MODELLING CROWD BEHAVIOR AND MOVEMENT: APPLICATION TO MAKKAH PILGRIMAGE

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ABSTRACT

Existing research on pedestrian movement deals mainly with steady state flows under normal conditions. Virtually no scientific basis appears to be available in the literature to design and analyze strategies for the management of very large crowds associated with special events. The present paper develops a model of the dynamics of crowd behavior and movement, with particular reference to the "Jamarat" system, which is part of the annual Hajj (Muslim pilgrimage to Makkah, Saudi Arabia), where over two million pilgrims converge every year at the same time to perform this religious duty. The Jamarat system is a notorious crowd bottleneck, composed of three stone monuments, each surrounded by a ring, where pilgrims are supposed to throw pebbles in a given sequence. The dynamics of crowd behavior and movement around an individual Jamarah ring are modelled. The model consists of a set of simultaneous differential equations describing the principal processes that govern the dynamics of the system. These equations are solved numerically using a discretization of space and time, yielding a profile of the system's evolution. The use of the model is illustrated by examining the effect of different loading strategies and other possible control measures on the system's throughput. Limited field data provided the basis for calibrating the principal relations, including a speed-density relation for multidirectional movement, for the purpose of illustrating the model.

INTRODUCTION

Crowding occurs in many human activities. High volumes of pedestrians are processed daily through transportation terminals, high rise buildings, stadia, and various public facilities. Special events such as street festivals (e.g., Carnaval in Rio de Janeiro, Mardi Gras in New Orleans, Gion Matsuri in Kyoto) may generate millions of celebrating pedestrians in only a few city blocks. Efficient crowd management and control is a determinant of the quality and safety of the human experience in these settings. Crowd disasters in which people are seriously injured, sometimes fatally, can occur, and have occurred at sports events, religious gatherings and rock music concerts. Examples of these are reviewed in Fruin [1981].

Surprisingly little scientific work has been directed at the characterization of pedestrian traffic flow and crowd behavior, leaving considerable gaps in the knowledge base and methodological capabilities pertinent to this problem. The limited body of related studies can be classified, for convenience, into two broad categories: 1) steady-state pedestrian flows under normal conditions, and 2) crowding conditions, which correspond to the higher end of the concentration spectrum, and generally involve multidirectional flow patterns with strong interactions.

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The first class of studies deals primarily with unidirectional pedestrian flows encountered on a daily basis in high traffic areas such as passageways, stairways, plazas, sidewalks, etc. Following vehicular traffic flow models, passenger flows have been characterized in terms of macroscopic descriptors such as average speed, volume and density. Hankin and Wright [1958] established relations among these variables for unidirectional flow in subways. Other studies of these relations include Oeding [1963] on mixed urban traffic, Older [1968] on shoppers, Navin and Wheeler [1969] on students, Predtechnskii and Milinski [1978] on mixed mass movements, and Fruin [1971] on commuters. A synthesis of the above speed-density relations is shown in Figure 1, which depicts a family of curves abstracted from measurements reported by the investigators cited previously and converted to common units. As noted, all of these studies were restricted to streamlined and orderly pedestrian movements under steady-state conditions.

The second category of studies deals with the behavior of large crowds under non-routine circumstances, which often involve extremes of human emotion such as excitement, fear, religious fervor, anger or exaltation. Thus, movement is less orderly, multidirectional interactions are strong, and dynamic effects of greater importance than in the previous case. Because of the role of human emotion, several studies have been directed at the socio-psychological aspects of crowd behavior rather than its physical aspects [e.g., Freedman, 1975; Evans, 1975]. As such, they might have neglected the direct influence of a constructed environment on behavior, though they provide useful insights for the development of mathematical models of these phenomena.

Another group of studies have sought models of individual behavior in a crowd, recognizing the constructed as well as the behavioral environments. Wolpert and Zillman [1969] developed a simulation model of information dissemination and the resulting movement of individuals under panic. Stiltz [1970] attempted to relate pedestrian behavior patterns in high density congested circulation spaces to the physical attributes of these spaces. The study suggested a conceptual framework of how attributes of the built environment and those of pedestrians interact to determine the flow patterns, described in terms of route patterns, speed-density relations and configurations of standing people (static patterns). Baer [1974] proposed a disaggregate simulation model of multidirectional pedestrian movements in high density traffic. No explicit macroscopic speed-density model was employed, though individuals were assumed to continually adapt their velocity to the prevailing local density. Hirai and Tarui [1975] developed a mathematical model of crowd behavior in panic, and tried to capture factors such as mass psychology, the presence of signs or symbols to guide the crowd to exits in emergency, wall configurations, and the locations of emergency exits. Fruin [1981] proposed a conceptual framework to organize the factors contributing to crowd disasters, and to establish possible countermeasures. More recently, Harlow and Sandoval [1986] adapted a mathematical model originally developed for turbulence in fluids to describe the general mechanisms and processes underlying mob behavior.

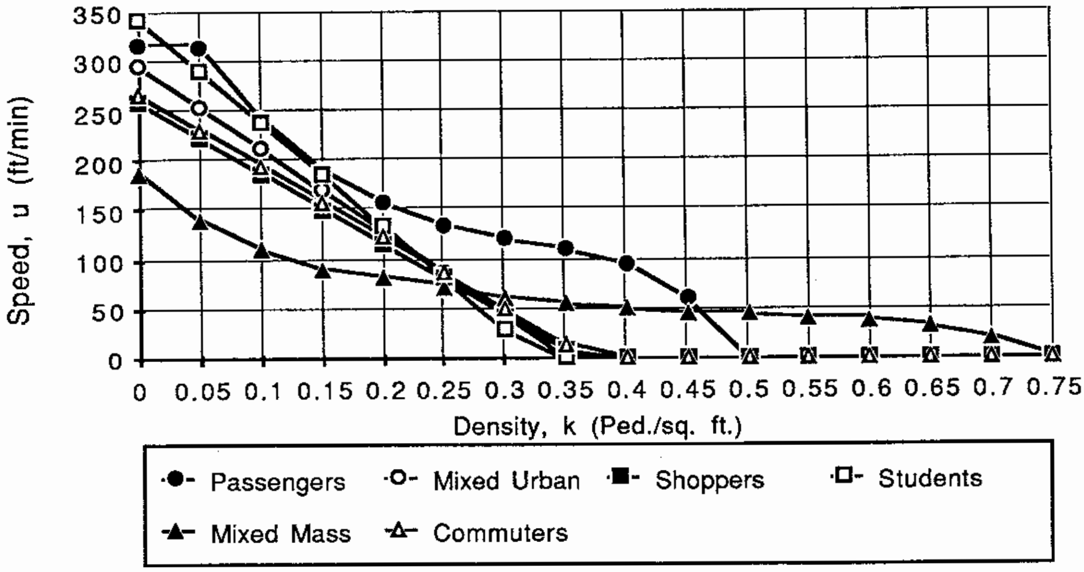

Figure 1. Speed-density relationships for normal steady-state pedestrian flows.

The limited body of disparate studies reviewed above does not provide a sufficient technical basis for dealing with special events' crowd movements. No models exist that describe the flow of dense crowds where the mean area occupied by an individual is less than 0.5 square meters. Such models are needed for design, safe operation and control of crowd facilities.

