Probability ,Conditional Probability and Independence

Addition rule: $\boldsymbol{P(A) + P(B) - P(A \cap B)}$

Disjoint Events:

$If\ A \cap B\ = \phi\ ;\ we\ say\ that\ A\ and\ B\ are\ \boldsymbol{disjoint}\ sets\ and\ P(A \cap B) = 0$

Complementary events: $P(A) = 1 - P(A^C)$

Conditional probability:

$P(A|B) = \frac{P(A \cap B)}{P(B)}\ \ ;\ \ P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(A^c|B) = 1 - P(A|B)$

Independent Events: $\begin{cases} P(A \cap B) = P(A)P(B) \\ P(A|B) = P(A) \\ P(B|A) = P(B) \end{cases}$

De Morgan's laws

$P(A \cap B)^C = P(A^C \cup B^C)$

$P(A \cup B)^C = P(A^C \cap B^C)$

$P(A^C \cap B^C) = P(A \cup B)^C\ = 1 - P(A \cup B)$

Another rule:

$$P(A \cup B) = \begin{cases} P(A) + P(B) - P(A \cap B) \\ P(A) + P(A^C \cap B) \\ P(A^C \cap B^c)^c = 1 - P(A^C \cap B^C) \end{cases}$$

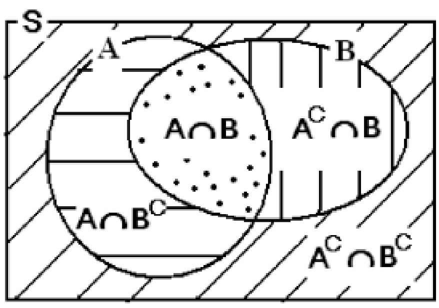
$$P(A \cap B) = P(A^C \cup B^C)^C = 1 - P(A^C \cup B^C) = 1 - [P(A^C) + P(B^C) - P(A^C \cap B^C)]$$

$P(A) = P(A \cap B) + P(A \cap B^C)$

$P(B) = P(A \cap B) + P(A^C \cap B)$

$P(A \cap B^C) = P(A) - P(A \cap B)$

$P(A^c \cap B) = P(B) - \ P(A \cap B)$



Q1: Consider the experiment of flipping a balanced coin **three** times independently.

1. The number of points in the sample space is..
   (A) 2 (B) 6 (C) 8 (D) 3 (E) 9
2. The probability of getting exactly two heads is…
   (A) 0.125 (B) 0.375 (C) 0.667 (D) 0.333 (E) 0.451
3. The events 'exactly two heads' **and** 'exactly three heads' are…
   (A) Independent (B) disjoint (C) equally likely (D) identical (E) None
4. The events 'the first coin is head' **and** 'the second and the third coins are tails' are…
   (A) Independent (B) disjoint (C) equally likely (D) identical (E) None

Solution of Q1:

$S = \{H, T\} \times \{H, T\} \times \{H, T\}$

$s = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

1) $n(s) = 2 \times 2 \times 2 = 2^3 = 8$

2) *Event A: getting exactly two heads*

$A = \{HHT, HTH, THH\} \rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{3}{8} = 0.375$

3) *Event B: exactly three heads* $, B = \{HHH\}$

$\therefore A \text{ and } B = A \cap B = \phi \rightarrow \therefore A \text{ and } B \text{ are disjoint}$

4) *Event C: the first coin is head* $, C = \{HHH, HHT, HTH, HTT\} \rightarrow P(C) = \frac{4}{8} = \frac{1}{2}$

Event D: the second and the third coins are tails, $D = \{HTT, TTT\} \rightarrow P(D) = \frac{2}{8} = \frac{1}{4}$

$C \text{ and } D = C \cap D = \{HTT\} \rightarrow P(C \cap D) = \frac{1}{8}$

$\text{as } P(C \cap D) \neq 0 \;\therefore C \text{ and } D \text{ are } \textbf{not} \text{ disjoint}$

$\text{as } P(C) \neq P(D) \therefore C \text{ and } D \text{ are } \textbf{not} \text{ equaly likely}$

$\text{as } P(C \cap D) = P(C)P(D) = \frac{1}{8} \;\therefore C \text{ and } D \text{ are independent.}$

