Final Examination : M203 Summer Semester 1439-1440 H Time : 3 Hours ; Max. Marks : 40	King Saud University College of Science Department of Maths.	THU 29-11-1440 H (1-8-2019)

Q#1 $[4+4+4]$, Q#2 $[4+4+4]$, Q#3 $[4+4+4+4]$

Q#1 (a) Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^n}{3^{n^2}}$ converges or diverges.

(b) Find the power series representation of the function $g(x) = \frac{1}{2+3x}$ and find the interval of convergence. Also , deduce the power series representation of the function $f(x) = \frac{1}{(2+3x)^2}$.

(c) Find the interval of convergence and the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{1}{n\, 2^n} (x+10)^n$.

Q#2 **(a)** Evaluate the integral $\int_0^1 \int_x^1 \text{Sin}(y^2)\, dy\, dx$.

(b) Evaluate the integral $\int_0^2 \int_0^{\sqrt{2y-y^2}} \sqrt{x^2+y^2}\, dx\, dy$.

(c) A solid has the shape of the region Q that lies inside the cylinder $x^2+y^2=a^2$, $where\ a>0$, within the sphere $x^2+y^2+z^2=4a^2$ and above the xy –plane.The density at the point $P \in Q$ is $\delta(x,y,z)=z$. Find the moment of inertia of the solid about the z –axis.

Q#3 (a) Show that the vector field $\vec{F}(x,y,z) = y^2\,\vec{i} + (2xy+e^{3z})\,\vec{j} + 3ye^{3z}\,\vec{k}$ is conservative on R^3 and evaluate the integral $\int_{(2,-1,0)}^{(-1,3,0)} \vec{F}\,.\,d\vec{r}$.

(b) Verify the Green 's theorem , where $\vec{F}(x,y,) = xy\,\vec{i} + 2x^2\,\vec{j}$ and C consists of the line segment from $(-2,0)$ to $(2,0)$ and upper half of the circle $x^2+y^2=4$.

(c) Evaluate the surface integral $\iint_S \vec{F}.\vec{n}\; dS$ by using the divergence theorem, where $\vec{F}(x,y,z) = x^3\,\vec{i} + y^3\,\vec{j} + z^3\,\vec{k}$ and S is the sphere $x^2+y^2+z^2=1$.

(d) Apply the Stokes' theorem to evaluate the surface integral $\iint_S curl\,\vec{F}.\vec{n}\, dS$ *where* $\vec{F}(x,y,z) = y^2z\,\vec{i} + xz\,\vec{j} + x^2y^2\,\vec{k}$ and S is the part of the paraboloid $z=x^2+y^2$ that lies inside the cylinder $x^2+y^2=1$, oriented upward.