A remarkable example of pedestrian overcrowding is that of the Hajj, or Muslims' pilgrimage to Makkah, Saudi Arabia, which provides the motivating context for the present study. The Hajj consists of a set of prescribed rites at specific hours and days in assigned locations in and around Makkah. Each year, over two million pilgrims converge on Makkah (a city of about 300,000 [Makky,1981]), to perform this religious duty at the same time. The result is a crowded event of extraordinary magnitude. Bottlenecks can turn into disasters. One of the most difficult such bottlenecks is the relatively small Jamarat system. One of the required rituals of the Hajj consists of throwing (seven) small pebbles at each of the three pillars in the tight Jamarat area. In performing this ritual, many pilgrims suffer considerable hardship because of congestion and overcrowding due to space and time constraints. For example, in the 1983 pilgrimage season, a total of 64 fatalities were reported in this area due to overcrowding [HRC, 1984].

This paper develops a model of the dynamics of crowd behavior and movement, with particular reference to the Jamarat area. First, a general crowd behavior and movement model is developed, and its main functions are described, followed by a description of the specific application of this model to the Jamarat system. A numerical solution method is then outlined with results and sensitivity analyses. These focus on the effect of different spatial and temporal loading patterns, and of possible control actions, on the dynamic properties and throughput of the system. The paper is concluded with a discussion of major findings and recommendations.

GENERAL CROWD BEHAVIOR AND MOVEMENT MODEL

The model consists of a set of simultaneous partial differential equations describing the principal processes that govern the dynamics of the system. To introduce the main features of the model, it is helpful to view the physical space where crowding occurs as composed of an infinite number of cells of infinitesimal dimensions. The pattern of discretization (i.e., along Cartesian vs. polar coordinates) depends on the specific application, particularly the major prevailing movement directions. Each member of the crowd population is classified into one of a finite number of user classes, on the basis of his/her immediate goal while in the crowd. For example, a model of a crowd leaving a stadium after a game might have just one class of users, all with the same goal of leaving the place. On the other hand, a model of a crowd in a rail transit terminal may have as many classes as the number of trains scheduled to depart during the period of interest, or may simply have just two classes, arriving and departing passengers.

For each class of users, the major applicable types of movements are identified. Generally, four types of movements are possible: forward, backward and lateral movements to either side of the infinitesimal cell, with corresponding speeds U^F, U^B, U^{LL}, and U^{LR}, respectively. These speeds are (possibly non-linear) functions of the prevailing concentrations of the various classes of users in the cell under consideration as well as neighboring cells, as discussed hereafter.

Consider a crowd in an area discretized into a rectangular grid with cells of length Δx and width Δy. Suppose that we have M classes of users, each of which can move in each of the possible four directions. We state a general balance law that constitutes the basic framework of our model. For simplicity, let us restrict ourselves to a situation in which all variation is with respect to the single spatial coordinate x. Denote by $K^m(x,t)$ the concentration of class m users in the cell centered at x, at time t. For convenience, assume that $K^m(x,t)$ varies smoothly with both t and x. In a rectangular cell C bounded by unit lengths at the sides x=xo and x=xo+Δx, the general balance law states that the rate of change of users in cell C equals the net creation rate of users in cell C plus the net rate at which users flow into cell C across its boundaries.

Let $S^m(x,t)$ be the net creation rate of class m users, defined as the birthrate per unit area $B^m(x,t)$ minus the corresponding death rate $D^m(x,t)$ in a given cell C. $D^m(x,t)$ consists of: 1) the number per unit time of class m users who change their goal, while in the crowd, and thus switch to another class of users in that same cell, $SW^m(x,t)$, plus 2) those that leave the system for whatever reason, $E^m(x,t)$, (i.e. $D^m(x,t)= SW^m(x,t) + E^m(x,t)$). Meanwhile, $B^m(x,t)$ is the sum of the respective rates with which all users from other classes switch to class m, plus any other source of class m users, $G^m(x,t)$. That is

$B^m(x,t) = \sum_{i=1}^{M} SW^{i,m}(x,t) + G^m(x,t)$, m=1, 2, ...,M, where $SW^{i,m}(x,t)$ is the switching rate of class i users into class m.

Let $Q^m(a,t)$ denote the flux density, defined as the net rate at which class m users cross a unit length at the line x=a in cell C. In general, $Q^m(a,t)$ is given by the product of the prevailing speed of movement and the level of concentration of class m users near the line x=a, i.e. $Q^m(a,t) = U^m(a,t) * K^m(a,t)$. A net crossing rate is positive if it is in the direction of increasing x.

Using the above definitions, the general balance law takes the mathematical form

$$\partial/\partial t \int_{xo}^{xo+\Delta x} K^m(x,t)dx = \int_{xo}^{xo+\Delta x} S^m(x,t)dx \quad + Q^m(xo,t) - Q^m(xo + \Delta x,t), \tag{1}$$

for any constants xo and Δx. Note that we have counted only the users' flow through the unit length boundaries x=xo and x=xo+Δx. Because of our assumption that there is no variation with y, then during any time interval exactly as many users flow into cell C through one of the other two sides (y=yo and y=yo+1) as flow out through the opposite side.

The balance law (eq. 1) can be rewritten as follows:

$$\partial/\partial t \quad [K^m(x1,t)\, \Delta x] = S^m(x2,t)\, \Delta x + Q^m(xo,t) + Q^m\,(xo+\Delta x,t) \tag{2}$$

where $xo \le x1 \le xo+\Delta x$, $xo \le x2 \le xo+\Delta x$, and $\Delta x > 0$. Dividing by Δx, taking the limit as $\Delta x \to 0$, and rearranging, we obtain the usual continuity equation, which is stated as follows:

$$\partial/\partial t \quad K^m(x,t) + \partial/\partial x \quad Q^m(x,t) = S^m(x,t),\ \forall\ x, \text{ and } m=1, 2, ...,M. \tag{3}$$

Now, if we relax the assumption of one dimensional variation of concentration, and consider the more general case of concentration variation along both spatial coordinates x and y, the corresponding continuity equation will be given by:

$$\partial/\partial t \ \ K^m(x,y,t) + \partial/\partial x\ Q^m(x,y,t) + \partial/\partial y\ Q^m(x,y,t) = S^m(x,y,t), \forall x,\ y, \text{ and } m=1, ..., M. \tag{4}$$

This system of governing partial differential equations can be solved numerically by discretizing in time and space. Such solution yields a profile of the system's evolution and allows the computation of various performance measures and figures of merit. The application of this general model to the Jamarat system of the Hajj is described next.

THE JAMARAT MODEL

We first describe the configuration of the Jamarat system and the nature of the crowd activities there, followed by the elements of the mathematical model of the underlying processes.

Description of Jamarat System Configuration

The Jamarat system is composed of three stone monuments, symbolizing the devil, which are supposed to be stoned (i.e. pebbles thrown at each of them) by pilgrims in a given sequence, starting with the first, "Small", second, "Middle", then the third, "Big" one, as shown in Fig. 2. Each monument is enclosed by a ring of about 16 meters (52 feet) diameter, referred to hereafter as the Jamarah ring, to collect the pebbles. Pilgrims squeeze through the crowd to a position from where to throw the pebbles to get into the ring, and, preferably, hit the monument. At the same time, others who have finished stoning squeeze and push their way out. On each of the three days of stoning at the Jamarat, a large number of pilgrims arrive at about the same time creating extremely crowded conditions, which occur at the same time every year [HRC, 1984]. To meet the demand of an enormously grown mass of pilgrims, an upper level was added to the Jamarat system in 1975, providing two levels from which stoning can be performed (Fig. 2).

The general model presented in the previous section is now applied to the Jamarat system, focusing on the processes taking place on the upper level around an individual Jamarah ring.