Q2. Suppose that a fair die is thrown twice independently, then

1. the probability that the sum of numbers of the two dice is less than or equal to 4 is;
   (A) 0.1667 (B) 0.6667 (C) 0.8333 (D) 0.1389
2. the probability that at least one of the die shows 4 is;
   (A) 0.6667 (B) 0.3056 (C) 0.8333 (D) 0.1389
3. the probability that one die shows one and the sum of the two dice is four is;
   (A) 0.0556 (B) 0.6667 (C) 0.3056 (D) 0.1389
4. the event A={the sum of two dice is 4} and the event B={exactly one die shows two} are,
   (A) Independent (B) Dependent (C) Joint (D) None of these.

Solution of Q2:

$$S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$$

$$= \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$

$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$

$$(3,1), \dots, (4,1), \dots, (5,1), \dots,$$

$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

1) $A:\ the\ sum\ of\ numbers\ of\ the\ two\ dice \leq 4$

$$A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\} \rightarrow P(A) = \frac{6}{36} = \frac{1}{6} = 0.1667$$

2) $B: at\ least\ one\ of\ the\ die\ shows\ 4$

$$B = \{(1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (4,1), (4,2), (4,3), (4,5), (4,6)\}$$

$P(B) = \frac{11}{36} = 0.3056$

3) $C: that\ one\ die\ shows\ (1)\ and\ the\ sum\ of\ the\ two\ dice = 4$

$$C = \{(1,3), (3,1)\} \rightarrow P(C) = \frac{2}{36} = 0.0556$$

4) $D: the\ sum\ of\ two\ dice = 4\ , D = \{(1,3), (3,1), (2,2)\} \rightarrow P(D) = \frac{3}{36} = 0.0833$

$E: one\ die\ shows\ 2$

$E = \{(1,2), (2,1), (2,3), (2,4), (2,5), (2,6), (3,2), (4,2), (5,2), (6,2)\}$

$P(E) = \frac{10}{36} = 0.2778$

$D \cap E = \{\ \} \rightarrow P(D \cap E) = 0\ \therefore D\ and\ E\ are\ disjoint.$

$P(D \cap E) = 0 \neq P(D)P(E) \rightarrow D\ and\ E\ are\ dependent.$

Q3. Assume that $P(A) = 0.3$, $P(B) = 0.4$, $P(A \cap B \cap C) = 0.03$, $and\ P(\overline{A \cap B}) = 0.88$, then

1. the events A and B are,

   (A) Independent (B) Dependent (C) Disjoint (D) None of these.

2. $P(C|A \cap B)$ is equal to,

   (A) 0.65 (B) 0.25 (C) 0. 35 (D) 0.14

Solution of Q3:

1) $P(A \cap B) = 1 - P(A \cap B)^C = 1 - 0.88 = 0.12 \neq 0$

$\therefore A\ and\ B\ are\ not\ disjoint$

$P(A).P(B) = 0.3 \times 0.4 = 0.12 = P(A \cap B)$

$\therefore A\ and\ B\ are\ independent$

2) $P(C|A \cap B) = \frac{P(C \cap (A \cap B))}{P(A \cap B)} = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{0.03}{0.12} = 0.25$

Q4. If the probability that it will rain tomorrow is 0.23, then the probability that it will not rain tomorrow is:

(A) −0.23 (B) 0.77 (C) −0.77 (D) 0.23

Solution of Q4:

A: it will rain tomorrow , $P(A) = 0.23 \rightarrow P(A^C) = P(\bar{A}) = 1 - P(A) = 1 - 0.23 = 0.77$

Q5. The probability that a factory will open a branch in Riyadh is 0.7, the probability that it will open a branch in Jeddah is 0.4, and the probability that it will open a branch in either Riyadh or Jeddah or both is 0.8. Then, the probability that it will open a branch:

1. in both cities is:
   (A) 0.1 (B) 0.9 (C) 0.3 (D) 0.8
2. in neither city is:
   (A) 0.4 (B) 0.7 (C) 0.3 (D) 0.2

Solution of Q5:

A: factory open a branch in Riyadh $\rightarrow P(A) = 0.7$

B: factory open a branch in Jeddah $\rightarrow P(B) = 0.4$

$P(A \cup B) = 0.8$

1) $P(A \cap B) = ??$
   Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   $0.8 = 0.7 + .04 - P(A \cap B)$

