King Saud University
Department Of Mathematics.
M-203 [Final Examination](Differential and Integral Calculus)
(I-Semester 1432/1433)

Max. Marks: 50 Time: 3 hrs

Marking Scheme: Q.No:1[3+4+5]; Q.No:2[4+4+4]; Q.No:3[4+4+4]; Q.No:4[4+5+5]

Q. No: 1 (a) Discuss the convergence of the sequence $\{\ln(n+1)-(\ln n)\}$, and find the limit if it exists.

(b) Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{2n}{2n^2-1}$ is absolutely convergent, conditionally convergent, or divergent.

(c) Find the interval of convergence and radius of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n \frac{2^n}{n^2}(x+2)^n$.

Q. No: 2 (a) Use the **Maclaurin** series for $f(x)=Sinx$ to approximate $\int_0^1 xSin(2x)dx$ (using three non-zero terms).

(b) Evaluate the integral $\int_0^1\int_{\sqrt{x}}^1 e^{-y^3}dydx.$

(c) Find the surface area of the surface bounded by the graphs of $x^2+y^2-2=z$, $z\le 0$.

Q. No: 3 (a) Find the Volume of the region bounded by z =0 and the paraboloid $z=16-(x^2+y^2)$ using triple integral.

(b) Use spherical coordinates to evaluate the integral $\iiint_Q (x^2+y^2+z^2)^{-3/2}dV$, where Q is a solid region bounded between the two spheres $x^2+y^2+z^2=1$, $x^2+y^2+z^2=4$ in the first octant.

(c) Find the work done by the force $\vec{F}=y\vec{i}-x\vec{j}$ in moving a particle from (1,1) to (-1,1) along the curve $x^2=y$.

Q. No: 4 (a) Use Green's theorem to evaluate the line integral $\oint_C xydx+xdy$, where C is the boundary of the region bounded by the graphs of the equations $y^2=x$ and $x^2=y$.

(b) Use the divergence theorem to find the flux of the vector field $\vec{F}(x,y,z)=(x+Siny)\vec{i}-(y+Cosz)\vec{j}+(2z-e^{-x})\vec{k}$ through the surface S, which is the portion of the cone $z=\sqrt{x^2+y^2}$ bounded by the plane $z=1$.

(c) Use Stokes's theorem to evaluate the integral $\iint_S (curl\vec{F})\bullet\vec{n}dS$, where $\vec{F}(x,y,z)=-y\vec{i}+z\vec{j}+x\vec{k}$, S is the paraboloid $z=2-(x^2+y^2)$ cut off by the plane $z=1$, and $\vec{n}$ is the unit upper normal vector to the surface S.

y

① a

$\ln(n+1) - \ln(n)$

$\Rightarrow \ln\left(\frac{n+1}{n}\right)$

$\lim_{n\to\infty} \ln\left(\frac{n+1}{n}\right)$

$\Rightarrow \cancel{\lim} \ln(1) = 0 \quad \therefore \text{c.g}$

3

② b $\sum_{n=1}^{\infty} (-1)^n \frac{2n}{2n^2-1}$

by using L.B.C

$b_n = \frac{2n}{2n^2} = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{2n}{2n^2-1} \cdot \frac{n}{2}$

$\lim_{n\to\infty} \frac{2n^2}{4n^2-2} = \frac{1}{2} > 0$

$b_n = \frac{1}{n}$ p.series $\rightarrow p=1$ harmonic series $\therefore$ ~~dg~~ ~~d.g~~ d.g

by using A.s.t $\sum(-1)^n \frac{2n}{2n^2-1}$

$f'(n) = \frac{2(2n-1) - 2n(4n)}{(2n^2+1)^2} = \frac{4n-1-8n^2}{(2n^2+1)^2} < 0 \quad \therefore \text{decresing}$

$\lim_{n\to\infty} \frac{2n}{2n^2-1} \overset{\lim_{n\to\infty}}{=} \frac{2}{2n-\frac{1}{n}} \overset{\lim_{n\to\infty}}{=} \frac{2}{\infty} = 0 \quad \therefore \boxed{\text{c.g}}$