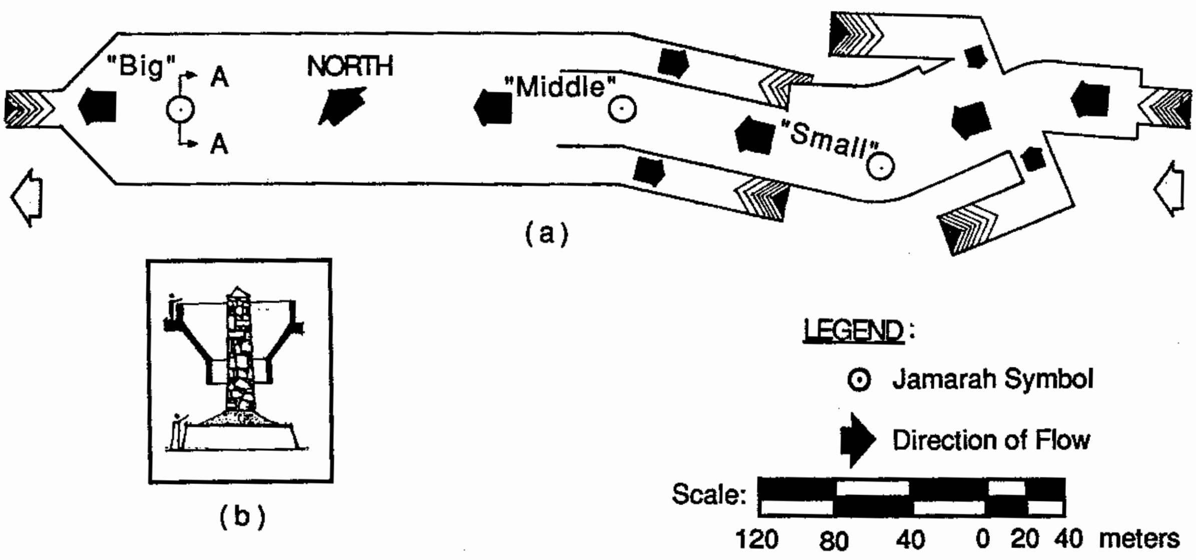

Figure 2. The Jamarat system: (a) plan view, (b) section A-A (not drawn to scale).

Cells: Basic Units of Analysis

In this application, the major movements are in the radial direction towards and away from the Jamarah ring. Thus, the space surrounding the Jamarah ring is divided into a number of rings, NRING, each one meter wide. Each ring is divided into a number of cells, NCELL, by taking NCELL equally spaced radials from the center of the Jamarah (stone monument) to the outer perimeter of the external ring (i=NRING+1) as shown in Fig. 3. Each cell is identified by its (i,j) index (i=2, 3, ..., NRING+1; j=1, 2, ...,NCELL; i=1 identifies the Jamarah ring itself). Note that the areas of cells in the same ring are equal, while the cell area increases with ring index i.

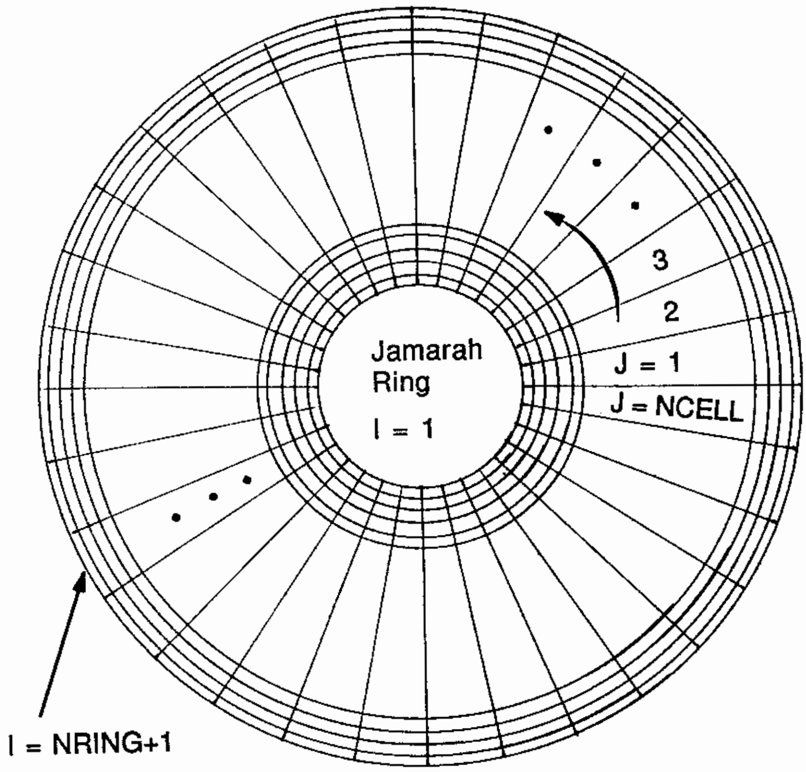

Figure 3. Space discretization around the Jamarah ring.

User Classification and Movements

The users of the Jamarah system (pilgrims) are classified into two major classes: class 1, who have the intention of performing stoning, or "Rajm", and class 2, who intend to leave the Jamarah, after having completed the rite. Accordingly, K(i,j), the total concentration of users (in persons/m^2) in cell (i,j) is the sum of the respective concentrations of the two classes, i.e., $K(i,j) = K^1(i,j) + K^2(i,j)$. For each class, two types of movements around the Jamarah can be identified: radial (forward and backward), and circumferential (clockwise and counter-clockwise). Correspondingly, four types of speeds are defined for each class (Fig. 4):

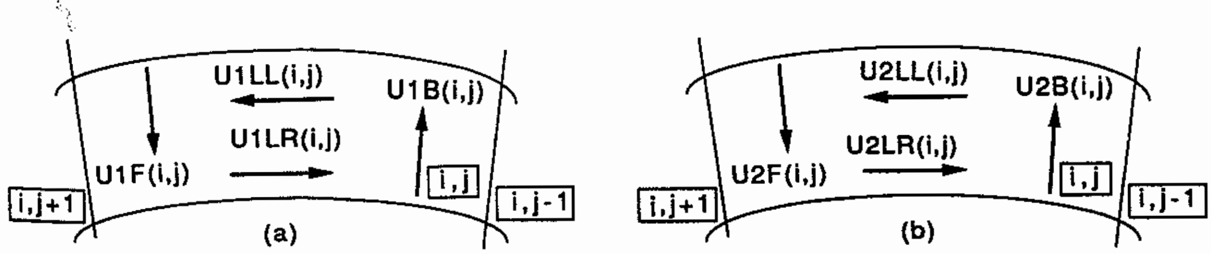

Figure 4. Types of speeds of movement in a typical cell (i,j) for:
(a) class 1, and (b) class 2 users.

U^{mF}_{ij} = speed of class m users (m=1, 2), in cell (i,j), moving towards the Jamarah ring (i.e. forward radial direction), in meters per hour.

U^{mB}_{ij} = speed of class m users in cell (i,j) moving away from the Jamarah ring (i.e. backward radial direction), in meters per hour.

U^{mLR}_{ij} = speed of class m users in cell (i,j) moving in the clockwise lateral direction.

U^{mLL}_{ij} = speed of class m users in cell (i,j) moving in the counterclockwise lateral direction.