$P(A \cap B) = 0.7 + 0.4 - 0.8 = 0.3$

2) $P(A^c \cap B^c) = P(A \cup B)^C = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

Or by table

	A	A^C	SUM
B	$P(A \cap B) = 0.3$	$P(A^C \cap B) = 0.1$	$P(B) = 0.4$
B^C	$P(A \cap B^C) = 0.4$	$P(A^C \cap B^C) = 0.2$	$P(B^C) = 0.6$
SUM	$P(A) = 0.7$	$P(A^C) = 0.3$	1

Q6. The probability that a lab specimen is contaminated is 0.10. Three independent specimens are checked.

1. the probability that none is contaminated is:
   (A) 0.0475 (B) 0.001 (C) 0.729 (D) 0. 3
2. the probability that exactly one sample is contaminated is:
   (A) 0.243 (B) 0.081 (C) 0.757 (D) 0. 3

Solution of Q6:

Event A: the lab specimen is contaminated , P(A)= 0.10

$$\therefore P(A^C) = 1 - P(A) = 1 - 0.1 = 0.9$$

We have three samples of lab specimen checked ***independent***

$S = \{A, A^C\} \times \{A, A^C\} \times \{A, A^C\}$

$S = \{AAA, AAA^C, AA^CA\ ,\ A^CAA\ ,\ A^CAA^C, A^CA^CA\ , AA^CA^C\ , A^CA^CA^C\}\ ;\ n(s) = 2^3 = 8$

1) Event B: none of lab specimen is contaminated , $B = \{A^CA^CA^C\}$

$$P(B) = P(\{A^C A^C A^C\}) = P(A^C)P(A^C)P(A^C) = 0.9^3 = 0.729$$

2) Event C:exactly one of lab specimen is contaminated, $C = \{ A^C A A^C, A^C A^C A, A A^C A^C\}$

$$P(C) = P(\{ A^C A A^C, A^C A^C A, A A^C A^C\}$$

and $\cong \cap \cong \times$ *or* $\cong \cup \cong +$

$$P(C) = P(\{ A^C A A^C\}) + P(\{A^C A^C A\}) + P(\{ A A^C A^C\})$$

$$= P(A^C)P(A)P(A^C) + P(A^C)P(A^C)P(A) + P(A)P(A^C)P(A^C)$$

$$= 3[P(A^C)P(A)P(A^C)] = 3[0.9 \times 0.1 \times 0.9] = 0.243$$

Q7. 200 adults are classified according to sex and their level of education in the following table:

Sex / Education	Male (M)	Female (F)
Elementary (E)	28	50
Secondary (S)	38	45
College (C)	22	17

If a person is selected at random from this group, then:

1. the probability that he is a male is:
   (A) 0.3182 (B) 0.44 (C) 0.28 (D) 78
2. The probability that the person is male given that the person has a secondary education is:
   (A) 0.4318 (B) 0.4578 (C) 0.19 (D) 0.44
3. The probability that the person does not have a college degree given that the person is a female is:
   (A) 0.8482 (B) 0.1518 (C) 0.475 (D) 0.085
4. Are the events M and E independent? Why?
   [P(M)=0.44 ≠ P(M|E)=0.359 ⇒ dependent]
5. Find $P(C^C \cap E), P(C^C \cup F), P(C^C \cap F^c)$ *and* $P(C^C \cup F^c)$?

Solutions of Q7:

	$M = F^c$	$F = M^C$	Sum
E	$n(M \cap E) = 28$	$n(F \cap E) = 50$	$n(E) = 78$
S	$n(M \cap S) = 38$	$n(F \cap S) = 45$	$n(S) = 83$
C	$n(M \cap C) = 22$	$n(F \cap C) = 17$	$n(C) = 39$
Sum	$n(M) = 88$	$n(F) = 112$	200

1) $P(M) = \frac{n(M)}{200} = \frac{88}{200} = 0.44$

2) $P(M|S) = \frac{P(M \cap S)}{n(S)} = \frac{n(M \cap S)}{n(S)} = \frac{38}{83} = 0.4575$