$\therefore \sum_{n=1}^{\infty} (-1)^n \frac{2n}{2n^2-1}$ is C.C

Conditionally convergent

4

Q1 (c) $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n^2} (x+2)^n$

$u_n = (-1)^n \frac{2^n}{n^2} (x+2)^n$ $\qquad u_{n+1} = \frac{(-1)^{n+1} 2^{n+1} (x+2)^{n+1}}{(n+1)^2}$

$\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right|$

$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} 2^n \cdot 2 \cdot (x+2)^n \cdot (x+2)}{(n+1)^2} \cdot \frac{n^2}{(-1)^n 2^n (x+2)^n} \right|$

$\Rightarrow \lim_{n\to\infty} \left| \frac{2n^2}{(n+1)^2} \right| |x+2|$

$\Rightarrow \lim_{n\to\infty} \left| \frac{2n^2}{n^2+2n+1} \right| |x+2|$

$-1 < 2|x+2| < 1$

$-\frac{1}{2} < x+2 < \frac{1}{2}$

$-\frac{1}{2} - 2 < x < \frac{1}{2} - 2$

$-\frac{5}{2} < x < -\frac{3}{2}$

$\boxed{\text{at } x = -\frac{5}{2}}$

$\sum (-1)^n \frac{2^n}{n^2} (x+2)^n$

$\sum (-1)^n \frac{2^n}{n^2} (-\frac{5}{2}+2)^n$

$\sum (-1)^n \frac{2^n}{n^2} (-\frac{1}{2})^n$

$\sum (-1)^n \frac{2^n}{n^2} (-1)^n \frac{1}{2^n}$

$\sum \frac{1}{n^2}$ p. series $\Rightarrow p = 2$

$\therefore$ c.g

at $x = -\frac{3}{2}$

$\sum (-1)^n \frac{2^n}{n^2} (x+2)^n$

$\sum (-1)^n \frac{2^n}{n^2} (\frac{1}{2})^n$

$\sum (-1)^n \frac{2^n}{n^2} \frac{1}{2^n}$

$\sum (-1)^n \frac{1}{n^2}$ by A.S.T

$f'(n) = \frac{-2}{n^3} < 0 \therefore$ decresing

~~lim~~

$\lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty} = 0$

$\therefore$ c.g

interval $\left[-\frac{5}{2}, -\frac{3}{2}\right]$

radius $= \frac{-\frac{3}{2} + \frac{5}{2}}{2} = \frac{1}{2}$

لا يكتب في هذا الهامش

Q ② [a]

$f(x) = \sin x$ $f(0) = 0$

$f'(x) = \cos x$ $f'(0) = 1$

$f''(x) = -\sin x$ $f''(0) = 0$

$f'''(x) = -\cos x$ $f'''(0) = -1$

$f''''(x) = \sin x$ $f''''(0) = 0$

$f^{5}(x) = \cos x$ $f^{5}(0) = 1$

M.S $f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \dots$

$= \cancel{0 + x + }$

$= 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!}$

$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

$\sin 2x = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+1}}{2n+1}$

$x \sin 2x = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+2}}{2n+1} = 2x^2 - \frac{2x^4}{3} + \frac{2x^6}{5}$

$\int_0^1 x \sin 2x = \int_0^1 2x^2 - \frac{2x^4}{3} + \frac{2x^6}{5}$

$= \frac{2}{3}x^3 - \frac{2}{15}x^5 + \frac{2}{35}x^7 \Big|_0^1$

$\int x \sin 2x \quad \frac{2}{3} - \frac{2}{15} + \frac{2}{35} = \frac{62}{105} = .59$

$\sin 2x = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+1}}{2n+1}$

$x \sin 2x = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+2}}{2n+1}$

$x \sin 2x = 2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!}$

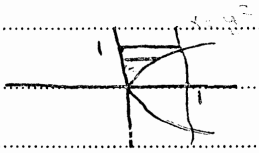
$= 2x^2 - \frac{2x^4}{6} + \frac{2x^6}{120}$

