King Saud University
Department of Mathematics
M-203
(Differential & Integral Calculus)
First Mid-Term Examination
(I-Semester 1434/35)

Max. Marks: 25 **Time: 90 minutes**

Marks: Q.1(4); Q.2(5); Q.3(6); Q.4(4); Q.5(6)

Q.No: 1 Determine whether or not the sequence $\left\{\dfrac{3^{n+2}}{5^n}\right\}_{n=1}^{\infty}$ converges, and if it converges find its limit

Q. No: 2 Test the convergence or divergence of the series $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)}$

Q.No: 3 Find the interval of convergence and the radius of convergence of the power series: $\displaystyle\sum_{n=1}^{\infty}\frac{(x+2)^n}{n4^n}$.

Q. No: 4 Test the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{\sin^2(n)}{2^n}$

Q. N0: 5 Find the Maclaurin's series for the function $f(x)=\sin(2x)$ and use its first three non-zero terms to approximate the integral

$$\int_0^1 \frac{\sin(2x)-x}{x}dx.$$

(1)

Q: 1 $a_n = 9\left(3/5\right)^n \longrightarrow 0$ as $n \to \infty$ [Mark:2]

$\Longrightarrow$ Conv. sequence (1)

Q: 2 $f(x) = \dfrac{1}{x \ln x}$ (i) Cont. on $[2, \infty)$ (1)

[Mark: 5] (ii) DEC " " (1)

$$\lim_{t \to \infty} \int_2^t (\ln x)^{-1} \left(1/x\right) dx \qquad \text{Put } u = \ln x, \; du = \tfrac{1}{x}\, dx$$

$$= \lim_{t \to \infty} \int_{\ln(2)}^{t} \frac{1}{u}\, du = \lim_{t \to \infty} \Big[\ln|u|\Big]_{\ln 2}^{t}$$

$$= \lim_{t \to \infty} \Big[\ln(t) - \ln(\ln 2)\Big] = \infty \quad \text{d'gt}$$

(2) (1)

Q: 3 $$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{(x+2)^{n+1}}{(n+1)\,4^{n+1}} \times \frac{n 4^n}{(x+2)^n}\right| = \frac{n}{4(n+1)} |x+2|$$ [Mark: 6]

$$\lim_{n \to \infty} \left|\frac{u_{n+1}}{u_n}\right| = \frac{1}{4}|x+2| \Longrightarrow \text{c'gt if } \frac{1}{4}|x+2| < 1$$

$$\Rightarrow |x+2| < 4 \Longrightarrow -4 < x+2 < 4 \Rightarrow \boxed{-6 < x < 2}$$

(3)

At $\boxed{x = -6}$

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n 4^n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \quad \text{c'gt}$$

(1)

$\boxed{\text{At } x = 2}$

$$\sum_{n=1}^{\infty} \frac{4^n}{n 4^n} = \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{d'gt}$$

(1)

$\Longrightarrow$ Interval $\boxed{-6 \le x < 2}$

$$\text{Radius} = \frac{2-(-6)}{2} = 4$$

(1)

2

Q:4 $\quad 0 \le \sin^2 x \le 1$ $\quad$ [Mark: 4]

$$\frac{0}{2^n} \le \frac{\sin^2 n}{2^n} \le \frac{1}{2^n} \qquad 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{2^n}$ is a c'gt G.S. (2) $\Rightarrow$ by the Basic Comparison test $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$ is c'gt (1)

Q:5 $\quad f(x) = \sin(2x) \Rightarrow f(0) = 0$ $\quad$ [Mark: 6]

$$f'(x) = 2\cos(2x) \Rightarrow f'(0) = 2$$

$$f''(x) = -2^2 \sin(2x) \Rightarrow f''(0) = 0$$

$$f'''(x) = -2^3 \cos(2x) \Rightarrow f'''(0) = -2^3$$

$$\Rightarrow f^{iv}(x) = 2^5 \cos(2x) \Rightarrow f^{iv}(0) = 2^5$$

$\boxed{\cancel{\sin(2x) \approx 2x < x}}$

$$\sin(2x) \approx x(2) - \frac{x^3}{3!}(2)^3 + \frac{x^5}{5!}(2)^5$$

$$= 2x - \frac{8}{6}x^3 + \frac{32}{120}x^5 \qquad 3$$

$$\sin(2x) - x \approx x - \frac{8}{6}x^3 + \frac{32}{120}x^5$$

$$\frac{\sin(2x) - x}{x} \approx 1 - \frac{8}{6}x^2 + \frac{32}{120}x^4$$

$$\int_0^1 \frac{\sin(2x) - x}{x}\,dx \approx \int_0^1 \left[1 - \frac{8}{6}x^2 + \frac{32}{120}x^4\right] dx$$

$$= \left[x - \frac{8}{18}x^3 + \frac{32}{120} \times \frac{x^5}{5}\right]_0^1 \qquad 2$$

$$= 1 - \frac{4}{9} + \frac{32}{600} = \frac{1800 - 800 + 96}{1800} = \frac{1096}{1800}$$

$$\approx 0.61$$

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