3) $P(C^c|F) = \frac{P(C^C \cap F)}{P(F)} = \frac{n(C^C \cap F)}{n(F)} = \frac{n(F \cap S)+n(F \cap E)}{112} = \frac{95}{112} = 0.8482$

Or $P(C^c|F) = 1 - P(C|F) = 1 - \frac{17}{112} = 0.8482$

4) We want to see that $P(M \cup E) = P(E)P(M)$

L.H.S= $P(M \cap E) = \frac{28}{200} = 0.14$

R.H.S= $P(E)P(M) = \frac{88}{200} * \frac{78}{200} = 0.1716$ $\quad \therefore L.H.S \neq R.H.S$

$$\therefore M \text{ and } E \text{ are } \mathbf{not} \text{ independent (dependent)}$$

5) $P(C^C \cap F) = \frac{50+45}{200} = \frac{95}{200}$

$$P(C^C \cup F) = P(C^C) + P(F) - P(C^C \cap F) = \frac{78+83}{200} + \frac{112}{200} - \frac{95}{200} = \frac{161+112+95}{200}$$

$$P(C^C \cap F^c) = P(C \cup F)^C = 1 - P(C \cup F)$$

$$= 1 - [P(C) + P(F) - P(C \cap F)] = 1 - \left[\frac{39+112-17}{200}\right] = 0.33$$

Or $P(C^C \cap F^c) = P(E \cap F^c) + P(S \cap F^c) = \frac{28+38}{200} = 0.33$

$$P(C^C \cup F^c) = P(C \cap F)^C = 1 - P(C \cap F) = 1 - \frac{17}{200} = 0.915$$

Q8. 1000 individuals are classified below by sex and smoking habit

		SEX	
		Male (M)	Female (F)
SMOKING HABIT	Daily (D)	300	50
	Occasionally (O)	200	50
	Not at all (N)	100	300

A person is selected randomly from this group.

1. Find the probability that the person is female. [P(F)=0.4]
2. Find the probability that the person is female and smokes daily. [P(F∩D)=0.05]
3. Find the probability that the person is female, given that the person smokes daily. [P(F|D)=0.1429]
4. Are the events F and D independent? Why? [P(F)=0.4 ≠ P(F|D)=0.1429 ⇒ dependent]

Solution of Q8:

	M	$M^C = F$	Sum
D	$n(D \cap M) = 300$	$n(D \cap F) = 50$	$n(D) = 350$
O	$n(O \cap M) = 200$	$n(O \cap F) = 50$	$n(O) = 250$
N	$n(M \cap N) = 100$	$n(N \cap F) = 300$	$n(N) = 400$
Sum	$n(M) = 600$	$n(F) = 400$	1000

1) $P(F) = \frac{n(F)}{1000} = \frac{400}{1000} = 0.4$

2) $P(F \cap D) = \frac{n(F \cap D)}{1000} = \frac{50}{1000} = 0.05$

3) $P(F|D) = \frac{P(F \cap D)}{P(D)} = \frac{n(F \cap D)}{n(D)} = \frac{50}{350} = 0.1429$

4) We want to see that $P(M \cap E) = P(E)P(M)$

L.H.S $P(M \cup E) = 0.05$

R.H.S $P(E)P(M) = \frac{400}{1000} \frac{350}{1000} = 0.14$

$\therefore L.H.S \neq R.H.S \quad \therefore M \text{ and } E \text{ are dependent}$

Q9. Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that the other engine will start is 0.6, then the probability that both will start is:

(A) 1 (B) 0.24 (C) 0.2 (D) 0.5

Solution :

A: the first engine start → P(A)=0.4

B: the second engine start → P(B)=0.6

the two engines operate independent $\therefore P(A \cap B) = P(A).P(B) = 0.4 * 0.6 = 0.24$

Q10. If P(B) = 0.3 and P(A| B) = 0.4 , then P(A∩ B) equals to;

(A) 0.67 (B) 0.12 (C) 0.75 (D) 0.3

Solution :

$P(A \cap B) = P(A|B)P(B)$; $\left[from\, P(A|B) = \frac{P(A \cap B)}{P(B)} \right]$