For simplicity, it is reasonable to assume that $U^{1B}_{ij} = U^{2F}_{ij} = 0$ (i.e. those who have yet to perform "Rajm" do not go backward, and those who have completed it no longer move towards the ring). Furthermore, the concentration fluxes in the radial direction are the dominant ones relative to the lateral fluxes (between a given cell and its neighbors in the same ring). The latter are assumed to occur only if the difference in the concentrations of the two adjacent cells exceeds a certain threshold, which can be obtained experimentally. In the next section, this lateral flux is specified as a function of the difference between the neighboring cell concentrations.

The radial speed of either class of users in cell (i,j) is thought to be a function of both types of concentrations, with a minimum shuffling speed U_{min} at or above jam concentration K_{jam} that is:

$$U^{1F}_{ij} = f_1(K^1(i,j), K^2(i,j)) \tag{5a}$$

$$U^{2B}_{ij} = f_2(K^1(i,j), K^2(i,j)) \tag{5b}$$

where the functional forms $f_1(.)$ and $f_2(.)$ need to be specified. The specific forms used in this model are described in the next section in connection with the numerical examples.

The general balance law derived in the previous section for a rectangular cell can be adapted to the Jamarat model, with its different spatial discretization pattern. It is expressed hereafter in difference form, as used in the numerical discrete time solution approach (Fig. 5):

For class 1 users:

$$A(i,j) * (K^1(i,j))_{t+\Delta t} = A(i,j) * (K^1(i,j))_t + \{Q^{1F}(i+1,j) - Q^{1F}(i,j) - SW(i,j) + Q^{1LR}(i,j+1) - Q^{1LL}(i,j) + Q^{1LL}(i,j-1) - Q^{1LR}(i,j)\}_{t+\Delta t},\ \forall i,j \tag{6a}$$

For class 2 users:

$$A(i,j) * (K^2(i,j))_{t+\Delta t} = A(i,j) * (K^2(i,j))_t + \{Q^{2B}(i-1,j) - Q^{2B}(i,j) + SW(i,j) + Q^{2LR}(i,j+1) - Q^{2LL}(i,j) + Q^{2LL}(i,j-1) - Q^{2LR}(i,j)\}_{t+\Delta t},\ \forall i,j \tag{6b}$$

where:

$A(i,j)$ = area of cell (i,j), in m^2.

$Q^{1F}(i,j)$ = net flux of class 1 users, in time step Δt, from cell (i,j) in the radial forward direction, in persons.

$Q^{2B}(i,j)$ = net flux of class 2 users, in Δt, from cell (i,j) in the radial backward direction, in persons.

$Q^{mLR}(i,j)$ = net flux of class m users, in Δt, from cell (i,j) in the lateral clockwise direction, in persons.

$Q^{mLl}(i,j)$ = net flux of class m users, in Δt, from cell (i,j) in the lateral counter-clockwise direction, in persons.

SW (i,j) = number of class 1 users switching into class 2 users, in cell (i,j) in Δt, after finishing stoning. This quantity acts as a sink term for class 1 and a source term for class 2 users.

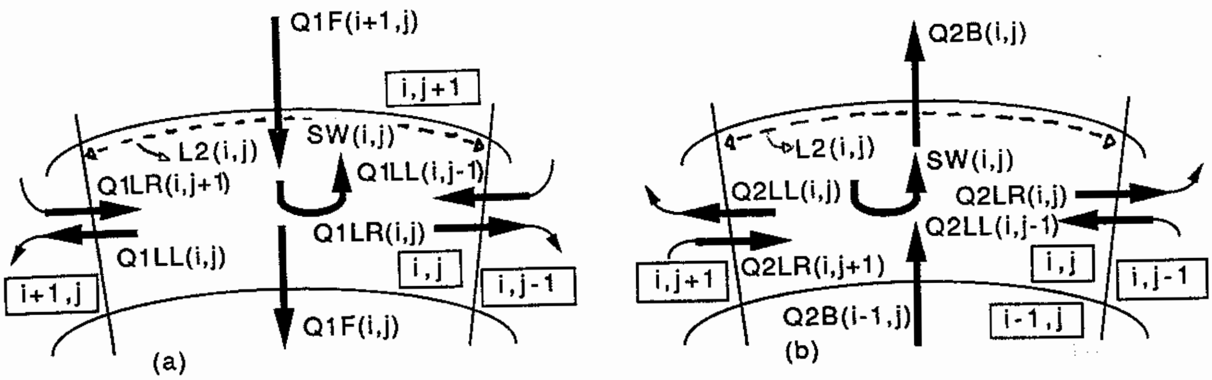

Figure 5. Operating fluxes at a typical cell (i,j) for: (a) class 1, and (b) class 2 users.

Switching Mechanism

The number of class 1 users switching to class 2 in cell (i,j) in a given Δt, SW(i,j), depends on two principal factors: 1) the fraction of class 1 users in the cell who perform stoning from that cell, FR(i,j); and 2) the time it takes an individual to perform "Rajm", TRAJM(i,j). The relation between SW(i,j), FR(i,j) and TRAJM(i,j) can be expressed as follows:

$$SW(i,j) = A(i,j) * K^1(i,j) * FR(i,j) * \Delta t / TRAJM(i,j) \tag{7}$$

where all terms are as previously defined.

The fraction of class 1 users switching to class 2 users in cell (i,j), FR(i,j), depends on two main factors: the distance r_i from the Jamarah ring and the prevailing pilgrim concentration level in cell (i,j). This is expressed as:

$$FR(i,j) = \begin{cases} 1 \text{ if } r_i \leq 1 \\ g_1(r_i)\, g_2(K(i,j)) \quad \text{otherwise} \end{cases} \tag{8}$$

Equation 8 states that the fraction is initially equal to unity at the rim of the Jamarah ring, where all class 1 users have to switch regardless of the prevailing concentration level. Beyond the first ring, the fraction is expressed as the product of a function $g_1(r_i)$ that decreases with distance from the rim until it equals zero at the maximum possible throwing distance from the Jamarah ring, R_{max} (i.e. at ring i= R_{max} +1). On the other hand, $g_2(K(i,j))$ is expected to range from unity at maximum concentration, K_{max}, to zero at a threshold concentration, KFR?, at which pilgrims would prefer to proceed further to perform Rajm from a closer ring.

The duration of the stoning process (pilgrims residence time), TRAJM(i,j), is anticipated to be an increasing function of the distance from the Jamarah ring, because of the extra time that may be needed for repeated orientation and targeting to finish throwing the seven pebbles as one moves further away from the ring.

Flux Constraints

The flux quantities defined above need further specification. Let us consider cell (i,j), shown in Fig. 5. The flux of class 1 users from cell (i+1,j) to cell (i,j), $Q^{1F}(i+1,j)$, is given by the product of the prevailing speed and concentration of class 1 users in cell (i+1,j), i.e., $Q^{1F}(i+1,j)=U^{1F}(i+1,j)*K^1(i+1,j)*L1(i+1,j)*\Delta t$, where L1(i,j) is the length of the interior edge of cell (i+1,j) in meters, subject to the following two constraints:

1) the total number of pilgrims, transferred out of a given cell in a given Δt cannot exceed the number of pilgrims available to transfer from that cell.
2) the total number of pilgrims, transferred into a given cell in a given Δt cannot exceed the number of spaces available to accommodate them in the destination cell.

Since we have fluxes interacting simultaneously over space, the numerical solution must check for these constraints and include a logic for allocating available space among all competing movements. In this case, available space at the end of each Δt is allocated in proportion to the prevailing concentration in its origin cell. On the other hand, when the total number of users who wish to transfer exceeds the number of users available in the cell, the transfer fluxes are recalculated in proportion to the initial values.

