

King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second Mid-Term Examination
(Summer Semester 1433/1434)

Max. Marks: 25

Time: 90 Minutes

Marking Scheme: Q.1:(5), Q.2:(5), Q.3:(5), Q.4:(5), Q.5:(5)
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Q. No: 1 Evaluate the integral

$$\int_0^1 \int_{y^{\frac{1}{2}}}^1 \sqrt{y} e^{x^2} dx dy.$$

Q. No: 2 Find the area of the surface $z = 1 - x - y$ lying above the circle $x^2 + y^2 = 2x$.

Q. No: 3 Find the mass of the cube with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ if its density at a point (x, y, z) is proportional to the square of its distance from the origin.

Q. No: 4 Evaluate the integral $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2 + y^2) dz dy dx$ by changing to cylindrical coordinates.

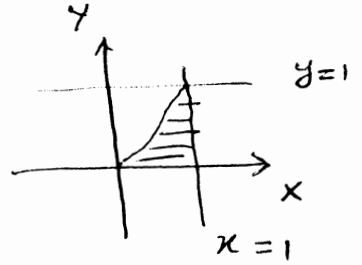
Q. No: 5 If spherical coordinates of a point are $(\rho, \varphi, \theta) = \left(2, \frac{\pi}{3}, \frac{\pi}{4}\right)$, find its

- (a) rectangular coordinates, and
- (b) cylindrical coordinates.

①

Q. No ① $\int_0^1 \int_{y^{3/2}}^1 \sqrt{y} e^{x^2} dx dy$ [Marks: 5]

$0 \leq y \leq 1$
 $y^{3/2} \leq x \leq 1$



$= \int_0^1 \int_{y^{3/2}}^1 \sqrt{y} e^{x^2} dy dx = \int_0^1 \left[\frac{y^{3/2}}{3/2} \right]_0^{x^{2/3}} e^{x^2} dx$

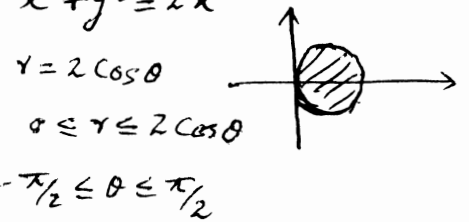
$= \frac{2}{3} \int_0^1 (x^{2/3})^{3/2} e^{x^2} dx = \frac{2}{3} \int_0^1 x e^{x^2} dx$

$= \frac{1}{3} \int_0^1 e^{x^2} (2x) dx = \frac{1}{3} [e^{x^2}]_0^1 = \frac{1}{3} (e-1)$ ② #

Q. ② S. A. = $\iint_{R_{xy}} \sqrt{1 + g_x^2 + g_y^2} dA$ ①
 [Marks: 5]

$z = 1 - x - y = g(x, y) \Rightarrow g_x = -1, g_y = -1$

Region $x^2 + y^2 = 2x$



$= \iint_{R_{xy}} \sqrt{3} dA$ ①

$= \sqrt{3} \int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r dr d\theta$ ②

$= \sqrt{3} \int_{-\pi/2}^{\pi/2} \left[\frac{r^2}{2} \right]_0^{2 \cos \theta} d\theta = \frac{\sqrt{3}}{2} \int_{-\pi/2}^{\pi/2} [2 \cos \theta]^2 d\theta = \frac{\sqrt{3}}{2} \int_{-\pi/2}^{\pi/2} 4 \cos^2 \theta d\theta$

$= 2\sqrt{3} \int_{-\pi/2}^{\pi/2} \frac{1 + \cos 2\theta}{2} d\theta = \sqrt{3} [\theta + \sin 2\theta]_{-\pi/2}^{\pi/2}$

$= \sqrt{3} [\pi/2 + \pi/2] = \sqrt{3} \pi$ ① #

(2)

Q: 3 mass = $\iiint_Q \delta \, dv$ [Marks: 5] $\delta \propto (x^2 + y^2 + z^2)$
 $\delta = k(x^2 + y^2 + z^2)$ ①

$$= k \int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) \, dz \, dx \, dy = k \int_0^1 \int_0^1 [x^2 z + y^2 z + \frac{z^3}{3}]_0^1 \, dx \, dy$$

③

$$= k \int_0^1 \int_0^1 (x^2 + y^2 + \frac{1}{3}) \, dx \, dy = k \int_0^1 [\frac{x^3}{3} + y^2 x + \frac{x}{3}]_0^1 \, dy$$

$$= k \int_0^1 [\frac{1}{3} + y^2 + \frac{1}{3}] \, dy = k [\frac{2y}{3} + \frac{y^3}{3}]_0^1 = k [\frac{2}{3} + \frac{1}{3}] = k$$

①
#

Q: 4 $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2 + y^2) \, dz \, dy \, dx$ [Marks: 5] $-2 \leq x \leq 2$
 $-\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}$
 $\sqrt{x^2+y^2} \leq z \leq 2$

$$= \int_0^{2\pi} \int_0^2 \int_0^2 r^2 r \, dz \, dr \, d\theta$$

③ $\Rightarrow \begin{cases} r \leq z \leq 2 \\ 0 \leq r \leq 2 \\ 0 \leq \theta \leq 2\pi \end{cases}$

$$= \int_0^{2\pi} \int_0^2 r^3 [z]_r^2 \, dr \, d\theta = \int_0^{2\pi} \int_0^2 (2r^3 - r^4) \, dr \, d\theta$$

$$= \int_0^{2\pi} [\frac{r^4}{2} - \frac{r^5}{5}]_0^2 \, d\theta = \int_0^{2\pi} [\frac{16}{2} - \frac{32}{5}] \, d\theta = (8 - \frac{32}{5}) 2\pi = (\frac{8}{5}) 2\pi = \frac{16\pi}{5}$$

②

Q: 5 $(\rho, \varphi, \theta) = (2, \pi/3, \pi/4) \Rightarrow \rho = 2, \varphi = \pi/3, \theta = \pi/4$
 [Marks: 5]

$$\begin{aligned} z &= \rho \cos \varphi = 2 \cos(\pi/3) = 2(\frac{1}{2}) = 1 \\ r &= \rho \sin \varphi = 2 \sin(\pi/3) = 2(\frac{\sqrt{3}}{2}) = \sqrt{3} \end{aligned} \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow (r, \theta, z) = (\sqrt{3}, \pi/4, 1)$$

②

$$\begin{aligned} x &= r \cos \theta = \sqrt{3} \cos(\pi/4) = \frac{\sqrt{3}}{\sqrt{2}} \\ y &= r \sin \theta = \sqrt{3} \sin(\pi/4) = \frac{\sqrt{3}}{\sqrt{2}} \end{aligned} \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow (x, y, z) = (\frac{\sqrt{3}}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}, 1)$$

③