

King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second Mid-Term Examination
(II-Semester 1435/1436)

Max. Marks: 25

Time: 90 Minutes

Note: All questions carry equal marks

Q. No: 1 Reverse the order of integration, and evaluate the resulting integral:

$$\int_0^1 \int_{y^2}^1 \sec^2\left(\frac{\pi}{4}x^{3/2}\right) dx dy$$

Q. No: 2 Find the area of the surface $x^2 - 2y - 2z = 0$ that lies above the triangular region bounded by the lines $x = 2$, $y = 0$, and $y = 3x$ in xy -plane.

Q. No: 3 Let Q be the solid region bounded by the graphs of $z = 1 - x^2$, $x = 0$, $y = 0$, $z = 0$, and $x + y = 1$. Find the volume of the solid.

Q. No: 4 Using cylindrical coordinates, find the mass of the solid bounded by $z = x^2 + y^2 - 1$ and $z = 0$ having density $\delta(x, y, z) = 1 + x^2 + y^2$.

Q.No: 5 Use spherical coordinates to evaluate the integral

$$\int_0^5 \int_0^{\sqrt{25-r^2}} \int_{\sqrt{r^2+y^2}}^{\sqrt{50-r^2-y^2}} \sqrt{x^2 + y^2 + z^2} dz dy dx$$

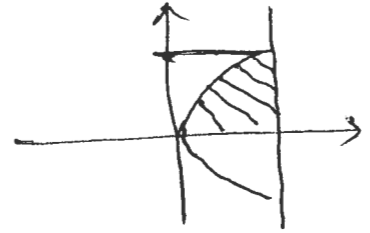
①

$$\underline{Q:1} \quad \int_0^1 \int_{y^2}^1 \sec^2\left(\frac{\pi}{4} x^{3/2}\right) dx dy$$

$$0 \leq y \leq 1$$

$$y^2 \leq x \leq 1$$

$$= \int_0^1 \int_0^{\sqrt{x}} \sec^2\left(\frac{\pi}{4} x^{3/2}\right) dy dx$$



$$= \int_0^1 \sec^2\left(\frac{\pi}{4} x^{3/2}\right) x^{1/2} dx$$

$$= \frac{8}{3\pi} \int_0^{\pi/4} \sec^2 u du = \frac{8}{3\pi} \left[\tan u \right]_0^{\pi/4}$$

$$0 \leq x \leq 1$$

$$0 \leq y \leq \sqrt{x}$$

Put $u = \frac{\pi}{4} x^{3/2}$

$$\frac{2}{3} \times \frac{4}{\pi} du = x^{1/2} dx$$

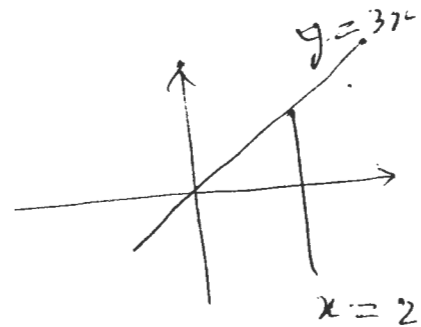
$$= \frac{8}{3\pi}$$

$$\underline{Q:2} \quad S.A = \iint_{R \times y} \sqrt{1 + f_x^2 + f_y^2} dA$$

$$z = \frac{1}{2}(x^2 - 2y)$$

$$= f(x, y)$$

$$= \int_0^2 \int_0^{3x} \sqrt{1 + x^2 + 1} dy dx$$



$$= \int_0^2 \sqrt{2+x^2} (3x) dx$$

$$= \frac{3}{2} \int_0^2 \sqrt{2+x^2} (2x) dx = \frac{3}{2} \left[\frac{(2+x^2)^{3/2}}{3/2} \right]_0^2$$

$$= (6)^{3/2} - 2^{3/2} = 14.697 - 2.828 = 11.869$$

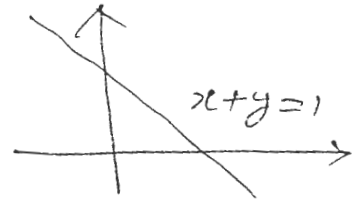
(2)

Q.3Using double integral

$$V = \iint f(x, y) dA$$

$$= \int_0^1 \int_0^{1-x} (1-x^2) dy dx = \int_0^1 (1-x)(1-x^2) dx$$

$$= \int_0^1 (1-x-x^2+x^3) dx = \left[x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} \right]_0^1 = 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} = \frac{5}{12} = 0.42$$



$$0 \leq x \leq 1$$

$$0 \leq y \leq 1-x$$

Using a triple integral

$$V = \iiint_Q dv = \int_0^1 \int_0^{1-x} \int_0^{1-x^2} dz dy dx = \iint (1-x^2) dy dx$$

Q.4

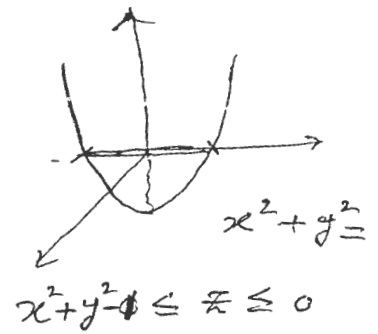
$$m = \iiint (1+x^2+y^2) dv$$

$$= \int_0^{2\pi} \int_0^1 \int_0^0 (1+r^2) r dz dr d\theta$$

$$= \int_0^{2\pi} \int_0^1 (1+r^2) r [-r^2+1] dr d\theta$$

$$= \int_0^{2\pi} \int_0^1 (r-r^5) dr d\theta = \int_0^{2\pi} \left[\frac{r^2}{2} - \frac{r^6}{6} \right]_0^1 d\theta = \int_0^{2\pi} \left[\frac{1}{2} - \frac{1}{6} \right] d\theta$$

$$= \frac{1}{3} (2\pi) = \frac{2\pi}{3} = 2.094$$



$$x^2+y^2 \leq r \leq 0$$

$$0 \leq r \leq 1$$

$$0 \leq \theta \leq 2\pi$$

③

$$\text{Q.5} \int_0^5 \int_0^{\sqrt{25-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{50-x^2-y^2}} dz dy dx$$

$$= \int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sqrt{50}} \rho \rho^2 \sin \varphi d\rho d\varphi d\theta$$

$$\begin{array}{l} 0 \leq \rho \leq \sqrt{50} \\ 0 \leq \varphi \leq \pi/4 \\ 0 \leq \theta \leq \pi/2 \end{array}$$

$$= \int_0^{\pi/2} \int_0^{\pi/4} \left[\frac{\rho^4}{4} \right]_0^{\sqrt{50}} \sin \varphi d\varphi d\theta$$

$$= \frac{2500}{4} \int_0^{\pi/2} \left[-\cos \varphi \right]_0^{\pi/4} d\theta = \frac{2500}{4} \left[-\frac{1}{\sqrt{2}} + 1 \right] \frac{\pi}{2}$$