Department of Mathematics
King Saud University
Final Examination
M-203(Differential & Integral Calculus)
Summer Term (1433/1434)

Max. Marks: 40 **Time**: 180 minutes

Marking Scheme: Q.No:1[3+3+3+3], Q.No:2[3+3+3+3], Q.No:3[4+4+4+4]

Q. No: 1 (a) Determine whether the sequence $\left\{\sqrt{n+1}-\sqrt{n}\right\}$ converges or diverges and if it converges, find its limit.

(b) Test the convergence of the series $\sum_{n=1}^{\infty} n^2 \tan^2\left(\frac{1}{n}\right)$.

(c) Find the radius of convergence and interval of convergence for the power series

$$\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n^2}(x-1)^n .$$

(d) Use Maclaurin Series of $\ln(1+x)$ to approximate the integral $\int_1^{1.1} \frac{\ln(x^2+1)}{x}dx$ using first two non-zero terms.

Q. No: 2 (a) Evaluate $\int_0^1 \int_y^1 \frac{\sin x}{x} dx\,dy$.

(b) Find the surface area of portion of **upper half** of the sphere $x^2+y^2+z^2=2$ cut off by the cylinder $x^2+y^2=1$.

(c) A solid is bounded by the cone $z=\sqrt{x^2+y^2}$ and the plane $z=2$ where the density at any point $P(x,y,z)$ is $\sqrt{x^2+y^2}$. Find the mass of the solid.

(d) Use **spherical coordinates** to evaluate the integral

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} \frac{z^2}{x^2+y^2+z^2}dz\,dy\,dx .$$

Q. No: 3 (a) Show that the line integral is independent of path by finding the potential function and hence find its value $\int_{(-1,1,1)}^{(0,e,2)} e^{-x}\ln y\,dx - \frac{e^{-x}}{y}dy + z\,dz$.

(b) Use **Green's theorem to evaluate the line integral** $\oint_C xy\,dx + (\frac{1}{2}x^2 + xy)dy$, where C is the Closed curve consisting of upper half of the ellipse $x^2+\frac{y^2}{1/4}=1$ and the interval [-1,1].

(c) Use the **Divergence theorem** to evaluate the integral $\iint_S \vec{F}\cdot\vec{n}\,dS$ where $\vec{F}=\sin y\,\vec{i}+y\,\vec{j}+e^x y\,\vec{k}$ and the surface S is bounded by $z=1-x^2-y^2$ and $z=0$.

(d) Use Stokes's theorem to evaluate $\iint_S (curl\,\vec{F})\cdot\vec{n}\,dS$, if $\vec{F}(x,y,z)=yz\,\vec{i}+xz^2\,\vec{j}-xyz\,\vec{k}$ and S is the surface $z=x^2+y^2$, $0\le z\le 4$.