

Name:

Sequence Number:

Teacher's Name:

Section:

<u>Question</u>	Mark
Question I	
Question II	
Question III	
Question IV	
Total	

Question Number	1	2	3	4	5	6	Total
Answer							

Question I:

Choose the correct answer, then fill in the table above:

(1) The truth value of " $5 + 1 = 4$ if and only if $5 + 1 = 6$ " is

- (a) True (b) False (c) None of the previous

(2) Let $Q(x, y)$ be the statement " $x = y + 5$ " then

- (a) $Q(5,0)$ is true (b) $Q(0,5)$ is true (c) $Q(3,2)$ is true (d) None of the previous

(3) If $P(x)$ is the statement " $2x = x, x \in \mathbb{Z}$ ", then the truth value of $\exists x P(x)$ is

- (a) True (b) False (c) None of the previous

(4) The inverse of the statement "*if $x^2 < x$ then $0 < x < 1$* " is

- (a) *if $x^2 > x$ then $x \leq 0 \wedge x \geq 1$* (b) *if $0 < x < 1$ then $x^2 < x$*
(c) *if $x^2 \geq x$ then $x \leq 0 \vee x \geq 1$* (d) None of the previous

(5) $\neg[\forall x (x < y \wedge x \text{ is rational})]$ is equivalent to

- (a) $\forall x (x \geq y \wedge x \text{ is irrational})$ (b) $\exists x (x \geq y \vee x \text{ is irrational})$
(c) $\exists x (x \geq y \wedge x \text{ is irrational})$ (d) None of the previous

(6) The power set of the set $\{\emptyset\}$ is

- (a) $\mathcal{P}(\{\emptyset\}) = \{\emptyset\}$ (b) $\mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$ (c) $\mathcal{P}(\{\emptyset\}) = \emptyset$ (d) None of the previous

Question II:

A. **Without using truth tables** prove the following:

$$\neg p \rightarrow (\neg p \wedge q) \equiv \neg p \rightarrow q.$$

B. Is the following argument valid or invalid? Justify your answer:

$$\begin{array}{c} p \rightarrow (\neg q \wedge p) \\ \neg[s \leftrightarrow (r \vee \neg q)] \\ q \leftrightarrow r \\ \hline \therefore s \vee q \end{array}$$

Question III:

A. If n is an integer, prove that " $7n + 4$ is even if and only if n is even".

B. Prove that the statement "*for every $x, y \in \mathbb{R}$, $(x + y)^2 = x^2 + y^2$* " is **false**.

Question IV:

A. Prove **by cases** that " $2n^2 + 3$ is odd", for every integer number n .

B. If the universal set $U = \{1,2,3,4,5\}$ and $A = \{1,4\}$, $B = \{2,4\}$ and $C = \{5\}$, then answer the following:

(1) Find the sets $\overline{B \cup C}$ and $A - B$.

(2) Find $A \times B$.

(3) $|\mathcal{P}(A \cup C)|$

(4) Is the set $D = \{4\}$ a subset of $A \cap B$?

Good Luck☺