min?

N = 3.00×10^{20} electrons

Quiz 2

A small flashlight bulb draws 300 mA from its 1.5-V battery.

(a) What is the resistance of the bulb?

(b) (b) If the battery becomes weak and the voltage drops to 1.2 V, how would the current change?

a) R = 5.0 Ω.
b) the current will drop to 240 mA.

Quiz 3

Birds resting on high-voltage power lines are a common sight. The copper wire on which a bird stands is 2.2 cm in diameter and carries a current of 50 A. If the bird's feet are 4.0 cm apart, calculate the potential difference across its body.

ΔV = 89 μV

Quiz 4

Calculate the diameter of a 2.5 cm length of tungsten filament in a small light bulb if its resistance is 0.047 Ω (The electrical resistivity of tungsten is 5.6×10^{-8} Ω .m)

d = 0.18×10^{-2} m

summary

Average current	$\bar{I} = \frac{\Delta Q}{\Delta t}$
Instantaneous current	$I = \frac{dQ}{dt}$
Number of the electrons	$N = \frac{\Delta Q}{e}$
Magnitude of the current	$I_{av} = nev_d A$
Resistance of a conductor	$R = \frac{V}{I}$
Resistance is determined by the geometry and by the resistivity of the material	$R = \rho \frac{L}{A}$

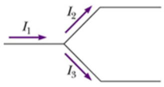
Home work:
1, 10, 11, 13 and 18

17.5 series and parallel resistors ;Kirchhoff's rules

Statement of Kirchhoff's Rules

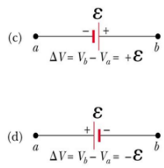
- Junction Rule ($\sum I = 0$)
  - The sum of the currents entering any point must equal the sum of the currents leaving that junction
    - A statement of Conservation of Charge

$$\sum I_{in} = \sum I_{out}$$



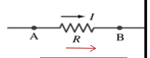
- $I_1 = I_2 + I_3$

- Loop Rule ($\sum V = 0$)
  - The sum of the potential changes around any closed circuit loop must be zero
  - You must go around the loop in one direction
  - The sum of the measured will equal zero

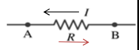


(a) The voltage across a battery is taken to be positive (a voltage rise) if traversed from – to + and and negative if traversed in the opposite direction.

(b) The voltage across a resistor is taken to be negative (a drop) if the loop is traversed in the direction of the assigned current and positive if traversed in the opposite direction.



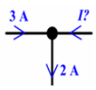
$V_{ba} = -IR$



$V_{ba} = IR$

Example

Calculate the current I flowing into the node



$$\sum I_{in} = \sum I_{out}$$

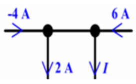
(3+ I) A = 2 A

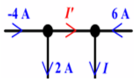
I = 2 -3 = -1 A

The current flowing into the node is – 1 A which is the same as +1 A flowing out of the node

Example

Calculate the current I *defined in the diagram*





I +2 A = - 4 A

I = (- 4 – 2) A = - 6 A

I is in the opposite direction

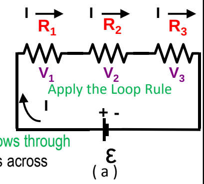
I + I = 6 A

I = (6 – 6) A = A

There are "two" ways to connect circuit elements.

1) Series combination:

Kirchhoff's rules :The sum of the potential changes around any closed circuit must be zero



The current is the same in resistors because any charge that flows through one resistor flows through the other but the potential differences across them are not the same

$$\varepsilon = I R_s \quad \Rightarrow \quad I = \frac{\varepsilon}{R_s}$$

$$\varepsilon - V_1 - V_2 - V_3 = 0$$

$$\varepsilon - I R_1 - I R_2 - I R_3 = 0$$

$$I = \frac{\varepsilon}{R_1 + R_2 + R_3}$$

$$\boxed{R_s = R_1 + R_2 + R_3} \qquad (17.12)$$

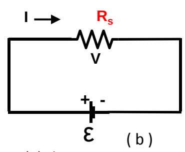


Figure 17- 10 (a) three resistors in series (b) the equivalent resistance R_s leads to the same current I,

2) Parallel combination

the current entering point A must equal that leaving

OR

$$I = I_1 + I_2 + I_3$$

$$I = \frac{\varepsilon}{R_p} \qquad I_1 = \frac{\varepsilon}{R_1} \qquad I_2 = \frac{\varepsilon}{R_2} \qquad I_3 = \frac{\varepsilon}{R_3}$$

$$I = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2} + \frac{\varepsilon}{R_3} = \varepsilon\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{\varepsilon}{R_p} = \varepsilon\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad (17.13)$$

