King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second Mid-Term Examination
(II-Semester 1435/1436)

Max. Marks: 25 Time: 90 Minutes

Note: All questions carry equal marks

Q. No: 1 Reverse the order of integration, and evaluate the resulting integral:

$$\int_0^1 \int_{y^2}^1 \sec^2\left(\frac{\pi}{4} x^{3/2}\right) dx\,dy$$

Q. No: 2 Find the **area of the surface** $x^2 - 2y - 2z = 0$ that lies above the triangular region bounded by the lines $x = 2$, $y = 0$, and $y = 3x$ in xy-plane.

Q. No: 3 Let Q be the solid region bounded by the graphs of $z = 1 - x^2$, $x = 0$, $y = 0$, $z = 0$, and $x + y = 1$. Find the volume of the solid.

Q. No: 4 Using cylindrical coordinates, find the mass of the solid bounded by $z = x^2 + y^2 - 1$ and $z = 0$ having density $\delta(x, y, z) = 1 + x^2 + y^2$.

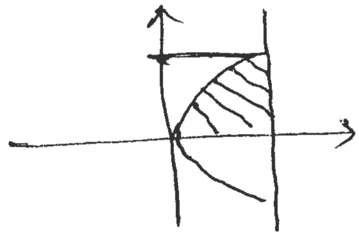
Q.No: 5 Use spherical coordinates to evaluate the integral

$$\int_0^5 \int_0^{\sqrt{25-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{50-x^2-y^2}} \sqrt{x^2+y^2+z^2}\; dz\,dy\,dx$$

Q:1 $\displaystyle \int_0^1 \int_{y^2}^1 \sec^2\left(\frac{\pi}{4} x^{3/2}\right) dx\, dy$

$0 \le y \le 1$

$y^2 \le x \le 1$



$$= \int_0^1 \int_0^{\sqrt{x}} \sec^2\left(\frac{\pi}{4} x^{3/2}\right) dy\, dx$$

$$\boxed{\begin{array}{c} 0 \le x \le 1 \\ 0 \le y \le \sqrt{x} \end{array}}$$

$$= \int_0^1 \sec^2\left(\frac{\pi}{4} x^{3/2}\right) x^{1/2}\, dx$$

Put $u = \frac{\pi}{4} x^{3/2}$

$\frac{2}{3} \times \frac{4}{\pi}\, du = x^{1/2}\, dx$

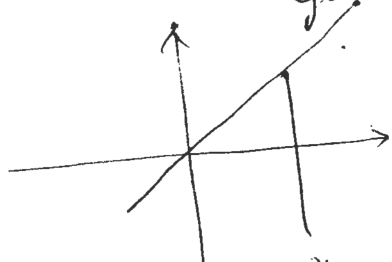
$$= \frac{8}{3\pi} \int_0^{\pi/4} \sec^2 u\, du = \frac{8}{3\pi} \Big[\tan u\Big]_0^{\pi/4}$$

$$= \frac{8}{3\pi}$$

Q: 2 $\displaystyle S.A = \iint_{R_{xy}} \sqrt{1 + f_x^2 + f_y^2}\, dA$

$z = \frac{1}{2}(x^2 - 2y)$

$= g(x,y)$



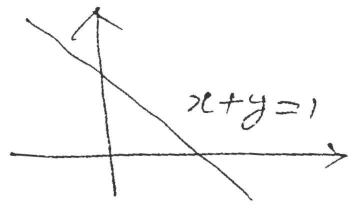
$$= \int_0^2 \int_0^{3x} \sqrt{1 + x^2 + 1}\, dy\, dx$$

$$= \int_0^2 \sqrt{2 + x^2}\, (3x)\, dx$$

$$= \frac{3}{2} \int_0^2 \sqrt{2 + x^2}\, (2x)\, dx = \frac{3}{2} \left[\frac{(2 + x^2)^{3/2}}{3/2}\right]_0^2$$

$$= (6)^{3/2} - 2^{3/2} = 14.697 - 2.828 = 11.869$$

Q:3 <u>Using double integral</u>



$$V = \iint_{R_{xy}} f(x,y)\, dA$$

$$= \int_0^1\int_0^{1-x} (1-x^2)\, dy\, dx = \int_0^1 (1-x)(1-x^2)\, dx$$

$0 \le x \le 1$

$0 \le y \le 1-x$

$$= \int_0^1 (1-x-x^2+x^3)\, dx = \left[x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}\right]_0^1 = 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$$

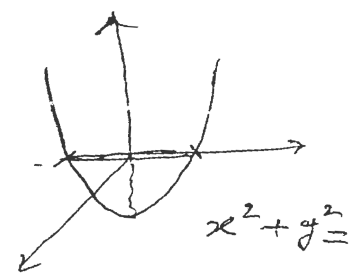
$$= \frac{5}{12} = 0.42$$

<u>Using a triple integral</u>

$$V = \iiint_Q dv = \int_0^1\int_0^{1-x}\int_0^{1-x^2} dz\, dy\, dx = \int_0^1\int_0^{1-x} (1-x^2)\, dy\, dx$$

Q:4

$$m = \iiint_{Q} (1+x^2+y^2)\, dv$$



$$= \int_0^{2\pi}\int_0^1\int_{r^2-1}^0 (1+r^2)\, r\, dz\, dr\, d\theta$$

$x^2+y^2-1 \le z \le 0$

$0 \le r \le 1$

$0 \le \theta \le 2\pi$

$$= \int_0^{2\pi}\int_0^1 (1+r^2)\, r\left[-r^2+1\right] dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 (r - r^5)\, dr\, d\theta = \int_0^{2\pi}\left[\frac{r^2}{2} - \frac{r^6}{6}\right]_0^1 d\theta = \int_0^{2\pi}\left[\frac{1}{2} - \frac{1}{6}\right] d\theta$$

$$= \frac{1}{3}(2\pi) = \frac{2\pi}{3} = 2.094$$

Q: 5 $\displaystyle\int_0^5 \int_0^{\sqrt{25-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{50-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx$

$$\boxed{\begin{array}{l} 0 \le \rho \le \sqrt{50} \\ 0 \le \varphi \le \pi/4 \\ 0 \le \theta \le \pi/2 \end{array}}$$

$$= \int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sqrt{50}} \rho\, \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{\pi/2} \int_0^{\pi/4} \left.\frac{\rho^4}{4}\right]_0^{\sqrt{50}} \sin\varphi\, d\varphi\, d\theta$$

$$= \frac{2500}{4} \int_0^{\pi/2} \left. -\cos\varphi \right]_0^{\pi/4} d\theta = \frac{2500}{4}\left[-\frac{1}{\sqrt{2}}+1\right] \pi/2$$