King Saud University
Department of Mathematics
M-203
(Differential and Integral Calculus)
Second Mid-Term Examination
(II-Semester 1432/1433)

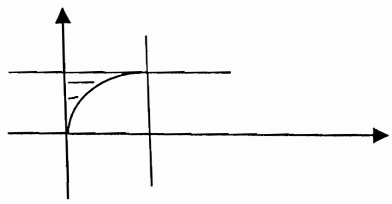
Max. Marks: 20 Time: 90 Minutes

Marking Scheme: **Q.1**(4), **Q.2**:(4), **Q.3**:(4), **Q.4**:(4), **Q.5**:(4)

Q. No: 1 Reverse the order of integration, and evaluate the resulting integral

$$\int_0^1\int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx.$$

Solution: Given $\sqrt{x} \le y \le 1, 0 \le x \le 1$



$$\text{We get } \int_0^1\int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx = \int_0^1\int_0^{y^2} \sqrt{y^3+1}\,dx\,dy = \int_0^1 \sqrt{y^3+1}\,[x]_0^{y^2}\,dy$$

2

$$= \int_0^1 \sqrt{y^3+1}\,(y^2)\,dy = \frac{1}{3}\left[\frac{(y^3+1)^{3/2}}{3/2}\right]_0^1 = \frac{2}{9}\left[(2)^{3/2}-1\right].$$

1 1

Q. No: 2 Use **polar coordinates** to evaluate the integral

$$\int_0^2\int_0^{\sqrt{2x-x^2}} 6y\,dy\,dx\ .$$

Solution: Here region is

$$0 \le y \le \sqrt{2x-x^2}, 0 \le x \le 2 \Rightarrow y^2 = 2x - x^2 \Rightarrow (x-1)^2 + y^2 = 1$$

Which is a circle whose center is (1,0). But $0 \le y \le \sqrt{2x-x^2} \Rightarrow$ its only upper half portion circle

In polar coordinates this region is $0 \le r \le 2\cos\theta, 0 \le \theta \le \frac{\pi}{2}$

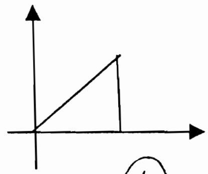
$$\int_0^2\int_0^{\sqrt{2x-x^2}} 6y\,dy\,dx = 6\int_0^{\pi/2}\int_0^{2\cos\theta} r\sin\theta\, r\,dr\,d\theta = 6\int_0^{\pi/2} \sin\theta\left[\frac{r^3}{3}\right]_0^{2\cos\theta}$$

2

$$=6\times\frac{8}{3}\int_0^{\pi/2}\cos^3\theta\sin\theta d\theta=-16\int_0^{\pi/2}\cos^3\theta(-\sin\theta)d\theta=-16\left[\frac{\cos^4\theta}{4}\right]_0^{\pi/2}=4.$$ 2

Q. No: 3 Find the surface area of the part of the surface $z=x^2+2y$ that lies above the triangular region R in xy-plane with vertices (0,0), (1,0), and (1,1).

Solution: Here region in xy-plane is



$$z=x^2+2y\ =g(x,y)\Rightarrow g_x=2x,g_y=2$$ 1

$$\textbf{Surface Area}=\iint_R\sqrt{1+(g_x)^2+(g_y)^2}dA=\int_0^1\int_0^x\sqrt{5+4x^2}dydx$$ 2

$$=\int_0^1 x\sqrt{5+4x^2}dx=\frac{1}{8}\int_0^1\left(5+4x^2\right)^{1/2}(8x)dx=\frac{1}{8}\left[\frac{\left(5+4x^2\right)^{3/2}}{3/2}\right]_0^1$$

$$=\frac{1}{12}\left[(9)^{3/2}-(5)^{3/2}\right].$$ 1

Q. No: 4 Use a triple integral to find the volume of the solid bounded by the graphs of $z=4-y^2$, $x+y=2$, $x=0$, and $z=0$.

Solution: 3

$$V=\iiint_Q dV=\int_{-2}^{2}\int_0^{4-y^2}\int_0^{2-y}dxdzdy=\int_{-2}^{2}\int_0^{4-y^2}(2-y)dzdy$$

$$=\int_{-2}^{2}(4-y^2)(2-y)dy=\int_{-2}^{2}(8-4y-2y^2+y^3)dy$$

$$=$$

$$=\left[8y-2y^2-\frac{2}{3}y^3+\frac{y^4}{4}\right]_{-2}^{2}=\left(16-8-\frac{16}{3}+4\right)-\left(-16-8+\frac{16}{3}+4\right)$$

$$=\frac{64}{3}.$$ 1

Q. No: 5 Use **cylindrical coordinates** to find the mass of the solid bounded by the paraboloids $z=2-x^2-y^2$, $z=x^2+y^2$ and having the area mass density $\delta=\left(x^2+y^2\right)^{3/2}$.

Solution: Using Cylindrical coordinates. Here region is

$$r^2 \le z \le 2 - r^2 , \quad 0 \le r \le 1, \quad 0 \le \theta \le 2\pi$$

$$\text{Mass} = m = \iiint_Q \delta(x,y,z)dV = \iiint_Q \left(x^2 + y^2\right)^{3/2} dV$$

$$3 \quad = \int_0^{2\pi} \int_0^1 \int_{r^2}^{2-r^2} (r^2)^{3/2} r dz dr d\theta = \int_0^{2\pi} \int_0^2 r^4 \left[(2 - r^2) - r^2\right] dr d\theta$$

$$= \int_0^{2\pi} \int_0^1 \left[2r^4 - 2r^6\right] dr d\theta = \int_0^{2\pi} \left[2\frac{r^5}{5} - 2\frac{r^7}{7}\right]_0^1 d\theta$$

$$= \frac{2}{35}(2\pi) = \frac{4\pi}{35}. \quad 1$$

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