EXERCISE SHEET MATH151

Section	Required Exercises
1.1	2, 3, 8(a,d,g), 11(a,c,e), 17, 28, 29(a,c), 31(c,e), 35(e), 40.
1.3	1(a), 3(a), 7, 9(c), 10(c), 11, 12, 14, 16, 19, 22.
1.4	1, 5, 7, 11, 14, 15, 19.
1.7	1, 3, 6, 9, 11, 15, 16, 17, 26, 31.
1.8	1, 3, 6, 9, 14, 19, 29, 34.
5.1	4, 5, 6, 8, 9, 12, 18, 20, 28, 31, 32.
5.2	**Q1**: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = 1,\ a_2 = 5, a_{n+1} = 2\,a_n + 3\,a_{n-1}\,, for\ all\ n \geq 2$. Prove that: $3^n \leq a_{n+1} \leq 2\,(3^n)\,, for\ all\ n \geq 1$ **Q2**: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_1 = a_2 = a_3 = 1, a_{n+2} = a_{n+1} + a_n + a_{n-1}$, for all $n \geq 2$. Prove that: a_n is an odd number for all n ≥ 1. **Q3**: Let $\{a_n\}$ be a sequence of integers defined inductively as: $a_0 = 1, a_{n+1} = a_n + 3^n\,, for\ all\ n \geq 0$. Prove that: $a_n = \frac{1}{2}\,(3^n + 1), for\ all\ n \geq 0$. **Q4**: Let $\{x_n\}$ be a sequence defined as: $x_1 = 1\,,$ $x_2 = 2,\ x_{n+2} = \frac{1}{2}\,(x_{n+1} + x_n), \forall\, n \geq 1$ Prove that: $1 \leq x_n \leq 2$. **Q5**: Let $\{y_n\}$ be a sequence defined as: $y_1 = 1, y_{n+1} = \frac{1}{4}(2\,y_n + 3), \forall\, n \geq 1$. Prove that: $(a) y_n < 2\,, for\ all\ n \geq 1$. $(b)\ y_n < y_{n+1}\,, for\ all\ n \geq 1$. **Q6**: Let $\{a_n\}$ be a sequence defined as: $a_0 = 2\,, a_1 = 4,\ a_2 = 6, a_n = 5\,a_{n-3}, \forall\, n \geq 3$. Prove that: $a_n\ is\ even, for\ all\ n \geq 0$. **Q7**: Let $\{b_n\}$ be a sequence defined as: $b_0 = 1, b_1 = 2, b_2 = 3, b_n = b_{n-1} + b_{n-2} + b_{n-3},$ $\forall\, n \geq 3$ Prove that: $b_n < 3^n\,, for\ all\ n \geq 1$.
9.1	1, 3, 6, 10, 11, 18, 26, 30, 32, 34(a,d,e), 36(d,e,h), 41, 50, 51, 52, 53, 56.
9.3	2(c,d), 3(a,b), 4(a,c), 7(a,b), 8(a,c), 13(c), 14(a,b,c), 18, 22, 24, 26, 27, 31, 32.
9.5	1, 3, 9, 16, 21, 22, 23, 26, 28, 36, 40(a), 42, 46, 47(b), 48(a), 55, 56(a,b).
9.6	1, 6, 9, 10, 11, 14, 20, 22.
10.1	3, 4, 5, 6, 7, 8, 9, 10.
10.2	1, 2, 3, 4, 5, 6, 20(a,b,c,d), 21, 22, 23, 24, 25, 26(a,b), 35, 36, 37, 38, 39, 40, 41, 53(a,b), 59, 60.
10.3	34, 35, 36, 37, 38, 39, 53, 54, 55.
10.4	1, 2, 3, 4, 5, 6.
11.1	2, 4, 6, 10, 16, 17.
11.2	1, 2.
11.3	None
11.4	2, 3, 4, 5, 6, 7(a, b, c, e, f), 8.
12.1	1, 2, 3(a), 4(a), 5(b,d), 6(c,d), 11, 28.
12.2	1, 2(a, b), 3(a, b), 11(a, b).
12.4	1, 2, 3, 4(c), 6(a,b), 12, 13, 14.