Brief solutions to selected questions

7.2.1

$n = 76$, $\bar{x} = 70.7$, $s = 14.6$, $\alpha = 0.01$

$H_0: \mu \geq 75$, $H_a: \mu < 75$

$Z_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{70.7 - 75}{14.6/\sqrt{76}}$ = -2.5657 < $Z_{0.01} = -2.33$, $p = 0.0051 < \alpha$=0.01

So, reject H_0 , and conclude that $\mu < 75$.

7.2.2

Assuming normal distribution

$n = 16$, $\bar{x} = 54.9375$, $s = 8.87295$, $\alpha = 0.05$

$H_0: \mu \geq 60$, $H_a: \mu < 60$

$t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{54.9375 - 60}{8.87295/\sqrt{16}}$ = -2.28222 < $t_{0.05,15} = -1.753$.

So, reject H_0 , and conclude that $\mu < 60$.

7.2.3

Assuming normal distribution

$n = 18$, $\bar{x} = 10.3$, $s = 7.3$, $\alpha = 0.1$

$H_0: \mu \leq 9$, $H_a: \mu > 9$

$t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{10.3 - 9}{7.3/\sqrt{18}}$ = 0.7555 < $t_{0.1,17} = 1.333$.

Do not reject H_0 , and conclude that $\mu \leq 9$.

7.2.4

$n = 25$, $\bar{x} = 4.8$, $s = 2$, $\alpha = 0.05$

$H_0: \mu \leq 4$, $H_a: \mu > 4$

$t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{4.8 - 4}{2/\sqrt{25}}$ = 2 > $t_{0.05,24} = 1.7109$.

So, reject H_0 , and conclude that $\mu > 4$.   Assuming normal distribution

7.3.1

$n_1 = 40$, $\bar{x}_1 = 22.41$, $s_1 = 1.27$, $\alpha = 0.01$

$n_2 = 35$, $\bar{x}_2 = 27.75$, $s_2 = 2.64$,

$H_0: \mu_2 \le \mu_1$, $H_a: \mu_2 > \mu_1$

$$Z_0 = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\frac{s_2^2}{n_2}+\frac{s_1^2}{n_1}}} = \frac{27.75-22.41}{\sqrt{\frac{(2.64)^2}{35}+\frac{(1.27)^2}{40}}} = 10.9127 > Z_{0.99} = 2.3264,$$

$p = 0.0 < \alpha$=0.01. So, reject H_0 , and conclude that $\mu_2 > \mu_1$.

7.3.2

$n_1 = 31$, $\bar{x}_1 = 76.9$, $s_1 = 12.6$, $\alpha = 0.05$

$n_2 = 31$, $\bar{x}_2 = 90.9$, $s_2 = 12.5$,

$H_0: \mu_2 \le \mu_1$, $H_a: \mu_2 > \mu_1$

$$Z_0 = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\frac{s_2^2}{n_2}+\frac{s_1^2}{n_1}}} = \frac{90.9-76.9}{\sqrt{\frac{(12.5)^2}{31}+\frac{(12.6)^2}{31}}} = 4.39184 > Z_{0.95} = 1.645,$$

$p = 0.0 < \alpha$=0.05 . So, reject H_0 , and conclude that $\mu_2 > \mu_1$.

7.3.7

Assuming normally distributed populations with equal variances

$n_1 = 10$, $\bar{x}_1 = 435$, $s_1 = 65$, $\alpha = 0.05$

$n_2 = 12$, $\bar{x}_2 = 645$, $s_2 = 80$,

$H_0: \mu_2 = \mu_1$, $H_a: \mu_2 \ne \mu_1$

$$s_p^2 = \frac{9(65)^2+11(80)^2}{9+11} = 5421.25 , \qquad s_p = 73.6291$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{435-645}{73.6291\sqrt{0.18333}} = -6.66114 < t_{0.025,20} = -2.086,$$

$p = 0.0 < \alpha$=0.05 ,   So, reject H_0 , and conclude that $\mu_2 \ne \mu_1$.

7.3.8

$n_1 = 50$, $\bar{x}_1 = 340$, $s_1 = 250$, $\alpha = 0.01$

$n_2 = 40$, $\bar{x}_2 = 45$, $s_2 = 25$,

$H_0: \mu_1 \leq \mu_2$, $H_a: \mu_1 > \mu_2$

$$Z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_2^2}{n_2} + \frac{s_1^2}{n_1}}} = \frac{340-45}{\sqrt{\frac{(25)^2}{40} + \frac{(250)^2}{50}}} = 8.1799 > Z_{0.99} = 2.3264, p = 0 < \alpha\text{=0.01}$$

So, reject H_0 , and conclude that $\mu_1 > \mu_2$.

7.3.9

$n_1 = 35$, $\bar{x}_1 = 8.5$, $s_1 = 5.5$, $\alpha = 0.05$

$n_2 = 40$, $\bar{x}_2 = 4.8$, $s_2 = 3.6$,

$H_0: \mu_1 = \mu_2$, $H_a: \mu_1 \neq \mu_2$

$$Z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{8.5-4.8}{\sqrt{\frac{(5.5)^2}{35} + \frac{(3.6)^2}{40}}} = 3.39423 > Z_{0.975} = 1.96,$$

$p = 6.9(10)^{-4} < \alpha$=0.05. So, reject H_0 , and conclude that $\mu_1 \neq \mu_2$.

