King Saud University
Department Of Mathematics.
M-203 [Final Examination]
(Differential and Integral Calculus)
(Summer Term (1438/1439)

Max. Marks: 40 Time: 3 hrs

Marking Scheme: Q.1 [2+3+4+3]; Q.2 [3+3+3+3]; Q.3 [4+4+4+4];

Q. No: 1

(a) Find the limit of the sequence $\left\{\frac{\sqrt[3]{8n^6+2n^4+5}}{n^2+n+1}\right\}$.

(b) Determine whether the series: $\sum_{n=1}^{\infty} \frac{2+\sin n}{n^{3/2}}$ converges or diverges.

(c) Find the interval of convergence and the radius of convergence of the power series $\sum_{n=2}^{\infty} \frac{(-1)^n(3x+5)^n}{n \ln n}$.

(d) Find the first four non-zero terms of a Taylor series for the function $f(x) = \cos x$ at $x = \pi/3$.

Q. No: 2

(a) Evaluate the integral $\int_0^{\pi/2} \int_{2y}^{\pi} \frac{\cos x}{x} dxdy$.

(b) Use polar coordinates to evaluate the integral $\iint_R \sqrt{10 - x^2 - y^2}\, dA$, where R is the plane region that lies inside the circle $x^2 + y^2 = 9$ and outside the circle $x^2 + y^2 = 1$.

(c) Find the volume and centroid of the region bonded by the paraboloid $\mathbf{z = 9 - x^2 - y^2}$ and the xy -plane.

(d) Evaluate the integral by changing it to spherical coordinates :

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} e^{(x^2+y^2+z^2)^{3/2}} dzdydx.$$

Q. No: 3

(a) Show that $\int_C \vec{F} \cdot d\vec{r}$ is independent of path by finding a potential function f for $\vec{F} = -2y^3 \sin x\vec{\imath} + (6y^2 \cos x + 5)\vec{\jmath}$.

(b) Use Green's theorem to evaluate $\oint_C xy\, dx + (y + x)\, dy$, where C is the circle $x^2 + y^2 = 1$

(c) If $\vec{F} = 4x\vec{\imath} - 4y\vec{\jmath} + z^2\vec{k}$, S is the surface of the region bounded by the cylinder $x^2 + y^2 =$ and the planes $z = 0$ and $z = 3$, find $\iint_S \vec{F}.\vec{n}\, dS$ by using divergence theorem.

(d) If $\vec{F} = z\vec{\imath} + x\vec{\jmath} + y\vec{k}$ is a force field and S is the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$, use Stokes' theorem to evaluate $\iint_S curl\vec{F}.\vec{n}\, dS$