$\int_0^1 x \sin 2x = \frac{2x^3}{3} - \frac{2x^5}{30} + \frac{2x^7}{840} \Big|_0^1$

$= \frac{2}{3} - \frac{2}{30} + \frac{2}{840}$

$= .6$

Q 2 [b] $\int_0^1 \int_{\sqrt{x}}^1 e^{y^3} \, dy \, dx$



$x = 0 \to 1$

$y = \sqrt{x}$ $y = 1$

$\int_0^1 \int_0^{y^2} e^{y^3} \, dx \, dy$

$\int_0^1 x e^{y^3} \Big|_0^{y^2} dy \Rightarrow \int_0^1 y^2 e^{y^3} \, dy$

$\Rightarrow$ $\Rightarrow \frac{1}{3} e^{y^3} \Big|_0^1$

Q 2 (c)

$\iint \sqrt{f_x^2 + f_y^2 + 1}\ dA$

$z = x^2 + y^2 - 2 \qquad z \le 0$

$f_x = 2x \Rightarrow f_x^2 = 4x^2$

$f_y = 2y \Rightarrow f_y^2 = 4y^2$

$\sqrt{f_x^2 + f_y^2 + 1} = \sqrt{4x^2 + 4y^2 + 1}$

$x^2 + y^2 = 2$

$r: 0 \to \sqrt{2}$

$\theta: 0 \to 2\pi$

$\int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{4r^2+1}\ r\,dr\,d\theta$

$\int_0^{2\pi}\int_0^{\sqrt{2}} (4r^2+1)^{\frac{1}{2}}\ r\,dr\,d\theta$

$\frac{1}{8}\int_0^{2\pi} \frac{(4r^2+1)^{3/2}}{3/2}\Big|_0^{\sqrt{2}} d\theta$

$\frac{1}{8}\cdot\frac{2}{3}\int_0^{2\pi} (9)^{3/2} - (1)^{3/2}\ d\theta$

$\frac{1}{12}\cdot 26\int_0^{2\pi} d\theta$

$\frac{13}{6}\ \theta\Big|_0^{2\pi}$

$= \frac{13}{6}\,2\pi = \frac{13}{3}\pi$

Q 3 (a)

$V = \iint f(x,y)\ dA$

$z = 16 - x^2 - y^2$

$x^2 + y^2 = 16$

$r^2 = 16$

$r = 4$

$r: 0 \to 4$

$\theta: 0 \to 2\pi$

$\int_0^{2\pi}\int_0^{4} (16 - r^2)\ r\,dr\,d\theta$

$\int_0^{2\pi}\int_0^{4} 16r - r^3\ dr\,d\theta$

$\int_0^{2\pi} 8r^2 - \frac{r^4}{4}\Big|_0^4\ d\theta$

$\int_0^{2\pi} 128 - 64\ d\theta$

$64\int_0^{2\pi} d\theta$

$64\ \theta\Big|^{2\pi}$

ρ^{2} $\rho^{-\frac{3}{2}}$ ρ^{3} $\frac{1}{2\rho}$ $\frac{1}{\rho}$

$\frac{2x}{x^2}$ $2x$ x^{-1} $2x^{-1}$ $-2x^{-2}$ $\frac{-2}{x^2}$ $2x$

لا يك

هذا ال

Q 3 $\boxed{b}$

$\iiint \rho \frac{1}{\rho^3} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \frac{1}{\rho^3} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \frac{1}{\rho} \sin\phi \, d\rho \, d\phi \, d\theta$

$\int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{2\rho}\Big|_1^2 \sin\phi \, d\phi \, d\theta$

$\int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{4}-\frac{1}{2} \sin\phi \, d\phi \, d\theta$