Additional Aspects of the Numerical Solution

So far in our model, we have studied the movement of pilgrims into and away from the Jamarah ring by considering a typical cell from each ring. Thus we are moving class 1 users along a path of variable, gradually diminishing, width. The funnelling effect is captured in this model by: 1) the width of the path (cell), which is used in finding flux quantities between adjacent rings, and 2) the decreasing cell area as we move toward the Jamarah ring, which results in an increase in concentration as a given number of class 1 users move in that direction.

The system can be loaded in any desired temporal and spatial pattern. As entering class 1 pilgrims approach the external ring, i = NRING+1, at the preset rate of NRATE pilgrims per

hour, it is possible that the receiving cell may not be able to accommodate all those desiring to enter in that Δt. In such case, those that could not enter the system are entered in a queue, instead of assuming that they are lost to the system. These waiting pilgrims will be entered into the cell in NRING+1 as soon as they can be accommodated in the subsequent time increments.

Finally, the choice of time step and space discretization scheme, when using numerical methods to solve partial differential equations, has important implications for the properties of the solution. Of course, it is generally desired that both Δr and Δt be small enough to approximate the assumed underlying smooth function of K(i,j) with respect to r and t. In addition, the ratio $\Delta r/\Delta t$ should be greater than the mean free speed U_{max} to ensure that no pilgrim transfer is possible from one cell to another non-adjacent one in any time step. The values used in the solution of this model are $\Delta r = 1$ meter, and $\Delta t = 0.0001$ hour, for a ratio of 10 km/hr (>> U_{max}).

NUMERICAL SOLUTION: RESULTS AND ANALYSIS

In this section, we operationalize the Jamarat system model and present the results of several numerical simulations to illustrate the dynamics of the solution, the effect of key variables and parameters, and the model's application to the investigation of the system's properties. The assumptions used in these simulations, including the functional forms and parameters of the principal underlying relations, are described next. Illustrative results are then presented for the following: 1) analysis of system throughput, 2) the dynamics of concentration build-up and dissipation, and 3) the effect of possible design and control strategies that might improve system performance.

Basic Assumptions

One of the principal relations used in the Jamarat model is the speed-concentration model (eq. 5), which differs from those reviewed in the introduction in that it should capture the interaction between user classes and apply to high concentration data. Field data, in the form of time lapse photography, were collected at the Jamarat system, and partially used to specify and calibrate a bi-directional speed-concentration relation to use for illustrative purposes in the operationalization of the Jamarat system model. The details of the empirical work are outside the scope of the present paper, and are reported elsewhere [Algadhi, 1989]. The function used here takes the following form:

$$U_{ij}^{1F} = 2096 * (1 - (K^1(i,j) + 0.56 * K^2(i,j))/K_{jam})) \quad (9)$$

where U_{ij}^{1F} is in meters per hour and K_{jam} was estimated to be 5.85 persons/m^2. One interesting aspect of this relation is that the impedance, on an individual's speed, of pilgrims moving in the same direction is about twice that of those moving in the opposing one. The speed of an individual moving within a group of people in one direction is controlled by the aggregate speed of the group, leaving the pilgrim little control over his/her own speed. On the other hand, an individual going against the main flow can often exert some degree of will in

maneuvering his way in the crowd. Note that the same functional specification was used for U_{ij}^{2B}, with $K^1(i,j)$ and $K^2(i,j)$ transposed.

As explained in the previous section, lateral movement is modelled differently. By analogy with the treatment of interlane concentration oscillations in multilane highway traffic [Gazis et al., 1962], we assume that the exchange of pedestrians between two neighboring cells is proportional to the difference of their concentrations, and only when this difference exceeds a given threshold, that is:

$$Q^{LR}_{ij} = \begin{cases} \alpha\ (K(i,j))^{\beta} & \text{if } K(i,j) - K(i,j\text{-}1) \geq \Delta K_{min} \\ 0 \text{ otherwise} \end{cases} \tag{10}$$

Analysis of limited relevant field data has suggested that the threshold level ΔK_{min} is about 0.1 persons/m^2, and yielded parameter estimates of α= 915.3 pilgrims/hour/meter and β= 0.06 for purposes of the present illustration [Algadhi, 1989]. It should be noted that this relation can only be expected to behave reasonably well if the difference in concentration between adjacent cells is less than one person per square meter, which is generally the case in crowded situations such as ours.

Another unique relation used in the Jamarat system model is the mechanism of switching from user class 1 to class 2. As explained earlier, the rate of switching in a given cell(i,j), SW_{ij}, depends on the two quantities FR(i,j) and TRAJM(i,j). Field data collected to calibrate this relation involved tracking pilgrims as they performed the required rite. From these trajectories, it was possible to obtain tentative estimates of the quantities governing the switching mechanism. The following relations were obtained and used in our simulation runs (see eq. 8):

$$g_1(r_i) = e^{(0.5(1-r_i))}, \text{ for } r_i=2, 3, \ldots, R_{max} \tag{11a}$$

$$g_2(K(i,j)) = \begin{cases} 0, & \text{for } K(i,j) \leq KFR2 \\ (K(i,j) - KFR2)/(K_{max} - KFR2), & \text{for } KFR2 \leq K(i,j) \leq K_{max} \end{cases} \tag{11b}$$

where R_{max} is the maximum throwing distance, which was found to be 12 meters from the perimeter of the Jamarah ring; KFR2 is the the threshold concentration level defined earlier, for which a value of 3 persons/m^2 is used here; and K_{max} is the maximum concentration possible under packed conditions (a value of 7 persons/m^2 is used hereafter).[1]

The other component of the switching mechanism, namely stoning time TRAJM, was obtained from field data as follows:

[1]Note that K_{max} is different from K_{jam}, which is a calibrated parameter of the speed-density model, as defined in connection with Eq. (5).

$$TRAJM(i,j) = 12.5 + 2.5\, r_i, \qquad \text{for } r_i=1, 2, 3, \ldots, R_{max} \qquad (12)$$

where TRAJM(i,j) is in seconds. Thus the average stoning time varies from 15 seconds at the rim of the Jamarah ring to about 43 seconds at the maximum throwing distance of 12 meters.

Loading Strategies and Throughput Analysis

The Jamarat system was simulated under different pilgrim loading strategies, defined in terms of their spatial and temporal characteristics. Two different spatial patterns are considered. In the first, pilgrims approach the Jamarah from all surrounding directions; in the second, they load only from the ring's half side that first faces the pilgrims approaching from the main entrance. For a given spatial pattern, a variety of temporal loading patterns can be simulated. In the present exploration of maximum system throughput, we loaded the system continuously at a uniform rate until a steady state was reached.

Figure 6 shows the maximum throughput as a function of loading rate when loading only from one side of the Jamarah. For this spatial loading pattern, and using the parameter values and relations described above, the maximum system throughput achieved is about 59,000 pilgrims per hour. Loading at rates higher than the optimum causes the throughput to decline slightly, with some fluctuations, but does not appear to shut off the system. Similarly, the maximum throughput of the system for the case of uniform spatial loading (from all directions) was found to be about 72,000 pilgrims per hour. However, loading at higher rates caused the system to break down. Overall system behavior in both cases appears reasonable; in the first case, one half of the Jamarah is used for loading while the other half functions as an evacuation outlet, whereas in the second case, the system gets locked up beyond a maximum loading rate which it can absorb, because masses of each class of users eventually block the other's progress.

