1428-1429 H **202 Math**

Stat true or false:-

(i) $2i-5j$ is a unit vector.

(ii) $ax+by+d=o$, is orthogonal to the xy-plane.

(iii) $(i\times j)\times k=1$.

(iv)Area of the circle $r=5, 0\le\theta\le 2\pi$, is $\frac{1}{2}\int_0^{\pi} 5^2\,d\theta$.

(V)The graph of $r=\theta$ is acircle .

(vi) $\left(\vec{a}\times\vec{b}\right)^2=a^2b^2-\left(\vec{a}.\vec{b}\right)^2$.

(i)Find the length of the curve , $x=2-\ln t$, $y=3\ln t$,sketch the graph of the curve , and indicate

the orientation $1\le t\le e$.

(ii)Find apolar equation for $\left(x^2+y^2\right)^2+4xy=0$, sketch the graph .

(iii)Find area of the triangle determined by P $(1,0,-5)$,Q $(-2,1,0)$,and R $(3,2,1)$.

(i)Find an equation of the plane through A $(0,4,9)$, and B $(0,-2,6)$ and perpendicular . to $xy-plane$, Find the distance from C $(1,0,-1)$ to the plane .

(ii)Sketch the graph of $r=1+2\cos\theta$,find the slope of the tangent line at $\theta=\pi/2$, and find the area of the regien bounded by $r=1+2\cos\theta$.

(iii)If $\vec{a}=3i-j-4k$, $\vec{b}=2i+5i-2k$, and $\vec{c}=-i+6k$, find

* $comp_{\vec{b}}\left(\vec{b}\times\vec{c}\right)$.

*A vector having opposite direction of $\vec{a}$ and twice magnitude .

*the volume of the box determined by $\vec{a},\vec{b}, and\ \vec{c}$.

Q1- *Prove or disprove:-*

(i) $\left\| \vec{a} + \vec{b} \right\| \leq \left\| \vec{a} \right\| + \left\| \vec{b} \right\|$.

(ii) The graph of the polar equation $r = 2\cos 4\theta$, *has 4 loops.*

(iii) The distance from the point $(1,0,-2)$ *to the plane :* $2x - 3y + z = 6$, *is* $\frac{6}{\sqrt{14}}$

(iv) $r = 4\cos\left(\theta - \frac{\pi}{2}\right)$ *is a circle.*

(v) The curvature of $x = 2\cos t$ *and* $y = 2\sin t$ *is 2 for all* t *.*

(vi) $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$.

(vii) The line $x = 1 + 5t, y = 1 - 2t, z = 4 + t$, *and the plane* $2x + 3y - 4z = 5$, *are perpendicular .*

(iix) If $\vec{v}$ *is a nonzero vector,* $a = 0$ *be scalar, If* $\vec{u}$ *is any vector then*

$proj_{\vec{v}} \vec{u} = proj_{\vec{av}} \vec{u}$.

(ix) $\oint_c \left(y - 7e^{x^3}\right)dx + \left(x + \ln\sqrt{y}\right)dy = 0$.

Where c *is the ellipse ,* $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(x) $\iiint_Q \nabla.\vec{n}\, dV = S$, *where* $\vec{n}$ *is a unit normal vector to any closed surface.*

Q2-

(i) Use Green's Theorem to evaluate $\oint_c xydx + \sin ydy$, *where* c *is the graph of the triangle with vertices* $(1,0),(1,1)$ *and* $(0,0)$.

(ii) Find velocity, acceleration at $t = 1$, *for the curve,* $\vec{r}(t) = e^{2t}i + e^{-2t}j$, *and sketch them.*

(iii) Find the point on the curve at which the tangent line is, horizontal, vertical: $x = t^3 - 4t, y = t^2 - 4$.

--**Q3-**

(i) Use the divergence theorem to evaluate $\iint_s \vec{F}.\vec{n}ds$, *over the closed surface bounded by* $x^2 + y^2 = 9, z = 0$, *and* $z = 5$.

$\vec{F} = (x^2 + z^2)i + (y^2 - 2xy)j + (4z - 2yz)k.$

(ii) If $\vec{F} = y^2 i + 2xj + 5yk$ *,verify stokes theorem, where* s *: is the hemisphere*

$z = (4 - x^2 - y^2)^{\frac{1}{2}}$.

