

Q#1 [4 + 4 + 4] , Q#2 [4 + 4 + 4] , Q#3 [4 + 4 + 4 + 4]

Q#1 (a) Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{3n^2}$ converges or diverges.

(b) Find the power series representation of the function $g(x) = \frac{1}{2+3x}$ and find the interval of convergence. Also , deduce the power series representation of the function $f(x) = \frac{1}{(2+3x)^2}$.

(c) Find the interval of convergence and the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{1}{n 2^n} (x + 10)^n$.

Q#2 (a) Evaluate the integral $\int_0^1 \int_x^1 \sin(y^2) dy dx$.

(b) Evaluate the integral $\int_0^2 \int_0^{\sqrt{2y-y^2}} \sqrt{x^2 + y^2} dx dy$.

(c) A solid has the shape of the region Q that lies inside the cylinder $x^2 + y^2 = a^2$, where $a > 0$, within the sphere $x^2 + y^2 + z^2 = 4a^2$ and above the xy -plane. The density at the point $P \in Q$ is $\delta(x, y, z) = z$. Find the moment of inertia of the solid about the z -axis.

Q#3 (a) Show that the vector field $\vec{F}(x, y, z) = y^2 \vec{i} + (2xy + e^{3z}) \vec{j} + 3ye^{3z} \vec{k}$ is conservative on R^3 and evaluate the integral $\int_{(2,-1,0)}^{(-1,3,0)} \vec{F} \cdot d\vec{r}$.

(b) Verify the Green 's theorem , where $\vec{F}(x, y,) = xy \vec{i} + 2x^2 \vec{j}$ and C consists of the line segment from $(-2,0)$ to $(2,0)$ and upper half of the circle $x^2 + y^2 = 4$.

(c) Evaluate the surface integral $\iint_S \vec{F} \cdot \vec{n} dS$ by using the divergence theorem, where $\vec{F}(x, y, z) = x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k}$ and S is the sphere $x^2 + y^2 + z^2 = 1$.

(d) Apply the Stokes' theorem to evaluate the surface integral $\iint_S \text{curl } \vec{F} \cdot \vec{n} dS$ where $\vec{F}(x, y, z) = y^2 z \vec{i} + xz \vec{j} + x^2 y^2 \vec{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 1$, oriented upward.