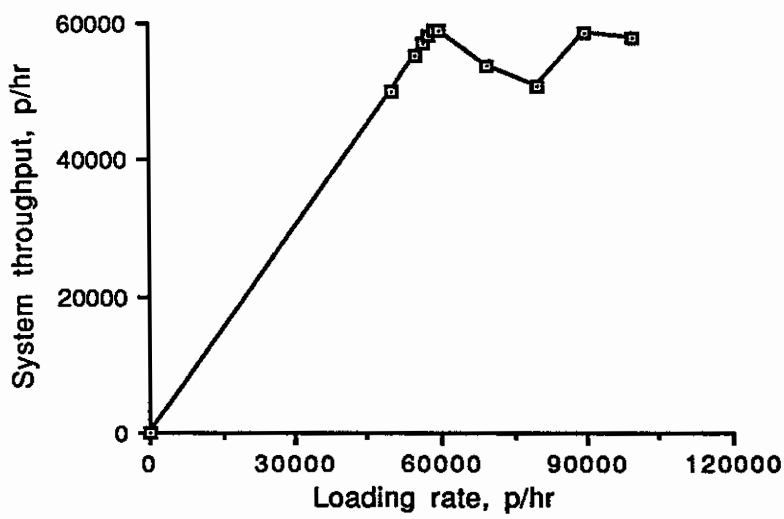

Figure 6. Maximum throughput of the Jamarah for semi-uniform loading.

It is interesting to point out that a field study conducted independently at the Jamarat measured the maximum throughput of the system at 69,000 pilgrims per hour (top level only)

for the case of loading from all directions (during the first day of stoning when pilgrims are required to stone only the third Jamarah which can be approached from any direction), and at 43,000 pilgrims per hour for loading only from one half of the Jamarah (during second and third days of stoning)[HRC, 1985]. Our findings are in close agreement with these measurements, even though the measurement procedure by the HRC is not directly comparable to the manner in which throughput is calculated in the simulation.

Dynamic Behavior

The dynamic behavior of the Jamarah is examined by injecting pilgrims into the system at a constant rate over a given period of time (sufficient to illustrate concentration build-up), then shutting off the inflow and simulating until the system is completely evacuated. The crowd build-up and dissipation processes are illustrated here for different loading rates for the two spatial loading patterns described previously.

When loading uniformly from all directions around the ring, the model solution includes pilgrim movement in the radial direction only. No lateral movement occurs because of the assumption that lateral fluxes take place only when the difference in concentration levels between adjacent cells (in the circumferential direction) exceeds a certain threshold. Thus the concentration profiles, depicting the variation of concentration with distance from the ring, within a given sector, are identical for all sectors. Figure 7a shows the concentration profile of class 1 and class 2 users for a typical sector at a selected time (t=0.3 hours). Figure 7b depicts the temporal evolution of the concentration levels of each user class in a typical cell at the rim of the Jamarah ring (i=2). Both plots correspond to loading at the near optimal rate of 72,000 pilgrims per hour. Figure 7a illustrates the levelling off of concentration beyond the third ring. In addition, at steady state, users of both classes share the space in rings 5 through 21 equally, while rings closer to the Jamarah ring have different mixes of the two classes.

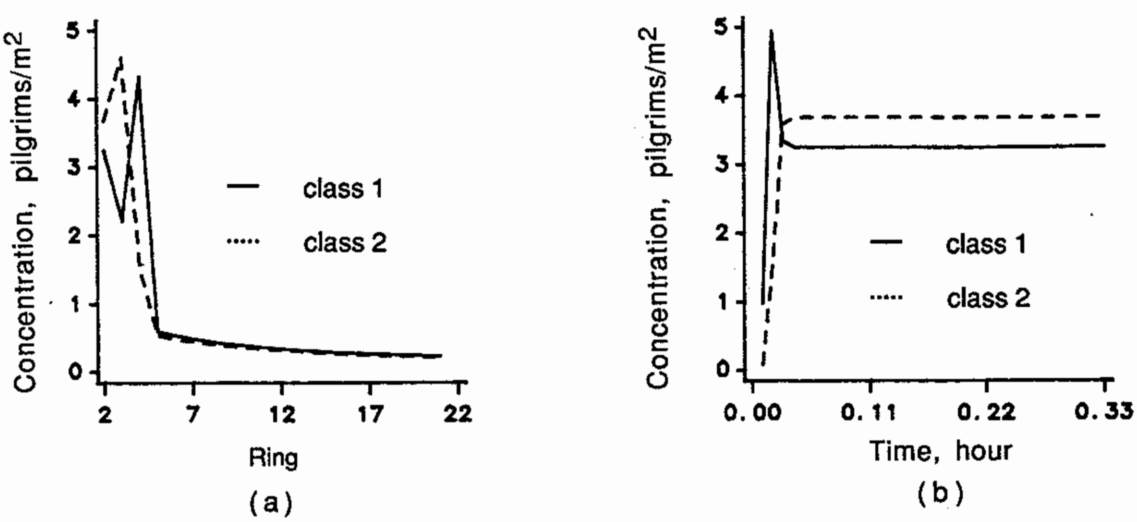

Figure 7. Concentration profiles of class 1 and class 2 users, for 72,000 pilgrims/hr. uniform loading, at: a) a typical sector at 0.3 hrs., and b) a typical cell in ring 2.

The other spatial loading pattern (i.e., from one side only) exhibits more interesting dynamic features than the uniform case. Two representative runs were made for this purpose, one with a loading rate of 50,000 pilgrims per hour, which is less than the maximum system throughput (59,000), and the other with 100,000 pilgrims per hour. For the first solution, Figs. 8(a-c) depict the dynamics of the concentration profiles of both classes, class 1, and class 2 users, respectively, at the rim of the Jamarah ring (i=2). It can be noted that at the rim of the Jamarah, class 1 users tend to spill over to the less congested space on the other side of the Jamarah, where no loading is taking place, to get a closer position for performing stoning. On the other hand, class 2 users try to leave this ring in the radial direction, rather than laterally, in order to escape the crowd pressure.

The dynamic behavior of the system under extreme congestion is presented in Figs. 9 and 10 for a selected ring (i=2) and sector (j=13), respectively, while choking the system at a rate of 100,000 pilgrims per hour. The tendency of class 1 users to go laterally and class 2 users to go radially, at the rim of the Jamarah ring, is seen in Fig. 9. This figure illustrates another interesting aspect of the system's dynamic behavior: after about 6 minutes of loading, a slight drop in the concentration takes place at the rim of the Jamarah, but only for the middle sectors of the loading side; this trend continues until loading is stopped (at 20 minutes) at which time concentration starts building up again in those cells. This may be explained by examining the relative proportions of each class of users in that area. Over time, the proportion of class 2 users increases in this middle part due to switching (i.e., completing the stoning), thereby encouraging arriving pilgrims to go sideways around the Jamarah to a less congested space, in order to avoid high interaction with those leaving the rim of the Jamarah ring. Figure 10 shows that the concentration of class 1 users, in sector 13, decreases as they proceed towards the Jamarah ring while that of class 2 users increases. Note that sector 13 is located on the loading side of the Jamarah, and thus is representative of the situation discussed above.

Design and Control Strategies

To attain the maximum throughput of the Jamarah system under the current situation, the analysis presented here suggests that one should maintain the input rate at the optimum level. However, to improve this maximum throughput, we may consider changing the design of the physical system and/or affecting the behavior of the crowd in one way or another so as to result in an improvement to the efficiency of the system.

