P242 - 243 الاختبار النزلي (٣) (١)

السؤال

for the following mass function (2)

$$f_x(x) = \frac{x}{6} \qquad x = 1, 2, 3$$

find

(1) $F(x) = ??$

x	1	2	3
$f(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$
$F(x)$	$1/6$	$3/6$	1

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{6} & 1 \le x < 2 \\ \frac{3}{6} & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

2) Moment Generating function of x $M_x(t)$

$$M_x(t) = E(e^{tx}) = \frac{1}{6}e^{1t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$$

$$M_x(t) = \frac{1}{6}e^{t} + \frac{1}{3}e^{2t} + \frac{1}{2}e^{3t}$$

3) Find Moment generating function of Y $M_y(t)$

where $y = 3x + 1$

$$M_y(t) = M_{3x+1}(t) = e^{t} M_x(3t)$$

$$= e^{t}\left[\frac{1}{6}e^{3t} + \frac{1}{3}e^{6t} + \frac{1}{2}e^{9t}\right]$$

$$= \frac{1}{6}e^{4t} + \frac{1}{3}e^{7t} + \frac{1}{2}e^{10t}$$

P242 - 243 الاختبار الأول ٣ (2)

4) The function of Y $f_Y(y) = ?$ تكملة سؤال (2)

(1) $Y = 3X + 1$

$X = \frac{Y-1}{3}$

2) $X = 1, 2, 3$

$y = 4, 7, 10$

3) $f_y(y) = f_x(g^{-1}(y)) = f_x\left(\frac{y-1}{3}\right)$

$$= \frac{\frac{y-1}{3}}{6} = \frac{y-1}{18}$$

$\therefore f_y(y) = \frac{y-1}{18}$ $y = 4, 7, 10$

For the following moments find the name (3) of the distribution and Expected value for each

(i) $M_x(t) = e^{-2(1-e^t)} = e^{2(e^t - 1)}$

Poisson dist with $\lambda = 2 \rightarrow E(x) = \lambda = 2$

iii) $M_X(t) = \left(\frac{4}{4-t}\right)^2 = \left(\frac{\beta}{\beta-t}\right)^{\alpha}$

Beta dist $\alpha = 2,\ \beta = 4$

$E(X) = \frac{\alpha}{\beta} = \frac{2}{4} = \frac{1}{2}$

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iv) $M_X(t) = (1-2t)^{-6} = (1-2t)^{-v/2}$

chi_ dist^n with $\frac{v}{2} = 6 \Rightarrow v = 12$

$E(X) = v = 12$

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v) $M_X(t) = e^{3t+2t^2} = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Normal dist^n with $\mu = 3,\ \frac{\sigma^2}{2} = 2 \Rightarrow \sigma^2 = 4$

$E(X) = \mu = 3$

(ب) If M.g.f of random variable x, y is اثبت ان X ④

$M_{x,y}(t_1, t_2) = e^{t_1^2 + t_2^2} \quad -\infty < t_1, t_2 < \infty$

find $M_X(t_1) = M_{X,Y}(t_1, 0) = e^{t_1^2}$

$M_Y(t_2) = M_{X,Y}(0, t_2) = e^{t_2^2}$

Prove x, y indep if satisfies that

$M_{X,Y}(t_1, t_2) \stackrel{?}{=} M_X(t_1)\, M_Y(t_2)$

$\Downarrow$ كما بالسؤال $e^{t_1^2 + t_2^2}$

$e^{t_1^2} \times e^{t_2^2} = e^{t_1^2 + t_2^2}$

الطرفين متساويين

بما ان الطرفين متساويين x, y indep.

الاختبار النهائي (٣)

5

x	0	2	5
$f(x)$	0.4	0.4	0.2

y	1	2	3	4
$f(y)$	0.2	0.3	0.3	0.2

1) $E(XY) = \cancel{1\times 0\times 0.1} + \cancel{2\times 0\times 0} + \cancel{3\times 0\times 0.2} + \cancel{4\times 0\times 0.1}$
$+ \cancel{2\times 1\times 0} + 2\times 2\times 0.2 + 3\times 2\times 0.1 + 4\times 2\times 0.1$
$+ 5\times 1\times 0.1 + 5\times 2\times 0.1 + \cancel{5\times 3\times 0} + \cancel{5\times 4\times 0}$

$= 0.8 + 0.6 + 0.8 + 0.5 + 1 = 3.7$

2) $E(X) = 0\times 0.4 + 2\times 0.4 + 5\times 0.2 = 1.8$
$E(Y) = 1\times 0.2 + 2\times 0.3 + 3\times 0.3 + 4\times 0.2 = 2.5$

$Cov(X,Y) = E(XY) - E(X)E(Y)$
$= 3.7 - 1.8\times 2.5 = -0.8$

4) $\rho_{X,Y} = \dfrac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \dfrac{-0.8}{\sqrt{3.36\times 1.05}}$
$= -0.4259$