$-\frac{1}{4}\int_0^{\pi/2}\int_0^{\pi/2} \sin\phi \, d\phi \, d\theta$

$-\frac{1}{4}\int_0^{\pi/2} -\cos\phi\Big|_0^{\pi/2} \, d\theta$

$\frac{1}{4}\int_0^{\pi/2} (0-1) \, d\theta$

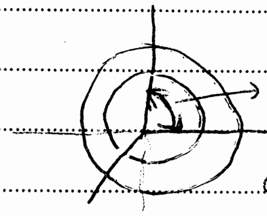
$-\frac{1}{4}\int_0^{\pi/2} d\theta$

$-\frac{1}{4} \theta\Big|_0^{\pi/2}$

$-\frac{1}{4}\frac{\pi}{2} = -\frac{\pi}{8}$

$x^2+y^2+z^2=1$ $\rho=1$

$x^2+y^2+z^2=4$ $\rho=2$



$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 2\rho \sin\phi \, d\rho \, d\phi \, d\theta$

$\int_0^{\pi/2}\int_0^{\pi/2} \rho^2\Big|_1^2 \sin\phi \, d\phi \, d\theta$

$\int_0^{\pi/2}\int_0^{\pi/2} 4-1 \sin\phi \, d\phi \, d\theta$

$3\int_0^{\pi/2} -\cos\phi\Big|_0^{\pi/2} \, d\theta$

$\int_0^{\pi/2} 0-1 \, d\theta$

$3(\theta)^{\pi/2}$

$3\frac{\pi}{2} = \frac{3\pi}{2}$

كتب في
الهامش

Q 3 c

$w = \int_c f \cdot dr$

$f \cdot dr = x\,dx - x\,dy$

~~Let~~

Let $x = t$, $y = t^2$

$\dot{x} = 1$ $\dot{y} = 2t$

$\int_{-1}^{1} t^2(1) - t(2t)\ dt$

$2\int_0^1 t^2 - 2t^2\ dt$

$2\int_0^1 -t^2\ dt$

$-2\int_0^1 t^2\ dt$

$-2\left(\frac{t^3}{3}\right)\Big|_0^1$

$-2\ \frac{1}{3} = -\frac{2}{3}\,J$

$\therefore w = -\frac{2}{3}\,J$

$f \cdot dr$

$\oint$ ~~$x\,dx - x\,dy$~~

$f \cdot dr = x\,dx - x\,dy$

Let $x = t$ $y = t^2$

$\dot{x} = 1$ $\dot{y} = 2t$

$(\bar{1}, 1)$ $(1, 1)$

3/4

لا يـ
هنا

Q 4 $\boxed{a}$

$$\oint_C M\,dx + N\,dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$$

$M = xy \Rightarrow \frac{\partial M}{\partial y} = x$

$N = x \Rightarrow \frac{\partial N}{\partial x} = 1$

$\int_0^1\int_{\sqrt{x}}^{x^2} 1 - x \, dy\, dx$

$\int_0^1 y - xy \Big|_{\sqrt{x}}^{x^2}$

$(x^2 - x^3) - (\sqrt{x} - x\sqrt{x})$

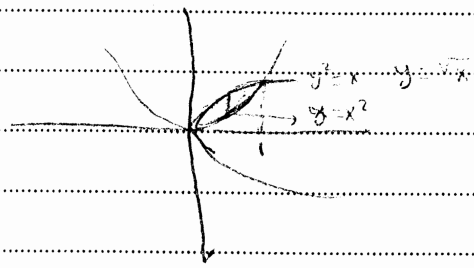
$\int_0^1 x^2 - x^3 - x^{\frac12} - x^{\frac32}\, dx$

$\frac{x^3}{3} + \frac{x^4}{4} - \frac{x^{\frac32}}{\frac32} + \frac{x^{\frac52}}{\frac52}\Big|_0^1$