7.3.10

Assuming normally distributed populations with equal variances

$n_1 = 12$, $\bar{x}_1 = 10.375$, $s_1 = 1.39602$, $\alpha = 0.05$

$n_2 = 12$, $\bar{x}_2 = 12.2083$, $s_2 = 1.44927$,

$H_0: \mu_2 = \mu_1$, $H_a: \mu_2 \neq \mu_1$

$$s_p^2 = \frac{11[(1.39602)^2 + (1.44927)^2]}{22} = 2.02462 , \quad s_p = 1.42289$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{6}}} = \frac{10.375-12.2083}{1.42289\sqrt{1/6}} = -3.15606 < t_{0.025,22} = -2.0739,$$

$p = 0.0045 < \alpha$=0.05 . So, reject H_0 , and conclude that $\mu_2 \neq \mu_1$.

7.4.1

d : $1, 0, 5, -1, 3, 5, 3, 2, 0, 0, 2, -1, 0, 3, 2$, $\alpha = 0.01$

$\bar{d} = 1.6$, $s_{\bar{d}} = 1.95667$, $H_0: \mu_d \leq 0$, $H_a: \mu_d > 0$

$$t_0 = \frac{\bar{d} - 0}{s_{\bar{d}} / \sqrt{n}} = \frac{1.6(3.87298)}{1.95667} = 3.167 > t_{0.99,14} = 2.6245.$$

So, reject H_0 , and conclude that $\mu_d > 0$.

7.4.5

d : $84, 50, 212, 181, 36, 248, 200$, $\alpha = 0.05$

$\bar{d} = 144.429$, $s_{\bar{d}} = 85.6774$, $H_0: \mu_d = 0$, $H_a: \mu_d \neq 0$

$$t_0 = \frac{\bar{d} - 0}{s_{\bar{d}} / \sqrt{n}} = \frac{144.429(\sqrt{7})}{85.6774} = \frac{144.429(2.64575)}{85.6774} = 4.46 > t_{0.975,6} = 2.447.$$

So, reject H_0 , and conclude that $\mu_d \neq 0$.

7.5.1

$N = 295, X = 90, \hat{p} = \frac{X}{N} = 0.305085$, $\alpha = 0.05$

$H_0: p \geq 0.35$, $H_a: p < 0.35$

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{N}}} = \frac{0.305085 - 0.35}{\sqrt{\frac{(0.35)(0.65)}{295}}} = -1.61739 > Z_{0.05} = -1.645,$$

$p = 0.0529 > \alpha$=0.05. Do not reject H_0 , and conclude that $p \geq 0.35$.

7.5.2

$N = 2428, X = 461, \hat{p} = \frac{X}{N} = 0.189868$, $\alpha = 0.05$

$H_0: p \leq 0.15$, $H_a: p > 0.15$

$$Z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{N}}} = \frac{0.189868 - 0.15}{\sqrt{\frac{(0.15)(0.85)}{2428}}} = 5.50169 > Z_{0.95} = 1.645,$$

$p = 0.0 < \alpha$=0.05. So, reject H_0 , and conclude that $p > 0.15$.

7.6.1

$\hat{p}_1 = \frac{X_1}{N_1} = \frac{72}{1222} = 0.0589198,\ \hat{p}_2 = \frac{X_2}{N_2} = \frac{30}{282} = 0.106383,$

$\bar{p} = \frac{72+30}{1222+282} = 0.0678191,\quad H_0: p_1 = p_2\ ,\quad H_a: p_1 \neq p_2,\quad \alpha = 0.01,$

$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}\bar{q}}{N_1} + \frac{\bar{p}\bar{q}}{N_2}}} = \frac{0.0589198 - 0.106383}{\sqrt{\frac{(0.067819)(0.932181)}{1222} + \frac{(0.067819)(0.932181)}{282}}} = -2.85737 < Z_{0.005} =$ $-2.5758.\quad p = 0.0316.$ So, reject H_0 , and conclude that $p_1 \neq p_2$.

7.6.2

$\hat{p}_1 = \frac{X_1}{N_1} = \frac{28}{175} = 0.16,\ \ \hat{p}_2 = \frac{X_2}{N_2} = \frac{43}{180} = 0.2389,\ \ \alpha = 0.05,$

$\bar{p} = \frac{28+43}{175+180} = 0.2,\qquad H_0: p_1 \geq p_2\ ,\qquad H_a: p_1 < p_2,$

$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}\bar{q}}{N_1} + \frac{\bar{p}\bar{q}}{N_2}}} = \frac{0.16 - 0.2389}{\sqrt{\frac{(0.2)(0.8)}{175} + \frac{(0.2)(0.8)}{180}}} = -1.85779 < Z_{0.05} = -1.645.$

$p - value = 0.0316$. So, reject H_0 , and conclude that $p_1 < p_2$.