King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second-Mid Term Examination
(First Semester 1435/1436)

Max. Marks: 25 Time: 90 minutes

Marking Scheme: **All questions carry equal marks.**

Q. No:1 **Reverse** the order of integration and evaluate the resulting integral $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx$.

Q. No:2 Use **polar coordinates** to evaluate the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx .$$

Q. No:3 Find the surface area of the part of the solid cut off from the sphere $z=\sqrt{25-x^2-y^2}$ by the planes $z=3$ and $z=4$.

Q. No:4 Use **cylindrical coordinates** to evaluate the integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \int_0^{\sqrt{x^2+y^2}} e^z\,dz\,dy\,dx$$

Q. No:5 Evaluate the integral (by using **spherical coordinates**)

$$\iiint_Q \sqrt{x^2+y^2+z^2}\,dv ,$$

where Q is the solid region bounded by the graphs of the equations

$$z=\sqrt{x^2+y^2} \quad \text{and} \quad z=\sqrt{2-x^2-y^2} .$$

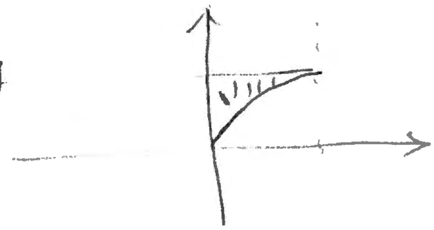
Q: 1 $\displaystyle\int_0^1\int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx$ [5 Marks]

$0 \le x \le 1$

$\sqrt{x} \le y \le 1$

$$= \int_0^1\int_0^{y^2} \sqrt{y^3+1}\,dx\,dy = \int_0^1 (y^3+1)^{1/2} y^2\,dy$$

4



$$= \frac{1}{3}\left.\frac{(y^3+1)^{3/2}}{3/2}\right]_0^1 = \frac{2}{9}\left[(2)^{3/2}-1\right]$$ 1

$0 \le y \le 1$

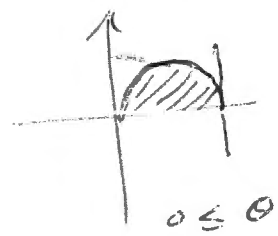
$0 \le x \le y^2$

[5 Marks]

Q: 2 $\displaystyle\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$

$0 \le x \le 2$

$0 \le y \le \sqrt{2x-x^2}$



$$= \int_0^{\pi/2}\int_0^{2\cos\theta} r\,(r\,dr\,d\theta)$$ 4

$0 \le \theta \le \pi/2$

$0 \le r \le 2\cos\theta$

$$= \int_0^{\pi/2} \left.\frac{r^3}{3}\right]_0^{2\cos\theta} d\theta = \frac{8}{3}\int_0^{\pi/2} \cos^3\theta\,d\theta$$

$$= \frac{8}{3}\int_0^{\pi/2} (1-\sin^2\theta)\cos\theta\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{8}{3}\int_0^1 (1-u^2)\,du = \frac{8}{3}\left[u - \frac{u^3}{3}\right]_0^1$$

$$= \frac{8}{3}\left[1-\frac{1}{3}\right] = \frac{16}{9}$$ 1

Q:3 [5 Marks]

$$S.A. = \iint_{R_{xy}} \sqrt{1+f_x^2+f_y^2}\, dA \quad (1)$$

$$f_x = \frac{-x}{\sqrt{25-x^2-y^2}}$$

$$f_y = \frac{-y}{\sqrt{25-x^2-y^2}} \quad (1)$$

$$= \iint_{R_{xy}} \frac{5}{\sqrt{25-x^2-y^2}}\, dA$$

$z = 3$, $z = 4$

$$\boxed{\begin{array}{c} 3 \le r \le 4 \\ 0 \le \theta \le 2\pi \end{array}}$$

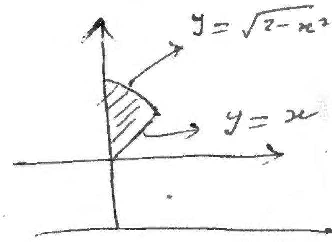
$$= \int_0^{2\pi}\int_3^4 \frac{5}{\sqrt{25-r^2}}\, r\, dr\, d\theta \quad (2)$$

$$= 5\int_0^{2\pi}\int_3^4 (25-r^2)^{-1/2}(r)\, dr\, d\theta = -\frac{5}{2}\int_0^{2\pi} \left.\frac{(25-r^2)^{1/2}}{1/2}\right]_3^4 d\theta$$

$$= -5\,[3-4]\int_0^{2\pi} d\theta = 10\pi \quad (1)$$

Q:4 [5 Marks]

$$\int_0^1\int_x^{\sqrt{2-x^2}}\int_0^{\sqrt{x^2+y^2}} e^z\, dz\, dy\, dx$$



$$\boxed{\begin{array}{c} x \le y \le \sqrt{2-x^2} \\ 0 \le x \le 1 \end{array}}$$

$$\boxed{\begin{array}{c} 0 \le r \le \sqrt{2} \\ \pi/4 \le \theta \le \pi/2 \\ 0 \le z \le r \end{array}}$$

$$= \int_{\pi/4}^{\pi/2}\int_0^{\sqrt{2}}\int_0^r e^z\, r\, dz\, dr\, d\theta \quad (3)$$

$$= \int_{\pi/4}^{\pi/2}\int_0^{\sqrt{2}} \left. e^z \right]_0^r r\, dr\, d\theta$$

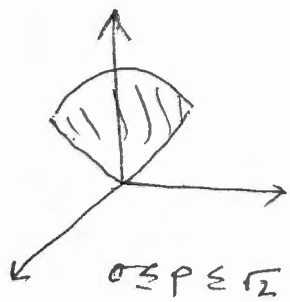
$$= \int_{\pi/4}^{\pi/2}\int_0^{\sqrt{2}} r(e^r - 1)\, dr\, d\theta = \int_{\pi/4}^{\pi/2}\left[\int_0^{\sqrt{2}} re^r\, dr - \int_0^{\sqrt{2}} r\, dr\right] d\theta$$

$$= \int_{\pi/4}^{\pi/2}\left[\{re^r - e^r\}_0^{\sqrt{2}} - \left\{\frac{r^2}{2}\right\}_0^{\sqrt{2}}\right] d\theta = \int_{\pi/4}^{\pi/2}\left[\sqrt{2}e^{\sqrt{2}} - e^{\sqrt{2}} + 1 - 1\right] d\theta$$

$$= \left(\sqrt{2}e^{\sqrt{2}} - e^{\sqrt{2}}\right)\left[\pi/4\right] = e^{\sqrt{2}}(\sqrt{2}-1)\frac{\pi}{4} \quad (2)$$

Q:5 [5 Marks]

$$\iiint_{Q} \sqrt{x^2+y^2+z^2}\, dV$$



$0 \le \rho \le \sqrt{2}$

$0 \le \varphi \le \pi/4$

$0 \le \theta \le 2\pi$

$$= \int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{2}} \rho\, \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta \quad 3+1= 4$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \left.\frac{\rho^4}{4}\right]_0^{\sqrt{2}} \sin\varphi \, d\varphi\, d\theta$$

$$= \int_0^{2\pi} -\cos\varphi \Big]_0^{\pi/4} d\theta = \int_0^{2\pi} \left[-1/\sqrt{2} + 1\right] d\theta$$

$$= \left[-1/\sqrt{2}+1\right] 2\pi = \sqrt{2}\pi(\sqrt{2}-1) \quad 1$$