$\frac13 + \frac14 - \frac23 + \frac25$

$= \frac{19}{60} = .317$

$y^2 = x \quad y = x^2$



$(x^2 - x) = 0$

$x(x-1) = 0$

$\int_0^1 \int_{x^2}^{\sqrt{x}} 1 - x \, dy\, dx$

$\int_0^1 y - xy \Big|_{x^2}^{\sqrt{x}}$

$\int_0^1 (\sqrt{x} - x\sqrt{x}) - (x^2 - x^3)\, dx$

$\int_0^1 x^{\frac12} - x^{\frac32} - x^2 + x^3\, dx$

$\frac{x^{\frac32}}{\frac32} - \frac{x^{\frac52}}{\frac52} - \frac{x^3}{3} + \frac{x^4}{4}\Big|_0^1$

$\frac23 - \frac25 - \frac13 + \frac14$

$\cancel{= \frac{23}{12} = 1.92}$

$= \frac{11}{60} = .183$

Q4 $\boxed{b}$

$$\iint_S F\cdot n\, dS = \iiint_Q \nabla\cdot F \, dV$$

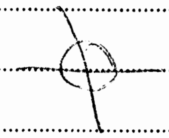
$$Z = \sqrt{x^2+y^2} \qquad z=1$$

$$z^2 = x^2+y^2$$

$$x^2+y^2=1$$

$$r^2=1$$

$$r=1$$



$$r \le z \le 1$$

$$0 \le \theta \le 2\pi$$

$$F(x,y,z) = (x+\sin y)i + (y+\cos z)j + (2z-e^x)k$$

$$\nabla\cdot F = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

$$= 1 + 1 + 2$$

$$= 4$$

$$4\int_0^{2\pi}\int_0^1\int_1^r 4\, r \, dz\, dr\, d\theta$$

$$4\int_0^{2\pi}\int_0^1 r z \Big|_1^r \, dr\, d\theta$$

$$4\int_0^{2\pi}\int_0^1 r^2 - r \, dr\, d\theta$$

$$4\int_0^{2\pi} \frac{r^3}{3} - \frac{r^2}{2}\Big|_0^1 \, d\theta$$

$$4\int_0^{2\pi} \frac{1}{3} - \frac{1}{2} \, d\theta$$

$$-\frac{1}{3}\int_0^{2\pi} d\theta$$

$$-\frac{1}{3}\left(\theta\right|_0^{2\pi}$$

$$-\frac{1}{3}\, 2\pi = -\frac{2}{3}\pi$$

Q4 $\boxed{c}$

$\iint_S (\text{curl}\,\vec{F})\cdot\vec{n}\,ds = \oint_C F\cdot dr$

L.H.S. $\oint_C F\cdot dr$

$F = -yi + zj + xk$

$F\cdot dr = -x\,dx + z\,dy + x\,d\overset{0}{z}$

$= -y\,dx + dy$

$z=1$
$dz=0$

~~$M=1 \quad \frac{\partial M}{\partial y}=0$~~

~~$N=-y \quad \frac{\partial N}{\partial x}=0$~~

~~$\int_0^{2\pi}\int_0^1 0-0\, r\,dr\,d\theta$~~

~~$=0$~~

~~$\therefore \iint (\text{curl}\,\vec{F})\cdot\vec{n}\,ds = \oint_C F\cdot dr = 0$~~

$M=-y \quad \frac{\partial M}{\partial y}=-1$

$N=1 \quad \frac{\partial N}{\partial x}=0$

$\iint_R \frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\,dA$

$\int_0^{2\pi}\int_0^1 r\,dr\,d\theta$

$\int_0^{2\pi} \frac{r^2}{2}\Big|_0^1 d\theta$

$\frac{1}{2}\int_0^{2\pi} d\theta$

$\frac{1}{2}(\theta)_0^{2\pi}$

$F = -yi + zj + xk$

$-x\,dx + z\,dy + x\,\cancel{dz}$

$z=1 \quad dz=0$

$-x\,dx + dy$

$x^2+y^2=2-1$

$x^2+y^2=1$

$r^2=1$

$r=1$

$r: 0\to 1$

$\theta: 0\to 2\pi$