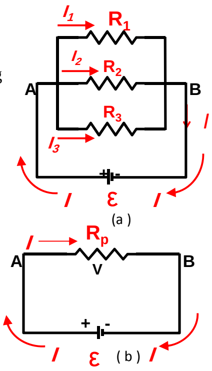


Figure 17 .11 (a) three resistors in parallel . (b) the equivalent single resistance R_p produces the same current *I*

2) Parallel combination

the current entering point A must equal that leaving

OR

$$I = I_1 + I_2 + I_3$$

$$I = \frac{\varepsilon}{R_p} \qquad I_1 = \frac{\varepsilon}{R_1} \qquad I_2 = \frac{\varepsilon}{R_2} \qquad I_3 = \frac{\varepsilon}{R_3}$$

$$I = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2} + \frac{\varepsilon}{R_3} = \varepsilon\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{\varepsilon}{R_p} = \varepsilon\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad (17.13)$$

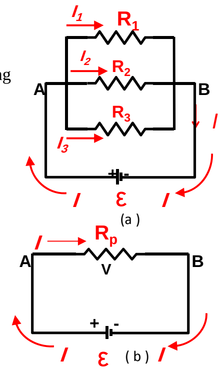
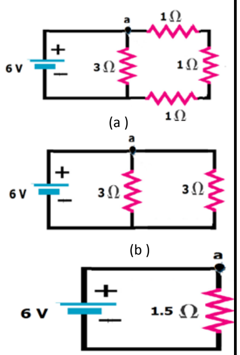


Figure 17 .11 (a) three resistors in parallel . (b) the equivalent single resistance R_p produces the same current *I*

Example 17.10

(a) find the equivalent resistance of the resistors in figure 17.10 a
(b) the current *I in each resistor*

Solved in the text book



Example

A) Find the current in the circuit shown in the figure.
B) Find the potential difference across each circuit element.

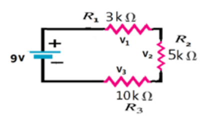
solution

In the figure ,we had a 3kΩ, 10 kΩ, and 5 kΩ resistor in series.

$$R_s = R_1 + R_2 + R_3$$

$$R_s = 3\ \text{k}\Omega + 10\ \text{k}\Omega + 5\ \text{k}\Omega = 18\text{k}\ \Omega$$

$$I = \frac{V}{R_s} = \frac{9}{18} = 0.5\,A$$



$$V_1 = IR_1 = 0.5 \times 3 = 1.5V$$

$$V_2 = IR_2 = 0.5 \times 10 = 5V$$

$$V_3 = IR_3 = 0.5 \times 5 = 2.5V$$

Example

From the figure find (a) I (total current) , R_p (total resistance)
(b) I_1 , I_2 , I_3

$$\frac{1}{R_p} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18} \quad \Rightarrow \quad R_p = \frac{18}{11} = 1.64\ \Omega$$

$$I_1 = \frac{V}{R_1} = \frac{18}{3} = 6\ A$$

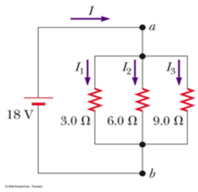
$$I_2 = \frac{V}{R_2} = \frac{18}{6} = 3\ A$$

$$I_3 = \frac{V}{R_3} = \frac{18}{9} = 2\ A$$

$$\sum I_{in} = \sum I_{out}$$

$$I = I_1 + I_2 + I_3$$

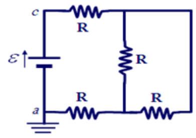
$$I = 6\ A + 3\ A + 2\ A = 11\ A$$



Example

Four resistors are connected as shown in figure. Find the equivalent resistance between points *a* and *c*.

A. 4 R.
B. 3 R.
C. 2.5 R.
D. 0.4 R.
E. Cannot determine from information given
.



Conceptual questions

- From the circuit with source voltage V and Total current I, which resistor will have the greatest voltage across it?

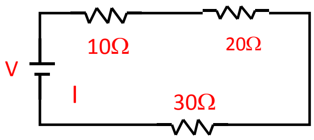
The resistor with the largest resistance (30 Ω)

- Which resistor has the greatest current flow through it?

Same for all because series circuit

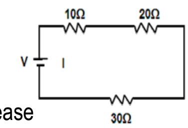
- If we re-ordered the resistors, what if any of this would change?

Nothing would change



- If we added a resistor in series with these, what would happen to the total resistance, total current, voltage across each resistor, and current through each resistor?