From the design standpoint, we can change the size of the Jamarah ring itself. The circumference of a circle increases linearly with the radius, so in order to allow more pilgrims to stone simultaneously at the Jamarah, one could increase the circumference of the Jamarah ring. The simulation model was used to explore the maximum throughput of a system with a Jamarah ring of 12 meters radius, instead of the present 8 meters). The maximum throughput was found to be about 100,000 pilgrims per hour when loading uniformly from all directions. So one way to increase the service rate of the system is by increasing the Jamarah ring size.

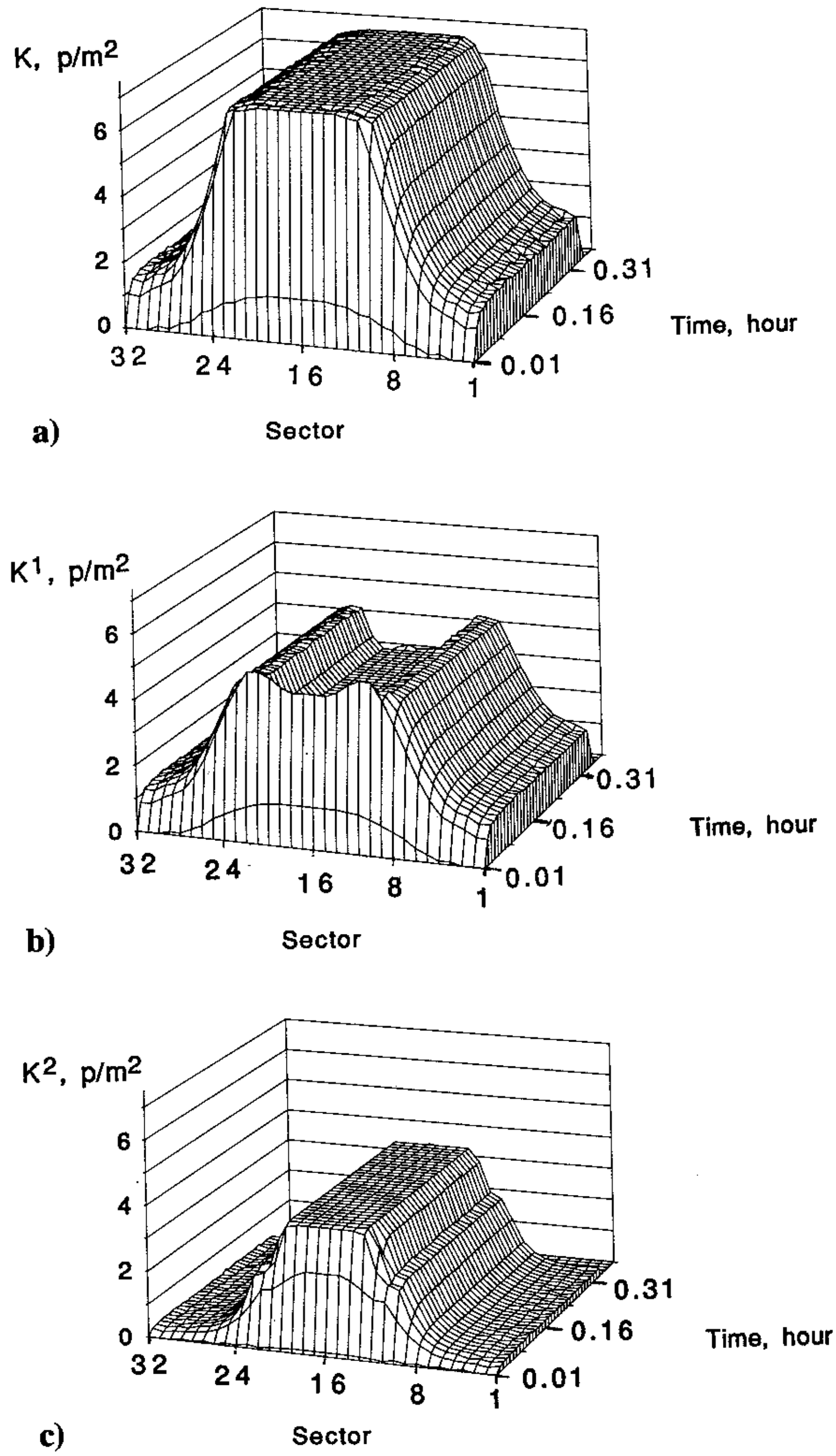

Figure 8. Concentration profiles dynamics, at ring 2, for 50,000 pilgrims/hr. semi-uniform loading, for: a) both classes, b) class 1, and c) class 2 users.

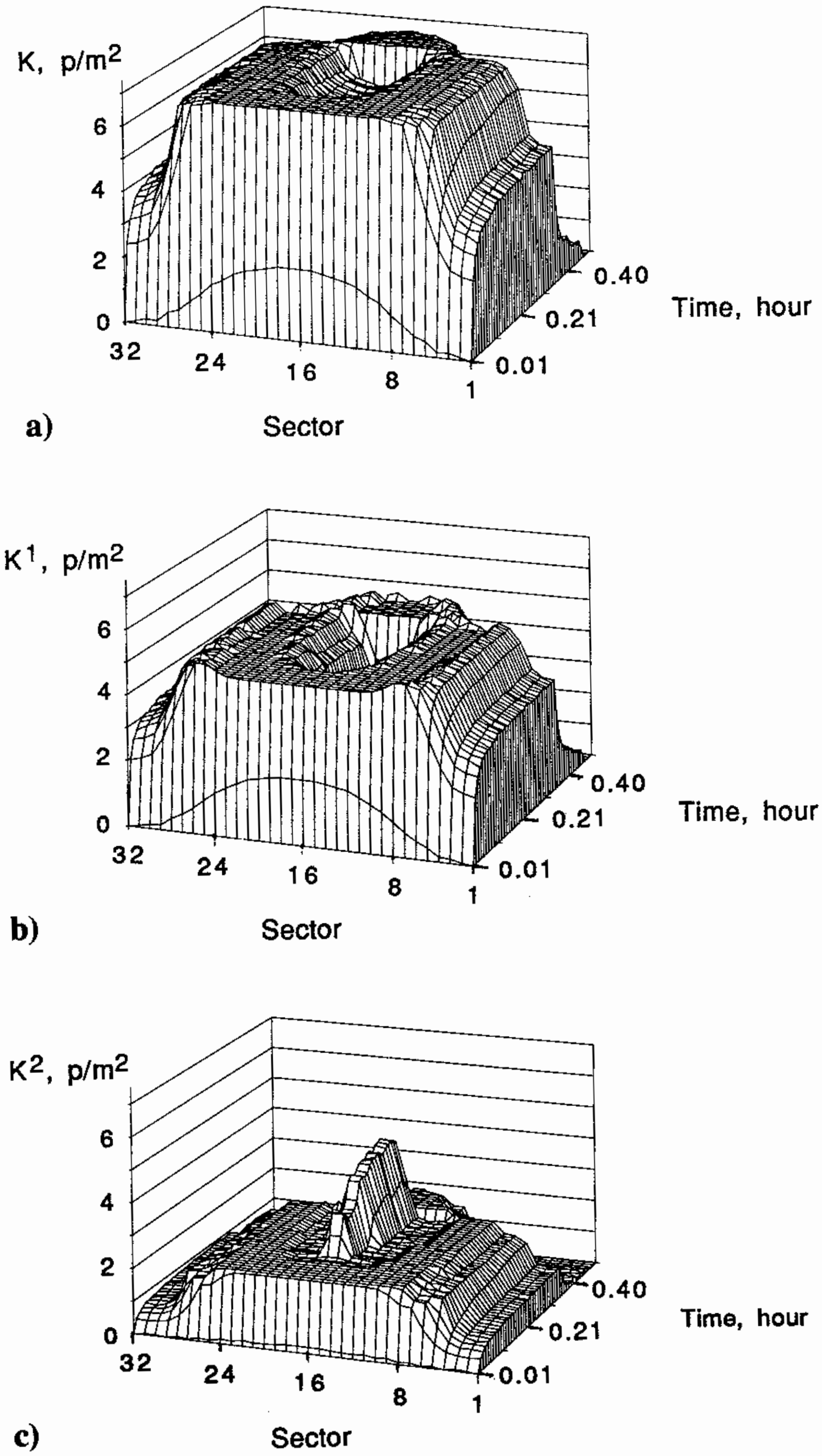

Figure 9. Concentration profiles dynamics, at ring 2, for 100,000 pilgrims/hr. semi-uniform loading, for: a) both classes, b) class 1, and c) class 2 users.