(iii) sketch the curve $r^2 = 4\cos 2\theta$. *Find the area of the region bounded by the above curve.*

--**Q4-***(i)*
Change the rectangular co-ordinate to a) spherical , b) cylindrical co-ordinates .
(without calculating the integral)

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{x^2+y^2}}^{1} dzdxdy \ .$$

(ii) Show that , $\int_{(2,1,2)}^{(-1,0,4)} (yz + 2)dx + (xz - 3)dy + (xy + 5)dz$, *is independent of path and find its volume.*

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M-202

FINAL EXAMINATION SEMESTER – II (1430-1431)

NOTE: ANSWER ALL QUESTION.

Q_1:

a) Use the spherical coordinates to find the volume of the solid bounded by cone $z=\sqrt{x^2+y^2}$ and the cylinder $x^2+y^2=4$.

b) Show that: i. $div\left(curl\vec{F}\right)=0$. ii. $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\left(\vec{a}\times\vec{b}.\vec{d}\right)\vec{c}-\left(\vec{a}\times\vec{b}.\vec{c}\right)\vec{d}$.

c) Find the initial point of $\vec{V}=-2i+4j$, if terminal point is $(2,0)$.

Q_2:

a) Find the unit vector perpendicular to each of the vectors $2i-j+k$ and $3i+4j-k$ and calculate the $\sin$ of the angle between the two vectors.

b) Find an equation of the plane through $P(4,2,-9)$ with normal vector $\overrightarrow{OP}$.

Q_3:

a) Find the area of the region that inside $r=2$, and outside $r=2+2\cos 0$.

b) If $\vec{r}(t)=\sin 2ti+\cos tj$ is the position vector for a point, find the tangential and normal components of acceleration at the time t.

c) Find the slope of the tangent line to the graph $x=\sin t, y=\cos t$ at $t=\frac{\pi}{4}$ and sketch the graph.

Q_4:

a) Show that the line integral $\int_{(0,0)}^{\left(1,\frac{x}{2}\right)} e^x \sin y dx + e^x \cos y dx$ is independent of paths and find its value.

b) Use Green's theorem to evaluate $\oint_C \left(x^2+4\right)dx+xydy$ where C is the cardiod $r=1+\cos\theta$.

c) Find the surface area of the part of the graph $z=xy$ that inside the cylinder $x^2+y^2=4$.

Q_5:

a) State the divergence theorem and show that $\frac{1}{3}\iint_S \vec{r}.\vec{n}ds = V$ where $\vec{r} = xi + yj + zk$ and V is the volume enclosed by the surface S.

Verity the Stoke theorem for $F = zi + xj + yk$ and S, where S is the hemisphere $Z = \left(a^2 - x^2 - y^2\right)^{\frac{1}{2}}$.

<u>1430-1431 H</u> **<u>202 Math</u>**

Q_1 :-State true or false:-

(i) $3i - j$ is a unit vector.

(ii) $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|^2 \|\vec{v}\|^2 - (\vec{u}.\vec{v})^2$

(iii) $(i \times j) \times k = 1$.

(iv)Area of the circle $r = 4, 0 \le \theta \le 2\pi$, is $\frac{1}{2}\int_0^{\pi} 16 d\theta$.

(V)The vectors $2i - 3j$ and $6i + 4j$ are perpendicular.

(vi) $|\vec{r}(t)|$ is constant if and only if $\vec{r}(t).\vec{r}\,'(t) = 0$.

Q_2 :-

(i)Find the length of the curve , $\vec{r}(t) = \sinh 2ti + \cosh 2tj + 2tk$, $-\frac{1}{2} \le t \le 1$.

(ii)Find apolar equation for $y^2 = 4px$, sketch the graph .

(iii)Find the area of the triangle determined by P $(3,2,-1)$,Q $(2,4,6)$,and R $(-1,2,7)$.

(iv)Find the area of the region outside the cardioid $r = 1 + \cos\theta$ and inside the circle $r = \sqrt{3}\sin\theta$.

Q_3 :-

(i) Find the angle between the vectors $\langle 4,-3,-2\rangle$ and $\langle -1,2,5\rangle$.

(ii)Find $proj_{\vec{v}}\vec{u}$ if $\vec{u}=-4i+j-2k$, $\vec{v}=i+3j-3k$.

