King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second Mid-Term Examination
(I-Semester 1434/1435)

Max. Marks: 25 Time: 90 Minutes

Marking Scheme: **Q.1**(4), **Q.2**:(4), **Q.3**:(4), **Q.4**:(4), **Q.5**:(4), **Q.6**:(5)

Q. No: 1 Reverse the order of integration, and evaluate the resulting integral

$$\int_0^4 \int_{\sqrt{y}}^2 y\cos(x^5)\,dx\,dy.$$

Q. No: 2 Use **polar coordinates** to evaluate the integral

$$\int_{-a}^{a} \int_{0}^{\sqrt{a^2-x^2}} (x^2+y^2)^{3/2}\,dy\,dx \quad \text{, where } a>0.$$

Q. No: 3 Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ lying between the planes $x = 0$, $y = 0$, $z = 0$, and $z = 4$.

Q. No: 4 Consider the **lamina** over the region R in the **first quadrant** bounded by the curves $y = x^3$ and $y = x$ with area mass density $\delta(x, y) = y$. Find the **moment of inertia** of the lamina about the $x-axis$.

Q. No: 5 Find the volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2 + y^2 = 16$ and the $xy-plane$.

Q. No: 6 Use **spherical coordinates** to evaluate the integral

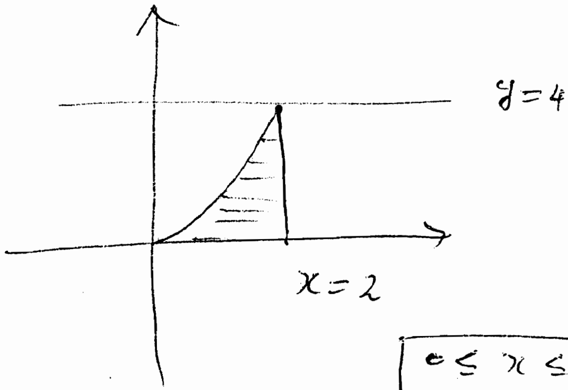
$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)\,dz\,dy\,dx$$

Q: 1

Marks: 4

$$= \int_0^2 \int_0^{x^2} y \cos(x^5)\, dy\, dx$$

(2)



$$\boxed{\begin{aligned} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{aligned}}$$

$$= \frac{1}{2}\int_0^2 x^4 \cos(x^5)\, dx$$

Put $u = x^5$

$\frac{1}{5}\, du = x^4 dx$

$$= \frac{1}{10}\int_0^{32} \cos u\, du = \frac{1}{10} \sin u\Big]_0^{32} + c$$

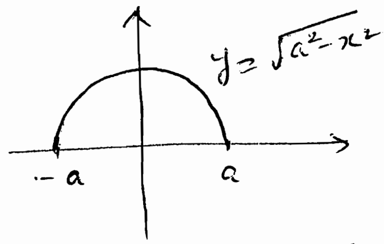
$$= \frac{1}{10} \sin(32) + c$$

(2)

Q: 2

[Marks: 4]

$$\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)^{3/2}\, dy\, dx$$



(2)

$$= \int_0^{\pi} \int_0^{a} r^3\, r\, dr\, d\theta = \int_0^{\pi} \frac{r^5}{5}\Big]_0^a d\theta$$

$0 \le r \le a$

$0 \le \theta \le \pi$

$$= \frac{a^5}{5}\int_0^{\pi} d\theta = \frac{a^5 \pi}{5}$$

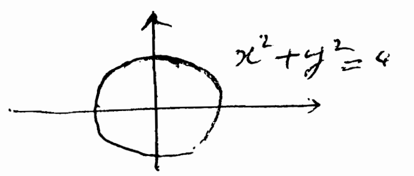
(1)

Q: 3

$$S.A = \iint_R \sqrt{1+f_x^2+f_y^2}\, dA$$

$z = f(x,y) = x^2 + y^2$

[Marks: 4]



(3)

$$= \int_0^{\cancel{\pi/2}\ 2\pi} \int_0^2 \sqrt{1+(2x)^2+(2y)^2}\; r\, dr\, d\theta$$