$P(A \cap B) = 0.4 * 0.3 = 0.12$

Q11. The probability that a computer system has an electrical failure is 0.15, and the probability that it has a virus is 0.25, and the probability that it has both problems is 0.10, then the probability that the computer system has the electrical failure or the virus is:

(A) 1.15 (B) 0.2 (C) 0.15 (D) 0.30

Solution :

Event A: The computer system has an electrical failure

Event B:The computer has a virus

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.15 + 0.25 - 0.1 = 0.3$

Q12. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:

(A) 6/27 (B) 2/27 (C) 12/27 (D) 4/27

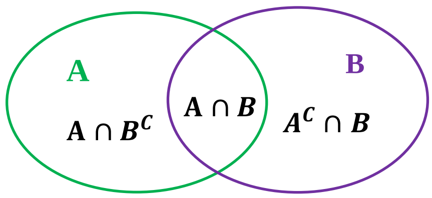
Solution of Q12:

3 balls (independently and with replaced)

g: green ball ; $P(g) = \frac{2}{6} = \frac{1}{3}$

b: black ball ; $P(b) = \frac{4}{6} = \frac{2}{3}$

$$S = \{bbb, bbg, bgb, gbb, ggb, gbg, bgg\}$$



$$P(2g \text{ and } 1b) = P(\{bgg, gbg, ggb\}) = P(b \cap g \cap g) + P(g \cap b \cap g) + P(g \cap g \cap b)$$

Since the balls are drawn **independently**

$$= P(b)P(g)P(g) + P(g)P(b)P(g) + P(g)P(g)P(b)$$

$$= 3\left[\left(\frac{4}{6}\right)\left(\frac{2}{6}\right)\left(\frac{2}{6}\right)\right] = 3\left[\frac{2}{27}\right] = \frac{6}{27}$$

Q13. If $P(A_1) = 0.4$, $P(A_1 \cap A_2) = 0.2$, and $P(A_3|A_1 \cap A_2) = 0.75$, then

1. $P(A_2|A_1)$ equals to
   (A) 0.00 (B) 0.20 (C) 0.08 (D) 0.50
2. $P(A_1 \cap A_2 \cap A_3)$ equals to
   (A) 0.06 (B) 0.35 (C) 0.15 (D) 0.08

Solution of Q13:

1) $P(A_2|A_1) = \frac{P(A_2 \cap A_1)}{P(A_1)} = \frac{0.2}{0.4} = 0.5$

2) $P(A_3|A_1 \cap A_2) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)} \rightarrow P(A_1 \cap A_2 \cap A_3) = P(A_3|A_1 \cap A_2)P(A_1 \cap A_2)$

$$= 0.75 * 0.2 = 0.15$$

Q14. If P(A)=0.9, P(B)=0.6, and P(A∩B)=0.5, then:

1. $P(A \cap B^C)$ equals to
   (A) 0.4 (B) 0.1 (C) 0.5 (D) 0.3
2. $P(A^C \cap B^C)$ equals to
   (A) 0.2 (B) 0.6 (C) 0.0 (D) 0.5
3. P(B|A) equals to
   (A) 0.5556 (B) 0.8333 (C) 0.6000 (D) 0.0
4. The events A and B are
   (A) independent (B) disjoint (C) joint (D) none
5. The events A and B are
   (A) disjoint (B) dependent (C) independent (D) none

Solution of Q14:

1) $P(A \cap B^C) = P(A) - P(A \cap B)$ $[from\ P(A) = P(A \cap B) + P(A \cap B^C)]$

2) $P(A^C \cap B^C) = P(A \cup B)^C = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$
$= 1 - [0.9 + 0.6 - 0.5] = 1 - 1 = 0$

3) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.5}{0.9} = 0.556$

4) $P(A \cap B) = 0.5$

5) $P(A \cap B) \neq P(A)P(B) \rightarrow 0.5 \neq 0.9 * 0.6 \therefore A \text{ and } B \text{ are dependent .}$

$P(A \cap B) \neq 0 \therefore A \text{ and } B \text{ are joint .}$

Q15. Suppose that the experiment is to randomly select with replacement 2 children and register their gender (B=boy, G=girl) from a family having 2 boys and 6 girls.

