King Saud University — Math 5501 — First term 1432/1433
Mathematics Department — Midterm — name:

I. Determine if the statement is true or false, and justify your answer

a) The Bisection method generates a sequence $\{p_n\}$ approximating p with rate of convergence $O(2^{-n})$ (   )

b) The sequence $x_{n+1} = x_n + 1 - \frac{x_n^{\ 2}}{5}$ will converge faster than the sequence $x_{n+1} = \frac{1}{3}\left[3x_n + 1 - \frac{x_n^{\ 2}}{5}\right]$ to $\sqrt{5}$. (   )

c) The Newton method guarantees quadratic convergent to the root p=0 of the function $f(x) = e^{2x} - 2x - 1$. (   )

d) The function $g(x) = \frac{2x}{x+1}$ has a unique fixed point in [-0.5, 0.5] but the convergence of $\boldsymbol{x_{n+1} = g(x_n)}$ is not guaranteed. (   )

II.

a) Show that the **secant method** applied to find the square root of a number **Q** with the equation $\mathbf{x^2 = Q}$ gives the iterative formula

$$x_{n+1} = \frac{(x_n - x_{n-1})x_n x_{n-1} + Q(x_n - x_{n-1})}{x_n^2 - x_{n-1}^2}$$

b) Find the approximation of the square root of **Q = 5**, correct to three significant digits using this formula and starting point $\mathbf{x_0=}$ **2.18**.compute **absolute error**?

III.

a) Use a quadratic convergent method to find the root **p=-1** of $x^2+2x+1=0$ with accuracy $1\text{x}10^{-3}$?

b) Find the **relative error** of your approximation?

c) Use **Horner method** to find the first iteration of Newton method on $x^2+2x+1=0$ starting with x_0= -0.75?

d) Use **Aitkens method** to find the first 2 terms of the sequence x_n that converge faster to x=1 than x_n=cos(1/n).