KING SAUD UNIVERSITY COLLEGE OF SCIENCE

M203 DEPARTMENT OF MATHEMATICS TIME: 3 hours

(SEMESTER 2, 1438-1439) Final Exam

Q1.

(a) Find the interval of convergence and the radius of convergence of the power series: **(5Marks)**

$$\sum_{n=0}^{\infty} \frac{n+1}{10^n}(x-4)^n.$$

(b) Find the MacLaurin series of $f(x) = \cos x$ and use the first three nonzero terms of the function $x\cos(x^3)$ to approximate the integral $\int_0^{0.5} x\cos(x^3)\,dx$. **(4Marks)**

Q2.

(a) Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{1+x^3}\,dxdy$. **(3Marks)**

(b) Let P be the plane given by $2x + 2y + z = 8$. Find the surface area of the portion of P that lies in the first octant. **(3Marks)**

(c) Evaluate the triple integral $\iiint_Q x\,dV$, where Q is the solid enclosed by $z = 0$, $z = x + y + 5$, $x^2 + y^2 = 4$, and $x^2 + y^2 = 9$. **(4Marks)**

(d) Evaluate the integral by using **spherical coordinates**: **(4Marks)**

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} y\sqrt{x^2+y^2+z^2}\,dzdydx.$$

Q3.

(a) Show that the line integral **(4Marks)**

$$\int_{(0,0,0)}^{(1,-1,3)} e^y dx + (xe^y + \sin z)dy + y\cos z\,dz$$

is independent of path and find its value.

(b) Use Green's theorem to evaluate the line integral $\oint_C xy\,dx + x^2y^3dy$, where C is the triangle with vertices $(1,3), (1,0),$ and $(0,0)$ with positive orientation. **(3Marks)**

(c) Let S be the surface of the sphere $x^2 + y^2 + z^2 = 4$. Use the divergence theorem to find the flux of the force $\vec{F} = \frac{1}{3}x^3\vec{i} + \frac{1}{3}y^3\vec{j} + \frac{1}{3}z^3\vec{k}$ through the surface S. **(4Marks)**

(d) Verify Stokes' theorem for the force $\vec{F} = y\vec{i} + x\vec{j} + 4\vec{k}$ and for S, where S is the portion of the cone $z = 4 - \sqrt{x^2+y^2}$, $z \geq 3$. **(6Marks)**