(iii)Find the volume of the box determined by $(-1,2,3)$, $(4,-1,2)$, $(5,6,3)$ and $(1,1,-2)$.

(iv)Find all vectors perpendicular to both vectors $-2i+j-4k$ and $3i-4j+5k$.

1430-1431 H **202 Math**

Q_1 :-State true or false:-

(i) $3i+4j$ is a unit vector.

(ii) $\|\vec{u}\times\vec{v}\|=\|\vec{u}\|^2\|\vec{v}\|^2-(\vec{u}.\vec{v})^2$

(iii) $i\times(j\times k)=0$.

(iv)Area of the circle $r=5, 0\le\theta\le\pi$, is $\frac{1}{2}\int_0^{\pi}25d\theta$.

(V)The vectors $2i+3j$ and $6i+4j$ are perpendicular.

(vi) $|\vec{r}(t)|$ is constant if and only if $\vec{r}(t).\vec{r}\,'(t)=0$.

Q_2 :-

(i)Find the length of the curve , $\vec{r}(t)=\sinh 2ti+\cosh 2tj+2tk$, $-\frac{1}{2}\le t\le 1$.

(ii)Find apolar equation for $x^2=4py$, sketch the graph .

(iii)Find the area of the triangle determined by P $(2,-1,1)$,Q $(1,0,-2)$,and R $(-1,1,3)$.

(iv)Find the area of one loop of $r=\sin 30$

Q_3:-

(i) Find the angle between the vectors $\langle 2,-2,1\rangle$ and $\langle -2,0,1\rangle$.

(ii)Find $proj_{\vec{v}}\vec{u}$ if $\vec{u}=2i+4+2k$, $\vec{v}=-i+2j+k$.

(iii)Find the volume of the box determined by $(1,-2,-1)$, $(2,0,-2)$, $(3,2,-1)$ and $(-1,2,-2)$.

(iv)Find all vectors perpendicular to both vectors $-3i+j+4k$ and $2i+4j-3k$.

First Midterm Examination

Second term 1430-1431 H

Course no,202 (calculus)

Answer all questions :

Q_1 : State "true" or "false" to the following statements :-

i. Orientation of a curve is determined by decreasing values of the parameter t.
ii. The dot product of two parallel vectors is zero.
iii. $\|\vec{a}+\vec{b}\|=\|\vec{a}\|+\|\vec{b}\|$ when $\vec{a}$ and $\vec{b}$ are orthogonal to each other.
iv. If any two of $\vec{a},\vec{b}$ and $\vec{c}$ are equal, then $\vec{a}.\vec{b}\wedge\vec{c}=0$.

Q_2 :

i. Sketch the graph of the polar equation $r=3+2\cos\theta$ and find its area. Also find the slope of the tangent line to the graph at $\theta=\frac{\pi}{2}$.
ii. Find the area of the triangle determined by $\vec{a}$ and $\vec{b}$ if $\vec{a}=3i-j-4k$ and $\vec{b}=2i+5j-2k$.

Q_2 :

i. Find the values of t for $\vec{u}=t^2i+6tj+tk$ and $\vec{v}=5ti-5tj+4t^2k$ are orthogonal.
ii. Find a vector of magnitude 4 that has the opposite direction of $\vec{a}=\langle 2,-5,0\rangle$.

Curve C is given parametrically $x = t^2 + 1, y = t^2 - 1, -2 \le t$. Setch the graph and indicate the orient

First semester **202 Math** **King saud university**

Second midterm **1430-1431 H**

Q_1

a) Answer true or false :-
   i. The line $x = 1 + 5t, \quad y = 1 - 2t, \quad z = 4 + t$ and the plane $2x + 3y - 4z = 1$ are perpendicular
   ii. A particle whose position $\vec{r}(t) = \cos ti + \cos tj + \sqrt{2}\sin tk$ moves with constant speed .
b) Fill in the blanks
   i. A vector that is normal to the plane $-6x + y - 7z + 10 = 0$ is ……………………………………
   ii. The equation of the plane that has $x - \mathrm{int}\, ercept$ and $z - \mathrm{int}\, ercept \quad at 8$ at 4, $y - \mathrm{int}\, ercept$ at 2 and $z - \mathrm{int}\, ercept$ at 8 is ……………………………………