$V(X) = EX^2 - (EX)^2$
$6.6 - 1.8^2$
$= 3.36$
$V(Y) = EY^2 - (EY)^2$
$= 7.3 - 2.5^2$
$= 1.05$

3) $Cov(2X, 3Y) = 2\times 3\ Cov(X,Y)$
$= 6\times -0.8 = -4.8$

5) $M_{X,Y}(t_1,t_2) = E(e^{t_1+t_2}) = 0.1\times e^{0t_1+1t_2} + 0\times e^{0t_1+2t_2}$
$+ 0.2\, e^{0t_1+3t_2} + \cdots\cdots + 0\times e^{5t_1+4t_2}$

اذا كانت القيمة الاحتمال داخل الجدول = صفر لا نحسبها

يجب كتابة $M_{X,Y}$ للنهاية بالامتحان

الاحصاء النظري ٣

6) $f_{X/Y}(0, 3) = \dfrac{f(x, y)}{f_Y(y)} = \dfrac{f(0, 3)}{f_Y(3)}$

$x = 0 \quad y = 3$

$= \dfrac{0.2}{0.3} = 2/3$

7) $F_{X/Y}(2/4) = P(X \leq 2 / Y = 4) =$

$= \dfrac{f(0,4) + f(2,4)}{f_Y(4)} = \dfrac{0.2}{0.2} = 1$

8) Are X, Y indep?

يجب ان تتحقق لكل

$f(x,y) = f(x) \times f(y) \quad \forall x \;\; \forall y$

$x=0$, $y=1$. نبدأ $f(0,1) \overset{??}{=} f_x(0) \times f_y(1)$

$0.1 \;|\; 0.4 \times 0.2$

$0.1 \neq 0.8$

$\therefore$ not indep

9) $f_Z(z) = ?$ $\quad Z = X + Y$

Z	1	2	3	4	5	6	$\cancel{8}$	8
f(z	0.1	0	0.2	0.3	0.1	0.2	0.1	0.1

$x = 0, 2, 5$
$y = 1, 2, 3, 4$

$Z = x + y = 1, 2, 3, 4,$
$3, 4, 5, 6, 6, \cancel{8},$
$8, 9$

$\therefore Z = 1, 2, 3, 4, 5, 6, \cancel{8}$
$8, 9$

الاختبار النهائي ٣

(6) $f(x,y) = \frac{8-x-y}{32}$ $\quad 0<x<4$, $\quad 1<y<3$

find

1) $f_x(x) = \int_1^3 f(x,y)\,dy = \int_1^3 \frac{8-x-y}{32}\,dy =$

$= \frac{1}{32}\left[8y - xy - \frac{y^2}{2}\right]_1^3 = \frac{1}{32}\left[8(3-1) - x(3-1) - \frac{1}{2}(3^2-1^2)\right]$

$= \frac{1}{32}[16-2x-4] = \frac{12-2x}{32} = \frac{6-x}{16} \quad 0<x<4$

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2) $f_y(y) = \int_0^4 f(x,y)\,dx = \int_0^4 \frac{8-x-y}{32}\,dx = \frac{1}{32}\int_0^4 (8-y)\,dx - \int_0^4 x\,dx$

$= \frac{1}{32}\left[(8-y)x\right]_0^4 - \left.\frac{x^2}{2}\right]_0^4 = \frac{1}{32}\left(4(8-y) - 8\right) = \frac{24-4y}{32}$

$f(y) = \frac{6-y}{8} \quad 1<y<3$

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3) $f(x/y) = \frac{f_{x,y}(x,y)}{f_y(y)} = \frac{(8-x-y)/32}{(6-y)/8} = \frac{1}{4}\,\frac{8-x-y}{6-y}$

$= \frac{8-x-y}{4(6-y)} \quad 0<x<4, \quad 1<y<3$

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4) $E(y) = \int_1^3 y f(y)\,dy = \int_1^3 y\,\frac{6-y}{8}\,dy = \frac{1}{8}\int_1^3 6y - y^2\,dy$

$= \frac{1}{8}\left[\frac{6y^2}{2} - \frac{y^3}{3}\right]_1^3 = \frac{1}{8}\left[18 - \frac{8}{3}\right] = \frac{23}{12}$

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5) $E(E(Y/X)) = E(Y) = \frac{23}{12}$

الاختبار النهائي 3

For the following density function

$$f_x(x) = e^{-x} \qquad x \geq 0$$

Find density function of Y $\qquad Y = \sqrt[3]{X}$

① $Y = \sqrt[3]{X} \longrightarrow X = g^{-1}(y) = y^3$ $\qquad X \geq 0$, $Y \geq 0$

2) $x \geq 0 \rightarrow y \geq 0$

3) $f_Y(y) = f_x(g^{-1}(y)) \cdot \left| \frac{d g^{-1}(y)}{dy} \right|$

$$= f_x(y^3) \left| 3y^2 \right|$$

$$= e^{-y^3} \cdot 3y^2$$

$$= 3y^2 e^{-y^3}$$

انتهت الاسئلة