1. The number of outcomes (elements of the sample space) of this experiment equals to
   (A) 4 (B) 6 (C) 5 (D) 125
2. The event that represents registering at most one boy is
   (A){GG, GB, BG} (B) {GB, BG} (C){GB}C (D) {GB, BG, BB}
3. The probability of registering no girls equals to
   (A) 0.2500 (B) 0.0625 (C) 0.4219 (D) 0.1780
4. The probability of registering exactly one boy equals to
   (A) 0.1406 (B) 0.3750 (C) 0.0141 (D) 0.0423
5. The probability of registering at most one boy equals to
   (A) 0.0156 (B) 0.5000 (C) 0.4219 (D) 0.9375

Solution of Q15:

2 Children with replacement (the events are considered to be independent)

G: *Girl* and **B**: *Boy*

$n = 8 ;\ P(G) = \frac{6}{8} = 0.75 ;\ P(B) = \frac{2}{8} = 1 - 0.75 = 0.25$

1) S={G,B}×{G,B}={GG,GB,BG,BB} ; n(S)=2×2=4

2) A= at most one boy={ GG,GB,BG}

5) P(A)=P({ GG , GB , BG})

and ≅∩≅× or ≅ ∪≅ +

P(A) =P({GG })+ P({GB })+ P({BG }) [because the independent]

=P(G)P(G)+ P(G)P(B)+ P(G)P(B)

$= (0.75)^2 + (2)(0.75)(0.25) = 0.9375$

3) C= no girls={BB}

P(C)=P({BB})=P(B).P(B)= 0.25(0.25)=0.0625

4) D= exactly one boy= {GB, BG}
P(D)=P({GB, BG})=P({GB})+P({BG}) =2 P(G)P(B)= (2)(0.75)(0.25)=0.375

Q16. A total of 36 members of a club play tennis, 28 play squash, 18 play badminton, 22 play both tennis and squash, 12 play both tennis and badminton, 9 play both squash and badminton, 4 play all 3. What is the probability that at least one a member of this club plays at least one sport. Assuming that the total number of members in the club is 50.

Solution of Q16:
T:people who play tennis ; n(T)=36
S:people who play squash ; n(S)=28
B:people who play badminton ; n(B)=18

$n(T\cap S)=22$; $n(T\cap B)=12$; $n(S\cap B)=9$; $n(T\cap S\cap B)=4$
number of members in the club= N=50

$$P(T\cup S\cup B)=P(T)+P(S)+P(B)-P(T\cap S)-P(T\cap B)-P(S\cap B)+P(T\cap S\cap B)$$
$$=\frac{36+28+18-22-12-9+4}{50}=\frac{43}{50}$$

NOTE:

$$P(E_1\cup E_2\cup\ldots\cup E_n)=\sum_{j=1}^{n}P(E_j)-\sum_{j_1<j_2}P(E_{j1}\cap E_{j2})+\cdots$$
$$+(-1)^{r+1}\sum_{j_1<j_2<\cdots<jr}P(E_{j1}\cap E_{j2}\cap\ldots\cap E_{jr})+\cdots+(-1)^{n+1}P(E_1\cap E_2\cap\ldots\cap E_n)$$

The summation $\sum_{j_1<j_2<\cdots<jr}P(E_{j1}\cap E_{j2}\cap\ldots\cap E_{jr})$ is taken over all of the $\binom{n}{r}$ possible subset of size (r) of the set {1,2,...,n}

Q17. A group of women in a certain hospital were selected it was found out that 18% were married, 2% of them have exceeded the age of 25, 81% are not married and didn't exceed the age of 25. A woman was selected at random

1) What is the probability that the women is married or exceeded the age of 25.
2) What is the probability that the exceeded the age of 25 given that she is married.
3) Is the married status and the age independent.

Solution of Q17: H.W

Q18. If the probability of passing course (A) is 0.6, passing course (B) is 0.7, passing course A or B is 0.9. Find:

1- Probability of passing course A and B.
2- Probability of passing course A only.
3- Probability of passing course B and not passing course A.
4- Probability of not passing course A and B.
5- Probability of passing course B or not passing course A.

Solution of Q18: H.W

Q19. Our sample space S is the population of adults in a small town. They can be categorized according to gender and employment status.