Q_2

Evaluate

i. $\int \frac{1}{1+t^2}\left(i + tj + \tan^{-1} tk\right)dt$

ii. $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5 + x^2 + y^2}\, dydx$

iii. Find the surface area of that portion of the paraboloid $z = x^2 + y^2$ that is below the plane $z = 2$

Q_3

i. Graph the curve and the velocity and acceleration vector of $\vec{r}(t) = t^2 i + \frac{1}{4} t^4 j$ at $t = 1$.

ii. Find the curvature and radius of curvature of $y = e^x - x$ at the point $(0,1)$

iii. Find parametric equations for the line of intersection of

$$2x - 3y + 4z = 1$$
$$x - y - z = 5$$

202 Math ***Second Term 1430H*** ***2 Hours*** ***Second Midtirm***

Q1 *Prove or disprove :-*

(i) The volume of the sphere $x^2 + y^2 + z^2 = 1$, is

$$V = \int_0^{2\pi} \int_0^{\pi} \int_0^{1} \rho^2 \sin\varphi \quad d\rho \quad d\theta \quad d\varphi \ .$$

(ii) Curvature at any point on the circle $r = 2\cos\theta$ is zero .

(iii) Every straight line has curvatre equal to 1 .

(iv) Let $\vec{F} = 2x\sin yi + x^2 \cos yj$ then $\nabla \times \vec{F}$ is $\vec{0}$.

(vi) If aparticle is moving at a constant speed then the tangential component at acceleration is zero .

Q2

(a) Convert $(0,-1,0)$ to :-

(i) Cylindrical , *(ii)* Spherical co-ordinates .

(b) Evaluate , $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{z^3}{\sqrt{x^2+y^2}} \, dz \quad dy \quad dx$

(c) Find the surface area of the part of the graph $z = xy$, that inside the cylinder $x^2 + y^2 = 4$.

(d) The position vector of aparticle at time t is $\vec{r}(t) = ti + t^2 j + t^3 k$

for $1 \le t \le 4$.

Find the tangential and normal components of acceleration at time $t = 1$.

Q3

(a) Sketch $\vec{r}(t), \vec{r}'(t)$ and $\vec{r}''(t)$ at $t = \frac{3\pi}{4}$, where $\vec{r}(t) = 4\cos ti + 7\sin tj,$ is the curve .

(b) Evaluate $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$.

(c) Find the curvalure , radius of curvature and centre of curvalure for $xy = 1,$ at the point $\left(2, \frac{1}{2}\right)$.

First Semester 1432-1433H | **King Saud University**

Final Examination | **Faculty of Science**

202 Math | **Mathematic Department**

Question. No.	1	2	3	4	5	6
Answer						

Q_1(i) Choose the correct answer:-

1) The vectors $\langle -4,-6,10\rangle$ and $\langle -10,-15,25\rangle$ are:

None of these (iii) Orthogonal (ii) Parallel (i)

2) The line $x = 1 + 5t, \quad y = 1 - 2t, \quad z = 4 + t$ and the plane $2x + 3y - 4z = 1$ are:

None of these (iii) Perpendicular (ii) Parallel (i)

3) If $\lim\limits_{t\to a} \vec{r}_1(t) = 2i + j$ and $\lim\limits_{t\to a} \vec{r}_2(t) = -i + 2j$, then $\lim\limits_{t\to a}\left(\vec{r}_1(t).\vec{r}_2(t)\right)$ is equal:

None of these (iii) 1 (ii) 0 (i)

4) If $f(x, y) = \cos y$, then:

None of these (iii) $\nabla f(x, y) = -\sin yj$ (ii) $\nabla f(x, y) = -\sin y$ (i)

5) If $\vec{r}(t) = \vec{a}\sin t + \vec{b}\cos t$, where $\vec{a}, \vec{b}$ are constant, then $\vec{r} \times \frac{d\vec{r}}{dt}$ is:

None of these (iii) $\vec{b} \times \vec{a}$ (ii) $\vec{a} \times \vec{b}$ (i)

6) $\oint_c \left(y - 7e^{x^2}\right)dx + \left(x + \ln\sqrt{y}\right)dy$, $c: 2(x-10)^2 + 9(y+13)^2 = 3$ is:

None of these (iii) 0 (ii) 1 (i)

