

King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second-Mid Term Examination
(First Semester 1437/1438)

Max. Marks: 25

Time: 90 minutes

Marking Scheme: All questions carry equal marks.

Q. No: 1 Evaluate the integral $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{x^3+1} \, dx dy$.

Q. No: 2 Evaluate the integral $\iint_R y \, dA$, where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 9$, the line $y = x$ and $y = 0$.

Q. No: 3 Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 1$.

Q. No: 4 Find the centre of mass of the solid bounded by the region that lies inside the sphere $x^2 + y^2 + z^2 = 4z$ and below the cone $z = \sqrt{3x^2 + 3y^2}$ having density $\delta(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$.

Q. No: 5 Find the volume of the solid bounded by the paraboloids $z = 4 - x^2 - y^2$, $z = x^2 + y^2$, and the cylinder $x^2 + y^2 = 1$.

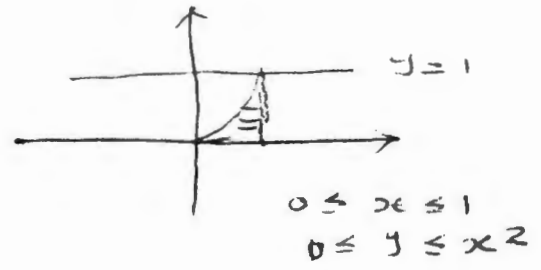
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II Mid-term Exam (I sem. 1437/1438)

Q. No. 1 $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1} dx dy$

$0 \leq y \leq 1$
 $\sqrt{y} \leq x \leq 1$ [Marks: 5]



$= \int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1} dy dx$ (3)

$= \int_0^1 (x^3+1)^{1/2} x^2 dx$

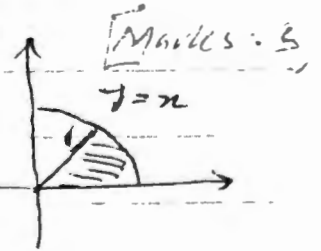
$= \frac{1}{3} \int_0^1 (x^3+1)^{1/2} (3x^2) dx = \frac{1}{3} \left[\frac{(x^3+1)^{3/2}}{3/2} \right]_0^1$

$= \frac{2}{9} [(2)^{3/2} - 1]$ # (2)

Q. No. 2

$\iint_R y dA$
 $= \int_0^{\pi/4} \int_0^1 r^2 \sin \theta dr d\theta$ (3)

$0 \leq r \leq 1$
 $0 \leq \theta \leq \pi/4$



$= \int_0^{\pi/4} \left[\frac{r^3}{3} \right]_0^1 \sin \theta d\theta = \frac{1}{3} [-\cos \theta]_0^{\pi/4} = \frac{1}{3} [-\frac{1}{\sqrt{2}} + 1]$

Q. No. 3

$SA = \iint_R \sqrt{1+4x^2+4y^2} dA = \int_0^{2\pi} \int_0^1 \sqrt{1+4r^2} r dr d\theta$ (3) [Marks: 5]

$= \frac{1}{2} \int_0^{2\pi} \left[\frac{(1+4r^2)^{3/2}}{3/2} \right]_0^1 d\theta = \frac{1}{12} [(5)^{3/2} - 1] 2\pi$

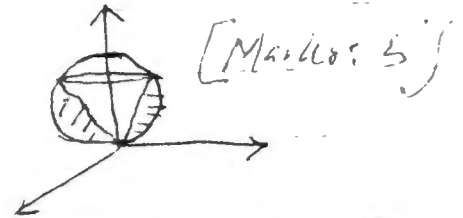
$= \frac{\pi}{6} [5^{3/2} - 1]$ # (2)

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$\rho^2 = 4z$ (2)

Q. No. 4

mass = $\iiint_Q \frac{1}{x^2+y^2+z^2} dv$



$= \int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_0^{4\cos\varphi} \frac{1}{\rho^2} \rho^2 \sin\varphi d\rho d\varphi d\theta$ (3)

$0 \leq \rho \leq 4\cos\varphi$
 $\pi/6 \leq \varphi \leq \pi/2$
 $0 \leq \theta \leq 2\pi$

$= \int_0^{2\pi} \int_{\pi/6}^{\pi/2} [\rho]_0^{4\cos\varphi} \sin\varphi d\varphi d\theta$

$= \int_0^{2\pi} \int_{\pi/6}^{\pi/2} 4 \sin\varphi \cos\varphi d\varphi d\theta = 2 \int_0^{2\pi} \left[\frac{1}{2} \cos 2\varphi \right]_{\pi/6}^{\pi/2} d\theta$

$= -[\cos 2\varphi]_{\pi/6}^{\pi/2} 2\pi = -[-1 - \frac{1}{2}] 2\pi$

$= \frac{3}{2} 2\pi = 3\pi$ (4)

$\int_0^{2\pi} \int_{\pi/6}^{\pi/2} \int_0^{4\cos\varphi} \frac{1}{\rho^2} \rho \cos\varphi \rho^2 d\rho d\varphi d\theta$

$= \int_0^{2\pi} \int_{\pi/6}^{\pi/2} \left[\frac{\rho^2}{2} \right]_0^{4\cos\varphi} \cos\varphi \sin\varphi d\varphi d\theta = 2 \int_0^{2\pi} \int_{\pi/6}^{\pi/2} \cos^3\varphi \sin\varphi d\varphi d\theta$

$= -2 \int_0^{2\pi} \left[\frac{\cos^4\varphi}{4} \right]_{\pi/6}^{\pi/2} d\theta = -2 \left[0 - \frac{(\sqrt{3}/2)^4}{4} \right] 2\pi$

$= 2 \left(\frac{9}{16} \right) 2\pi = \frac{9}{4} \pi$

$\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{\frac{9}{4}\pi}{\left(\frac{9}{4}\pi\right) 2\pi} = \frac{9/4\pi}{3\pi} = \frac{3}{4}$

(3)

Q: N5

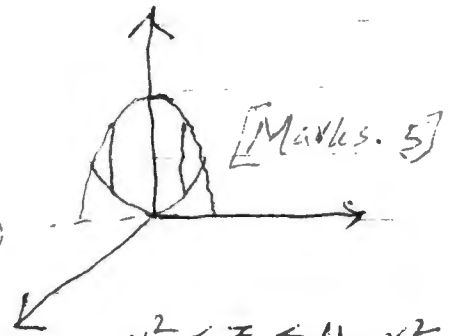
$$V = \int_0^{2\pi} \int_0^1 \int_{r^2}^{4-r^2} r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 r [4-r^2-r^2] \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 [4r - 2r^3] \, dr \, d\theta = \int_0^{2\pi} \left[\frac{4r^2}{2} - \frac{2}{4} r^4 \right]_0^1 \, d\theta$$

$$= \left(2 - \frac{1}{2}\right) 2\pi = 3\pi$$

(3)



$$r^2 \leq z \leq 4-r^2$$

$$0 \leq r \leq 1$$

$$0 \leq \theta \leq 2\pi$$

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