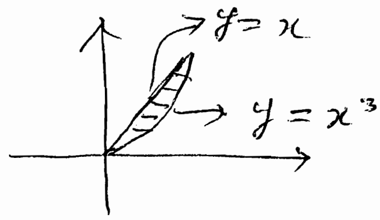
$$= \int_0^{\cancel{\pi/2}\ 2\pi} \int_0^2 \sqrt{1+4r^2}\; r\, dr\, d\theta$$

$$= \frac{1}{8}\int_0^{\cancel{\pi/2}\ 2\pi} \frac{(1+4r^2)^{3/2}}{3/2}\Big]_0^2 d\theta = \frac{1}{12}\left[17^{3/2} - 1\right] (2\pi)\ \cancel{\pi/2}$$

(1)

Q. 4 [Marks: 4]

$$I_x = \iint_R \delta y^2 \, dA = \iint_R y^3 \, dA$$



(2)

$$I_x = \int_0^1 \int_{x^3}^{x} y^3 \, dy \, dx = \frac{1}{4} \int_0^1 y^4 \Big]_{x^3}^{x} dx$$

$0 \le x \le 1$
$x^3 \le y \le x$

$$= \frac{1}{4} \int_0^1 [x^4 - x^{12}] \, dx = \frac{1}{4} \left[\frac{x^5}{5} - \frac{x^{13}}{13} \right]_0^1$$

$$= \frac{1}{4} \left[\frac{1}{5} - \frac{1}{13} \right] = \frac{1}{4} \frac{8}{65} = \frac{2}{65}$$

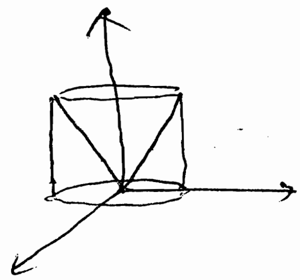
(1)

Q. 5 [Marks: 4]

$$V = \iiint_Q dV$$

Cylindrical

$0 \le z \le r$
$0 \le r \le 4$
$0 \le \theta \le 2\pi$



(3)

$$= \int_0^{2\pi} \int_0^4 \int_0^r r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^4 r^2 \, dr \, d\theta = \int_0^{2\pi} \frac{r^3}{3} \Big]_0^4 d\theta = \frac{64}{3} (2\pi) = \frac{128\pi}{3}$$

(1) #

Spherical

$0 \le \rho \le 4 \csc\varphi$
$\frac{\pi}{4} \le \varphi \le \pi/2$
$0 \le \theta \le 2\pi$

(3)

$$V = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^{4\csc\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \frac{\rho^3}{3} \Big]_0^{4\csc\varphi} \sin\varphi \, d\varphi \, d\theta$$

$$= \frac{1}{3} \int_0^{2\pi} \int_{\pi/4}^{\pi/2} 64 \csc^3\varphi \sin\varphi \, d\varphi \, d\theta = \frac{64}{3} \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \csc^2\varphi \, d\varphi \, d\theta$$

$$= \frac{64}{3} \int_0^{2\pi} -\cot\varphi \Big]_{\pi/4}^{\pi/2} d\theta = \frac{64}{3} \int_0^{2\pi} [-0 + 1] \, d\theta = \frac{128}{3} \pi$$

(1) #

Q.6

[Marks: 5]

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)\,dz\,dy\,dx$$

$0 \le \rho \le 2$

$0 \le \varphi \le \pi/4$

$0 \le \theta \le 2\pi$

(4) $$= \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} \rho^2 \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \left.\frac{\rho^5}{5}\right]_0^2 \sin\varphi \, d\varphi\, d\theta = \frac{32}{5}\int_0^{2\pi} \left.-\cos\varphi\right]_0^{\pi/4} d\theta$$

$$= \frac{32}{5}\left[-1/\sqrt{2}+1\right]\int_0^{2\pi} d\theta$$

$$= \frac{64}{5}\left(1-\frac{1}{\sqrt{2}}\right)\pi$$

(1) #