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

One individual is to be selected at random for a publicity tour.
The concerned events

- M: a man is chosen
- E: the one chosen is employed
- F: a Female is chosen
- U: The one chosen is unemployed

1- If the chosen is employed, what is the probability to be Female.
2- If the chosen is unemployed, what is the probability to be Female.
3- If the chosen is unemployed, what is the probability to be Male.
4- If the chosen is Male, what is the probability to be unemployed.

Solution of Q19: H.W

Q20. The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$.
Find the probability that a plane

1- arrives on time given that it departed on time.
2- departed on time given that it has arrived on time.
3- arrived on time given that it has not departed on time.

Solution of Q20: H.W

Q21. Suppose we have a fuse box containing 20 fuses of which 5 are defective D and 15 are non-defective N. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective.

Solution of Q21:

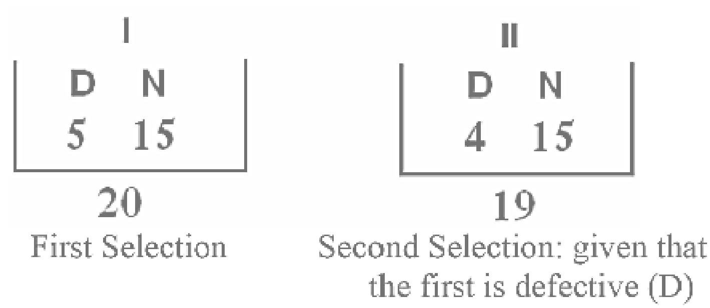
A={ the first fuse is defective } ; B={ the second fuse is defective }

The probability of first removing a defective fuse is P(A)=5/20

The probability of removing a second defective fuse after a defective first fuse was removed is $P(B|A) = \frac{4}{19}$

A∩B={ the event that **A** occurs and then **B** occurs after **A** occurred}
={both fuses are defective}

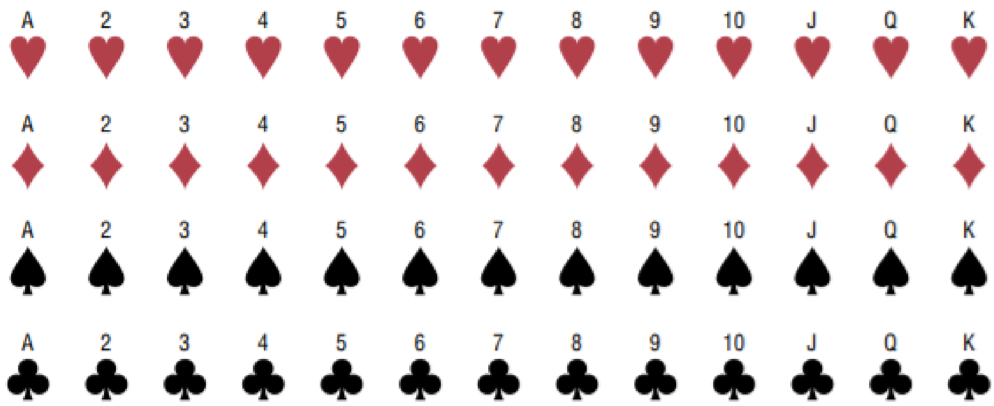
$$P(A \cap B) = P(A)P(B|A) = \frac{5}{20}\frac{4}{19} = 0.052632$$



Q22. Three cards are drawn in succession, **without replacement**, from an ordinary deck of playing cards. Fined $P(A_1 \cap A_2 \cap A_3)$, where the events A1, A2 , and A3 are defined as follows:

A1 = {the 1-st card is a red ace}
A2 = {the 2-nd card is a 10 or a jack}
A3 = {the 3-rd card is a number greater than 3 but less than 7}

Solution of Q22:



Playing card (52) { 13 cards of hearts ♥, 13 cards of diamonds ♦, 13 cards of spades ♠, 13 cards of clubs ♣ } ; every set has A,J,Q,K,2,3,4,5,6,7,8,9,10 .

Where A: Ace
J: Jack
Q:Queen
K:king

$$n(A_1) = 2 \ ; \ n(A_2) = 8 \ \ ; \ n(A_3) = 12$$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_3) = \frac{2}{52}\,\frac{8}{51}\,\frac{12}{50} = 0.0014479$$