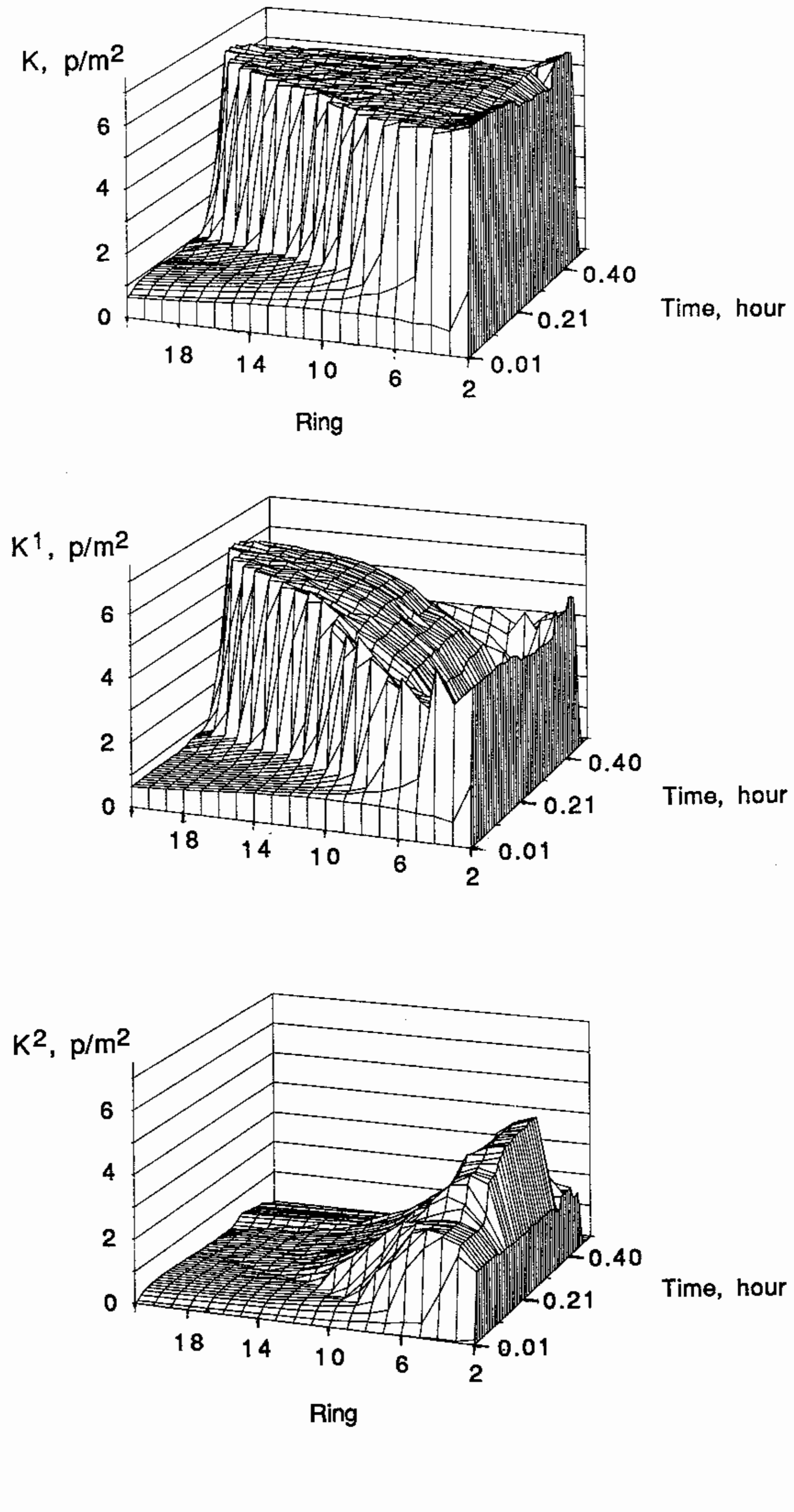

Figure 10. Concentration profiles dynamics, at sector 13, for 100,000 pilgrims/hr. semi-uniform loading, for: a) both classes, b) class 1, and c) class 2 users.

Even if we keep the current physical system unchanged, we can encourage pilgrims approaching the Jamarah to perform stoning sooner than is currently done (i.e. having a larger fraction of class 1 users switch from a given distance from the Jamarah ring). This can be reflected in our model by a different value of the parameter γ in the relation $g_1(r_i) = e^{\gamma(1-r_i)}$ given in Eq. 11a. In the current model we use $\gamma = 0.5$, but using values of 0.1 and 0.3 instead gave a maximum throughput of 85,000 and 77,000 pilgrims per hour, respectively, when loading from all directions. In practice this could be implemented through educating pilgrims to stone sooner, and possibly encouraging them to do so on the spot through a public address system. Also, one could elevate the position of the throwers by slightly inclining the surface towards the Jamarah ring.

CONCLUDING REMARKS

The efficient and safe operation of heavily crowded facilities used in special circumstances by unfamiliar users is a major concern to those responsible for the design and control of such public events. The theoretical and numerical solution framework presented in this paper consists of decomposing a complex system, such as a crowd and its environment, into its basic elements and underlying processes, and then capturing the principal interactions among these elements. The approach is not limited to the Jamarat system, though the primary contribution of the paper pertains to this particular system.

In the Jamarat model, the underlying variables were defined, relations among them were established and the principal processes that govern the dynamics of the system were described. This provides a basis for further and deeper understanding and characterization of crowd behavior and movement, which is possible only with extensive additional observational work to obtain measurements for the specification and calibration of the mathematical relations identified in the present work. To date, only a limited amount of observational work has been undertaken, primarily due to the lack of sufficient resources, but also in part due to the absence of an adequate theoretical framework to guide data taking and analysis. Considerable effort was spent on observing the overall system behavior in order to derive the basic conceptual aspects of the model and the principal underlying processes. Only limited measurements could be taken to calibrate the relations described in this paper in order to provide a realistic basis for illustrating the behavior of the model solution. While the speed-concentration relation obtained cannot be considered definitive without additional observations, it yields valuable insights into the relative effect of opposing movements. Overall, the behavior of the system appears reasonable, and the comparison with limited available data on throughput is favorable.

Many avenues of significant future research are opened by the present work. In addition to the essential observational studies just mentioned, a wealth of challenging questions are raised for further theoretical development. Crowd behavior is a complex phenomenon that has received very little attention to date; the present work was intended as a step towards better characterization of such phenomena.

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