(ii) **Prove or disprove:**

1) *If* $\vec{v}$ *is a nonzero vector,* $a \neq 0$ *be scalar, If* $\vec{u}$ *is any vector then,*

$$proj_{\vec{v}}\vec{u} = proj_{a\vec{v}}\vec{u}$$

2) $\iiint\limits_Q \nabla.\vec{n}\,dV = S$, *where* $\vec{n}$ *is a unit normal vector to any closed surface.*

3)*The distance from the point* $(1,0,-2)$ *to the plane :* $2x-3y+z=6$ *,is* $\frac{6}{\sqrt{14}}$

4) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)+\vec{b}\times\left(\vec{c}\times\vec{a}\right)+\vec{c}\times\left(\vec{a}\times\vec{b}\right)=\vec{0}$

Q2:

1. If $\vec{F}(x,y,z)=4xyzi+2x^2zj+2x^2yk$ prove that $div\left(curl\,\vec{F}\right)=0$

2. If $\vec{r}(t)=t^2i+2tj$ find $\vec{v}(t), \vec{a}(t)$ and sketch $\vec{r}(1), \vec{v}(1)$ and $\vec{a}(1)$

3.

(i) Convert $\left(8,\frac{\pi}{3},7\right)$ in cylindrical coordinates to rectangular coordinates.

(ii) Convert $\left(-\sqrt{2},\sqrt{2},1\right)$ in rectangular coordinates to cylindrical coordinates.

Q₃:

1. Determine whether the given pair of lines $L_1 : x = 1 + t,\ y = -3 + 2t,\ z = -2 - t$ and $L_2 : x = 17 - 3s,\ y = 4 - s,\ z = -8 + s$, has a point of intersection. If so, find it.

2. Find the curvature of the curve described by: $x = t - \sin t,\ y = 1 - \cos t$, at $t = \dfrac{\pi}{2}$.

3. Show that the given line integral is independent of path and find its value

$$\int_{\left(\frac{\pi}{2},0\right)}^{(\pi,1)} \left(e^{3y} - y^2 \sin x\right)dx + \left(3xe^{3y} + 2y\cos x\right)dy$$

Q4:

I. Use Green's Theorem to evaluate the line integral $\int_c (y+e^x)dx + (2x^2 + \cos y)dy$, where c is the boundary of the triangle with vertices (0,0), (1,1), and (1,0).

II. Use Divergence theorem to evaluate the integral $\iint_s \vec{F}.\vec{n}ds$, where $\vec{F} = x^3 i + y^3 j + z^2 k$, and S is the region enclosed by the cylinder $x^2 + y^2 = 4,\ z = 0$ and $z = 2$.

Q_5:

I. Verify Stoke's Theorem for the vector $\vec{F}(x,y,z)=2zi+3xj+5yk$, over the region.

$$z=4-2x-4y$$

II. Evaluate $\iint_s \vec{F}.\vec{n}ds$, where $\vec{F}(x,y,z)=xi+yj+zk$ and S is the paraboloid $z=1-x^2-y^2$ cut off by $z=0$.

First semester 1432-1433 King Saud University .III

2nd midterm exam Mathematical department .IV

202 Math .V

-- .VI

Q1. a) Find div $\vec{F}$ and curl $\vec{F}$ if $\vec{F}(x,y,z)=x^3z^4i+xyz^2j+x^2y^2k$ . .VII

b) Find the curvature of the curve described by $y=2x^2-x+2$. .VIII

.IX

Q2. a) Evaluate $\int_c (x-y)dx+xdy$, where c is the graph of $y^2=x$ from $(4,-2)$ to $(4,2)$. .X

b) Find the tangential and normal components of acceleration of a particle moving along the .XI

curve c described by $\vec{r}(t)=(t^2-1)i+(t+1)j+\frac{1}{2}(t^2+1)k$, $t=1$.

c) Find the surface area of the part of the paraboliod $z = 4 - x^2 - y^2$, that lies above the plane $z = 2$. .XII

.XIII

Q3. a) Show that the given line integral is independent of path and find its value .XIV

$$\int_{(-1,2)}^{(3,1)} \left(y^2 + 2xy\right)dx + \left(x^2 + 2xy\right)dy .$$

b) Find the equation for the plane through the point $(1,4,-5)$ and parallel to the plane $2x - 5y + 7z = 12$. .XV