Total resistance would increase
Total current would decrease
Voltage across each resistor would decrease (All voltage drops must still sum to total in series circuit; Kirchhoff's law of voltages)
Current through each resistor would be lower (total current decreased, but same through each one)



Conceptual questions

- from the circuit with source voltage V and Total current I, which resistor will have the greatest voltage across it?

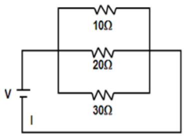
All the same in parallel branches

- Which resistor has the greatest current flow through it?

The "path of least resistance" (10Ω)

- What else can you say about the current through each branch?

They will sum to the total I (currents sum in parallel circuits; Kirchhoff's law of current)



- If we added a resistor in parallel with these, what would happen to the total resistance, total current, voltage across each resistor, and current through each resistor?

Total resistance would decrease
Total current would increase
Voltage across each resistor would still be V
Current through each resistor would be higher and would sum to new total

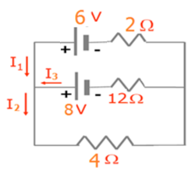
17.12 Kirchhoff's rules in complex circuits

Kirchhoff's rules permit us to analyze any dc circuit .including circuits too complex
Using the two rules
(1) the sum of all the potential drops around any closed path in a circuit is equal to zero.
(2) The current entering any point = The current leaving.

Example 17.15

Find the current in the circuit shown in the figure

Solved in the text book



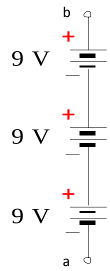
Example

Calculate ΔV_{ab}

ΔV_{ab}= 27V

ΔV_{ab} if one battery is reversed?

Δv_{ab}= 9v



Example

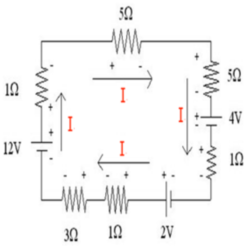
Calculate the current in the circuit.

$$\sum_{i=1}^{3} V_i = 0$$

$$12 - I - 5I - 5I - 4 - I + 2 - I - 3I = 0$$

$$10 - 16I = 0$$

$$I = \frac{10}{16} = 0.625A$$



Example

Find the current *I*, *r* and *ε*.

Junction rule at *a*: 2 A+1 A-*I*=0

I = 3 A

Loop (1): 12 V-*Ir* -(3 Ω)(2 A)=0

r =12 V/(3 A)-6 V/(3 A) =2 Ω

Loop (2):

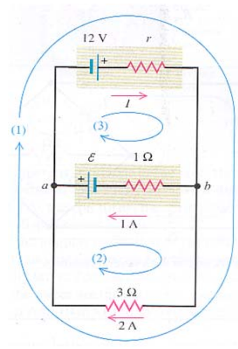
-*ε* +(1 Ω)(1 A)-(3 Ω)(2 A)=0

ε =-5 V (polarity is opposite!)

Check with loop (3):

12 V-(2 Ω)(3 A)-(1 Ω)(1 A)+*ε*=0

ε=-5 V



Conceptual questions

What is the current in branch P?

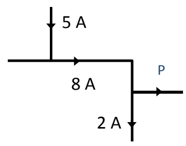
A) 2 A

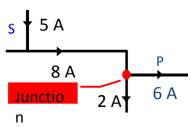
B) 3 A

C) 5 A

D) 6 A

E) 10 A





The current entering the junction in red is 8 A, so the current leaving must also be 8 A. One exiting branch has 2 A, so the other branch (at P) must have 6 A.

Conceptual questions

Which of the equations is valid for the circuit below?

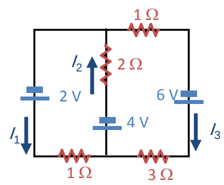
A) $2 - I_1 - 2I_2 = 0$

B) $2 - 2I_1 - 2I_2 - 4I_3 = 0$

C) $2 - I_1 - 4 - 2I_2 = 0$

D) $I_3 - 4 - 2I_2 + 6 = 0$

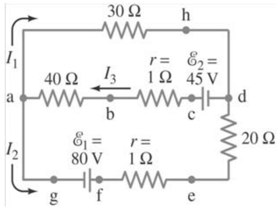
E) $2 - I_1 - 3I_3 - 6 = 0$



Quiz

Calculate the currents I_1, I_2, and I_3 in the three branches of the circuit in the figure.

I_1 = - 0.87 A.
I_2 = 2.6 A.
I_3 = 1.7 A.



summary

Kirchhoff's Rules

1- **Junction Rule** $\sum I_{in} = \sum I_{out}$

2- **Loop Rule** $\sum_i V_i = 0$

Series combination: $R_s = R_1 + R_2 + R_3 + ----$

Parallel combination: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ----$

Home work **45,46,71**