**King Saud University**

**College of Science**

**Department of Statistics and Operations Research**

**STAT 223**

**Theory of Statistics 1**

**Exercises**

**1438/1439**

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**Introduction**

1. Let A, B, and C be three events such that: , and . Then,
2. Consider the experiment of flipping a balanced coin three times independently. Find,
3. The number of points in the sample space.
4. The probability of getting exactly two heads.
5. The events exactly two heads and exactly three heads.
6. The events the first coin is head and the second and the third coins are tails.
7. 1000 individuals are classified below by sex and smoking habit.

|  |  |  |  |
| --- | --- | --- | --- |
|  | | Sex | |
| Male (M) | Female (F) |
| SMOKING  HABIT | Daily (D) | 300 | 50 |
| Occasionally (O | 200 | 50 |
| Not at all (N) | 100 | 300 |

A person is selected randomly from this group.

1. Find the probability that the person is female.
2. Find the probability that the person is female and smokes daily.
3. Find the probability that the person is female, given that the person smokes daily.
4. Are the events F and D independent? Why?
5. Let *X* be a discrete random variable with the probability distribution function:

.

1. Find the value of *k*.
2. Find the cumulative distribution function.
3. Using the cdf, find P (0.5 < X < 2.5).
4. Consider the probability density function:

Find:

1. The value of *k*.
2. The probability *P*(0.3 < *X* < 0.6).
3. The expected value of *X.*
4. Suppose that 4 out of 12 buildings in a certain city violate the building code. A building engineer randomly inspects a sample of 3 new buildings in the city.
5. Find the probability distribution function of the random variable *X* representing the number of buildings that violate the building code in the sample.
6. Find the probability that
7. none of the buildings in the sample violating the building code.
8. one building in the sample violating the building code.
9. at lease one building in the sample violating the building code.
10. Find the expected number of buildings in the sample that violate the building code.
11. Find Var(X).
12. On average, a certain intersection results in 3 traffic accidents per day. Assuming Poisson distribution,
13. what is the probability that at this intersection:
14. no accidents will occur in a given day?
15. More than 3 accidents will occur in a given day?
16. Exactly 5 accidents will occur in a period of two days?
17. what is the average number of traffic accidents in a period of 4 days?
18. If the random variable *X* has a uniform distribution on the interval (0,10), then
19. equals to
20. The mean of *X* is
21. The variance *X* is
22. Suppose that *Z* is distributed according to the standard normal distribution. Then,
23. the area under the curve to the left of 1.43 is:
24. the area under the curve to the right of 0.89 is:
25. the area under the curve between 2.16 and 0.65 is:
26. the value of *k* such that *P*(0.93< *Z* < *k*) = 0.0427 is:
27. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Find,
28. the proportion of rings that will have inside diameter less than 12.05 centimeters.
29. the proportion of rings that will have inside diameter exceeding 11.97 centimeters.
30. the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters.
31. Assume the length (in minutes) of a particular type of a telephone conversation is a random variable with a probability density function of the form:

.

Calculate:

1. *P*(3 < *x* <10).
2. The cdf of *X*.
3. The mean and the variance of *X*.
4. Find the moment-generating function of *X,* if you know that
5. For a chi-squared distribution, find
6. when .
7. when .
8. when .
9. Find:
10. when .
11. when .
12. when .
13. when .
14. when .
15. For an *F*-distribution, find:
16. with and .
17. with and .
18. with and .
19. If , find the distribution of .
20. If . Find the pdf of .
21. If 𝑋~𝑈𝑛𝑖𝑓𝑜𝑟𝑚(0,1), find the pdf of . Name the distribution and its parameter values.
22. Suppose independent random variables *X* and *Y* are such that . If , what is the distribution of *Y*.
23. Let and are two random variables have the joint probability distribution as follows:

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
| 1 | 0 |  | |
| 0.58 | 0.05 | 0 |  |
| 0.07 | 0.3 | 1 |

(a) Prove that and are dependent random variables.

(b) Find the pmf of the random variable

1. If 𝑋~𝑈𝑛𝑖𝑓𝑜𝑟𝑚(0,1) independent of 𝑌~𝐸𝑥𝑝𝑜𝑛𝑒𝑛𝑡𝑖𝑎𝑙(1), find the distribution of .
2. If and are independent random variables. Find the distribution of .